CHAPTER 4

MATHEMATICAL MODELLING BY USING EXTENDED RELIABILITY GROWTH MODEL FOR SECRETION OF GnRH IN BEEF COWS.

4.1. Introduction

Despite the fact that the use of reliability engineering tools is well established, their application to the medical industry is new. Most research in this area merely suggests how to assess or improve the reliability of devices in their design or manufacturing stages. In view to this point, the best maintenance strategies for medical equipment in their operating context have not been considered. The research reported in this thesis aims to address this gap and proposes various methods to improve the current maintenance strategies in the healthcare industry.

Nowadays the environment of compressed schedules and limited testing, every opportunity to identify and correct reliability deficiencies in a new design is a prime objective. A metric for tracking system reliability before development testing based on preemptive corrective actions for potential problem modes is discussed. The tradition reliability growth models provide assessments when the failure modes corrected are surface during the testing. In the test-fix-test strategy problem modes are found during testing and corrective actions for these problems are incorporated during the test. For the test-find-test strategy problem modes are found during testing but all corrective actions for these problems are delayed and incorporated after the completion of the test.

The focus of this paper is a combination of these two approaches, referred to as test-fix-find-test. This is the practical situation where some corrective actions are incorporated during the test and some corrective actions are delayed until the end of the test. Reliability growth assessments for the test-fix-find-test strategy are not provided with the widely used tradition models in international and U. S. ANSI standards. This paper presents an extended reliability growth model that provides assessments for the test-fix-find-test strategy and also allows for preemptive corrective actions.

The Extended Model preserves the properties of the traditional models and reduces to these models and strategies as special cases. The model also provides extensive metrics useful for managing the reliability program. The preemptive corrective actions strategy, the Extended Model, and the maturity metrics are all currently being used in industry and government. As a practical managing methodology, the model parameters and management metrics are simple and straightforward to calculate and interpret. These applications will be illustrated in this paper **[25, 29, 111].**

4.2. Parameter interpretation

λ	Scale parameter for Crow (AMSAA) model
β	Shape parameter for Crow (AMSAA) model
α	Growth rate for Crow (AMSAA) model
t	Test time
T	Total test time
MTBF	Mean time between failures
$r(t)$	Crow (AMSAA) model failure intensity
X_i	The i^{th} successive failure time
N	Total number of failures
λ_A	Type A modes failure intensity
λ_B	Type B modes failure intensity
λ_P	Projected failure intensity

M_P Projected MTBF

$\hat{\lambda}$ Scale projected model

$\hat{\beta}$ Shape projected model

4.3 A Mathematical model for Reliability Analysis of Test- Fix- Test

The Duane postulate, for reliability growth during test-fix- test development testing states that the instantaneous system MTBF at cumulative test time t is $M(t) = \left[\lambda\beta t^{\beta-1}\right]^{-1}$ where 0 < λ and 0<β are parameters.

Crow modeled the Duane postulate stochastically as a non-homogeneous Poisson process (NHPP) with intensity

$$r(t) = \lambda\beta t^{\beta-1} \quad 4.1$$

thus allowing for statistical procedures based on this process for reliability growth analyses. This model is applicable to test-fix-test data not test-fix-find-test. Estimation procedures, confidence intervals, etc.

The parameter λ is referred to as the scale parameter and β is the shape parameter. For β = 1, there is no reliability growth **[25, 32,111].** For β < 1, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. For β > 1, there is negative reliability growth.

Under the Crow (AMSAA) basic model the achieved or demonstrated failure intensity at time T, the end of the test, is given by r (T). Denote the achieved failure intensity by

$$\lambda_{CA} = r(T) \qquad 4.2$$

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0<X_1 < X_2 < \ldots X_N < T$ denote the N successive failure times on a cumulative time scale. Assume that the Crow (AMSAA) NHPP assumption applies to this set of data. Under the Crow (AMSAA) basic model the maximum likelihood estimates (MLEs) for λ and β (numerator of MLE for β adjusted from N to N-1 to obtain unbiased estimate) are

$$\hat{\lambda} = \frac{N}{T^{\beta}} \qquad 4.3$$

$$\hat{\beta} = \frac{N-1}{\sum_{i=1}^{n} \frac{T}{x_i}} \qquad 4.4$$

From the figure 4.1 curve-I $\hat{\lambda}$ = 0.101 and $\hat{\beta}$ = 1.713

$$\lambda_{CA} = \hat{\lambda}\hat{\beta}T^{\beta-1} = 1.062 \qquad 4.5$$

$$\hat{M}_{CA} = \frac{1}{\lambda_{CA}} = 0.942 \qquad 4.6$$

An effectiveness factor (EF) d_j is the fractional decrease in λ_j after a corrective action has been made for the j-th Type B mode. The failure rate for the i^{th} Type B failure mode after a corrective action is $(1-d_j)\,\lambda_j$. In practice, for application of the projection model, the EFs are assigned based on mathematical assessments, test results, etc. Studies indicate that an average EF d of about 3.4 is typical for a reliability growth program. Individual EFs may vary. The assigned EFs for distinct Type B modes are given in the figure 4.1 curve-III **[25, 27].**

For test-find-test the system failure intensity is constant, say, λ_S, during the testing and then jumps to a lower value due to the incorporation of corrective actions. The intensity at the end of the test T, before delayed corrective actions are introduced into the system, is the achieved intensity. The reciprocal of the intensity is the achieved MTBF M_S.

Estimation of the achieved failure intensity $\hat{\lambda}_s$ by

$$\hat{\lambda}_s = \hat{\lambda}_A + \hat{\lambda}_B \qquad 4.7$$

$$\hat{\lambda}_A = \frac{N_A}{T} \qquad 4.8$$

$$\hat{\lambda}_B = \frac{N_B}{T} \qquad 4.9$$

Based on the data in the figure 4.1 curve-II,

$$\hat{\lambda}_s = 0.105, \hat{\lambda}_A = 0.025, \hat{\lambda}_B = 0.08 \qquad 4.10$$

The estimated achieved MTBF M_S at time T = 30 before the jump is 1.021.

M_S = 1.021. Estimate the jump next.

The estimated projected failure intensity is

$$\hat{\lambda}_P = \hat{\lambda}_A + \sum_{j=1}^{M}\left(1-d_j\right)\frac{N_j}{T} + \bar{d}\hat{h}(T) \qquad 4.11$$

Where $\bar{d} = \frac{\sum_{j=1}^{M} d_j}{M}$ is the average EF, and

$$\hat{h}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \qquad 4.12$$

The projection model $\hat{\lambda}$ and $\hat{\beta}$ for equation (4.7) use only the M first occurrence failure times of the seen and unique Type B failure modes. These first occurrences are given in the figure 4.1 curve-III. Applying equation (4.3) to the first occurrence data in the figure 4.1 curve-III, results in **[111]**

$$\hat{\lambda} = 0.1820, \quad \hat{\beta} = .7472 \qquad 4.13$$

Based on the data in the figure 4.1 curve II & III, having M = 10,

$$\bar{d} = 0.72, \ \hat{h}(30) = 0.0299 \quad 4.14$$

Also

$$\bar{d}\hat{h}(T) = 0.0215, \ \hat{\lambda}_A = 0.025 \quad 4.15$$

$$\sum_{j=1}^{M}\left(1-d_j\right)\frac{N_j}{T} = 0.0196 \quad 4.16$$

The projection model estimates that the MTBF jumps to 1.021 hours from 0.40 hours due to the 4 distinct corrective actions.

4.4 Application

In addition to its well established role of causing luteolysis, prostaglandin $F_{2\alpha}$ ($PGF_{2\alpha}$) has been show to enhance fertility in cattle . In a study involving 5000 dariy cows, [115] reported a higher calving rate (69% vs. 60%) in $PGF_{2\alpha}$ treated cows than in untreated cows. $PGF_{2\alpha}$ may exert a fertility effect by stimulating a release of luteinizing hormone upon demise of the corpus luteum and reduced and reduced progesterone concentrations reported that equine LH and follicle stimulating hormone (FSH) concentrations were elevated in pituitary and jugular venous blood after the administration of the synthetic $PGF_{2\alpha}$ luprostiol. The objective to determine if $PGF_{2\alpha}$: (1) would cause a release of PH in the absence of progesterone in anestrous cows; (2) would affect the gonadotropin releasing hormone (GnRH)-induced LH release) would affect the GnRH - induced ovulation response of previously anestrous cows.

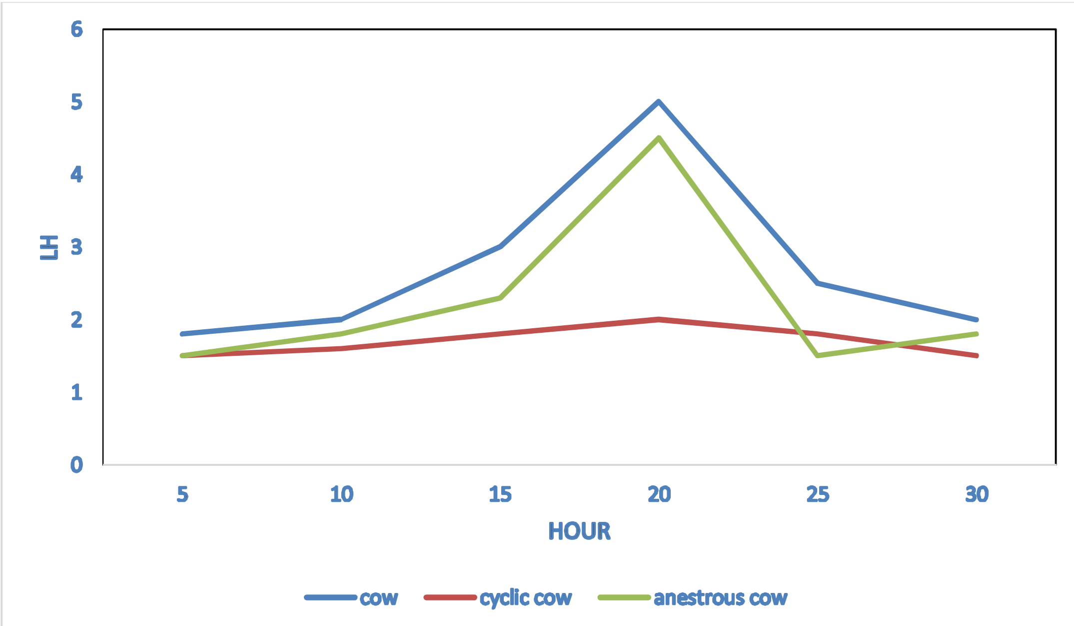

Figure 4.4.1

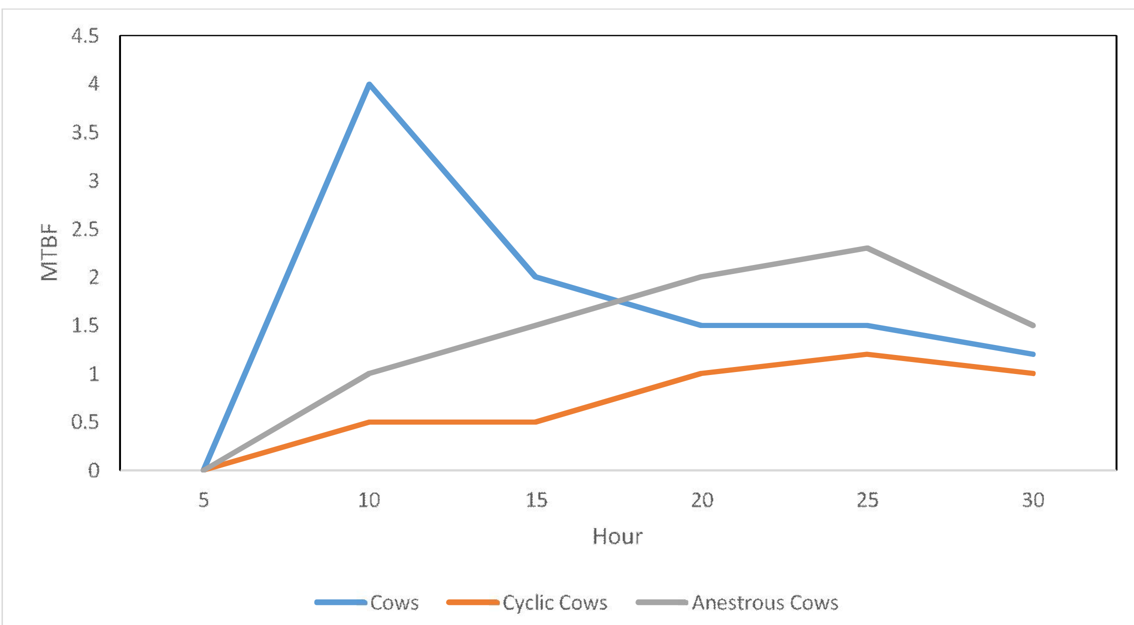

Figure 4.4.2 (Extensive model projected MTBF)

GnRH, another compound that exerts a fertility effect, has been demonstrated to hasten the first postpartum ovulation in beef cows. Although it has been shown that GnRH induced a high proportion of anestrous cows to ovulate (approximately 65%) the majority (approximately 85%) had subnormal luteal phases. Subnormal luteal phases have also been identified in humans, other primates and sheep. Suggested caused of subnormal luteal phases in cattle include a premature release of $PGF_{2\alpha}$. Another cause that has been identifies in humans and sheep is a preovulatory LH surge of less duration than a spontaneous LH surge; **[62].** The objective of experiment was to determine if the profile of the preovulatory LH surge was associated with the occurrence of subnormal luteal phases in cattle **[32,115]**

Previous research has demonstrated that LH concentrations increase after $PGF_{2\alpha}$ treatment due to withdrawal of progesterone during luterolysis. Data herein demonstrate that LH concentrations increase after $PGF_{2\alpha}$ treatment in anestrous cows in the absence of progesterone. Furthermore, LH release from the anterior pituitary in response to GnRH treatment was greater after $PGF_{2\alpha}$ priming and the ovulation response was also higher ($p<0.05$) in cows administered $PGF_{2\alpha}$ 30 h before GnRH treatment may be an explanation for the observed fertility effect of $PGF_{2\alpha}$ treatment.

Others have also observed fertility effects due to $PGF_{2\alpha}$ treatment **[32,115]** reported a 7% calving rate in cows previously with inactive ovaries administered a prostagandin analogue. **[115]** reported an ovulation response of 89% in previously anestrous cows after the second of two injections of $PGF_{2\alpha}$ administered 11 days apart.

Although no control cows were included in these two studies, in another study only 33% of the untreated cows, as compared to 71% of the $PGF_{2\alpha}$ (Lutalyse) treated cows (P=0.08), exhibited estrus within 7 days of a single injection of $PGF_{2\alpha}$ (or buffer) with similar (P>0.25) pregnancy rates (67%) from AI at that estrus (estrus within 7 days of treatment; unpublished data).Collectively, these data suggest that $PGF_{2\alpha}$ hastened fertile estrus in some previously anestrous cows**[32].**

4.5 Conclusion

Categorized data were analyzed by reliability growth models, LH concentrations of the $PGF_{2\alpha}$ treatment were analyzed by test- fix-best method with treatment ($PGF_{2\alpha}$) and time as a main effect. GnRH induced in LH concentrations 10hr after $PGF_{2\alpha}$ treatment were analyzed my effectiveness factor with treatment ($PGF_{2\alpha}$) and time as a main effect. Area under the response curve (determined mathematically) and GnRH induced peak amplitude were analyzed by reliability growth model.

Using Mathematical Modelling by Extended Reliability Growth, the LH concentration remained relatively constant and similar for anestrous and cyclic cows. LH concentrations increased after $PGF_{2\alpha}$ in both cyclic and anestrous cows but the increase was greater in anestrous cows where the predictability is easier considering the sample mathematically than the process of practical implementation which is a tedious process.