Chapter-1

Introduction

In this chapter the work done so far in the field of Fuzzy group, Fuzzy subgroup is presented as review of literature. In addition, some basic definitions related to Bipolar Intuitionistic M-Fuzzy group and anti M– Fuzzy group, Scope of the study and organization of the thesis are also presented.

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1.1 Fuzzy Group and M-Fuzzy group.

Among the various paradigm changes in science and mathematics, one such change concerns the concept of uncertainty. Identification of this important role of uncertainty by some researchers began the stage of transition from the traditional view of the modern view of uncertainty and such transition is characterised by the emergence of several new theories of uncertainty from possibility theory. An important point in the evolution of the modern concept of uncertainty was the publication of a seminal paper by Lotfi A. Zadeh. He has introduced a theory whose objects fuzzy sets are sets with boundaries that are not precise. The concept is being used and is also found to be more appropriate in solving problems of all disciplines.

The concept of fuzzy sets was initiated by L. A. Zadeh [36] in 1965 to represent and manipulate data represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision to many problems.

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterised by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

More often the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. However such
objects have an ambiguous status with respect to the class. The same kind of ambiguity arises in the case a number such as in the relation to the “class” of all real numbers which are much greater than 1. Yet, the fact remains that such imprecisely defined “classes” play an important role in human thinking, particularly in the domains of pattern recognition, communication of information and abstraction.

Then the theory of fuzzy sets has found wide ranging applications in such diverse fields as automata, control theory, decision theory, solve many problems in applied mathematics, information sciences, graph theory, medical sciences, etc..

In 1971 Rosenfeld [31] introduced the notion of a fuzzy group and showed that many concepts of group theory can be extended in the elementary manner to develop the theory of fuzzy groups. In particular, he characterised all the fuzzy subgroups of cyclic groups of prime order. Many authors investigated the theory of fuzzy algebra. In the definition of fuzzy subgroups he was assumed that the subset of group G are a fuzzy group.

In 1981, P. S. Das [9] introduced a similar characterisation of all fuzzy subgroup of finite cyclic groups. In this characterisation, he introduced the notion of level subgroups of a fuzzy subgroup which is based on the notion of a level subset introduced earlier by L. A. Zadeh. These level subgroups of a fuzzy subgroups form a chain. A fuzzy
subgroups whose chain of Level subgroups is maximal in the special cases when the group is direct product of cyclic groups of prime orders and abelian. However, the two fuzzy subgroups with an identical family of level subgroups are equal. Identical family of level subgroups are equal if and only if their image sets are equal.

The author **W.R. Zhang [37, 38]** commenced the concepts of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In fuzzy the membership degree of belonginess of elements to a fuzzy sets. The membership degree “1” indicates that an element completely belongs to its corresponding fuzzy set, and membership degree “0” indicates that an element does not belong to fuzzy set. The membership degree “0” means that the elements are irrelevant to the corresponding property. The membership degree (0,1] indicates that elements some what satisfy the property and the membership degree [-1,0) indicates that elements some what satisfy the implicit counter property.

**K. T. Atanassov [1 - 6]** introduced “necessity” and “possibility” operators, level operators, interior operators and special operators, operation on operators of intuitionistic fuzzy sets.

**Nandakumar. S et al [26 -29]** developed the necessity, possibility operators, level operators, interior operators and special operators on fuzzy primary and semi primary ideals of a ring.
Muthuraman, M. S et al [19, 20] introduced the bipolar M- Fuzzy group, a group with operators is an algebraic system. The author investigated the properties of a bipolar M- Fuzzy group.

Now we extended to the bipolar intuitionistic M- Fuzzy group and anti M- Fuzzy group and all the above concepts we apply and create a new algebraic structure of bipolar intuitionistic M- Fuzzy group and anti M- Fuzzy group with their operators are established by the researcher.

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1.2 Review of Literature

The concepts of fuzzy sets was initiated by L. Z. Zadeh [36] in 1965 to represent and manipulate data and represent uncertainty and vagueness and provided formalized tools for dealing with the imprecision to many problems. A fuzzy set is a class of objects with a continuum of grades of membership (characteristic) function which assigns to each object a grade of membership ranging between “zero” and “one”. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets and various properties of these notions in the context of fuzzy sets are established.

In 1971, Rosenfeld. A [31] gave the idea of fuzzy subgroup, he investigated fuzzy subgroupoid and fuzzy ideal. The author introduced some important results of homomorphic image and preimage of fuzzy group.

P. Sivaramakrishna Das [9] in 1981 introduced new idea of fuzzy groups and level subgroups, the level set \( A \) is a subset of \( S \) in the ordinary sense and the level set was initiated by L. A. Zadeh. Any subgroup of a group \( G \) can be realized as a level subgroup of some fuzzy subgroup of \( G \). If \( \tilde{A} \) be the collection of all fuzzy subgroups \( G \) and \( \tilde{B} \) be the collection of all level subgroups of members of \( \tilde{A} \) then there is a 1-1 correspondence between the subgroup of \( G \) and the equivalence classes of level subgroups.

In 1986 Atanassov. K. T [1-6] introduced intuitionistic fuzzy sets, various operators were defined on the intuitionistic fuzzy sets which are analogous to the model logic operators. The author investigated operations and relations over intuitionistic fuzzy sets relations and operations over fuzzy sets. He introduced necessity and possibility operators, level operator, interior operators and other type of operators $P_{α,β}$ and $Q_{α,β}$ and they were defined in 1983. The degree of membership of the elements of a given universe to its subset can be changed directly by these operators. This stimulated the author continue his research on the intuitionistic fuzzy sets

Mustafa akgul [18] in 1988, extended some theorems of P. S. Das. If G is commutative group, then every fuzzy subgroup of G is a fuzzy normal subgroup, and if G is also finite then it is a fuzzy level normal subgroup.


In 1991 Mohamed assad [16] investigated properties of groups and fuzzy sub groups.
**Rajeshkumar. A** [30] in 1993, provides a description of the fuzzy subgroup, besides a lattice structure is also given the set of all fuzzy subgroups of a group. Any constant fuzzy subset of a finite abelian group $G$ can be written as a product of a finite number of fuzzy sylow subgroups of the sylow subgroups of $G$, and he was investigated fuzzy ideals of a field are characterised and deals with fuzzy prime ideals, fuzzy maximal ideals and fuzzy semi prime ideals. A fuzzy prime ideals is always fuzzy irreducible in the reverse, it is fuzzy ideals which is both fuzzy semi prime and fuzzy irreducible is fuzzy prime. A fuzzy prime ideals new formulations enable us to characterize each of these fuzzy ideals in terms of their level ideals without imposing any condition on their image sets.

In 1994 **W. R. Zhang** [38] investigated bipolar fuzzy sets and relations a computational frame work for cognitive modelling and multiple decision analysis.

**Atanassov. K. T** in 1994, [1-6] introduced new operations defined over intuitionistic fuzzy sets operators over interval valued intuitionistic fuzzy sets and relation between the operators in intuitionistic fuzzy logics. In 1997, he introduced an extension of the universal operator over intuitionistic fuzzy sets and some of its properties.

**Chakrabartby. K** and **Biswa R. Nanda** [8] in 1997, developed the concept of fuzzy group a note on union and intersection of intuitionistic fuzzy sets.


In 2004 Xuehai Yuan and E. S. Lee [35] investigated fuzzy group based on binary operation. A New kind of definition of fuzzy binary operation in a different sense and a new kind of fuzzy group was introduced. Demirci’s smooth group may have an infinite number of identity elements and an element of smooth group may have an infinite number of inverses. In our definition, a fuzzy group has only one identity element and an element of fuzzy group has only one inverse element.


on bipolar anti Q-fuzzy groups and bipolar Q-fuzzy ideals under (t, s) norms. In 2012 Muthuraj .R and Sridharan. M [21] introduced bipolar anti fuzzy HX group and its lower level sub groups.


In 2012, Subramanian. S, Nagarajan. R and Chellappa. B [33] e analysed structure properties of M- fuzzy groups. If we take any two M- fuzzy groups then union of two M- fuzzy groups is also a M- fuzzy group for all natural numbers and the intersection of any two M- fuzzy groups is also a M- fuzzy group.

Palanivelrajan. M, Gunasekaran. K and Nandakumar. S in 2013 [28, 29] introduced level operators over intuitionistic fuzzy primary and semi primary ideals. The level operators and special operators were found in Atanassov. K. T [6]. The author developed all the operation of a level operators over intuitionistic fuzzy primary and semi primary ideals.

In 2014 Muthuraj. M, Muthuraman. M. S, and Sridharan. M [20] introduced, the concept of bipolar M- fuzzy group. A group with operator is an algebraic system consisting of a group, a set M is a function defined in the product set having values is in G.


1.3. Preliminary Definitions and Results.

Definition: 1.3.1

Let $G$ be a nonempty set, $A$ bipolar intuitionistic fuzzy set (IFS) $A$ in $G$ is an object of the form $A = \{ \langle x, \mu_A^+(x), \mu_A^-(x), \nu_A^+(x), \nu_A^-(x) \rangle \mid x \in G \}$ where $\mu_A^+: G \to [0,1]$ and $\nu_A^+: G \to [0,1]$, $\mu_A^- : G \to [-1,0]$ and $\nu_A^- : G \to [-1,0]$ is called degree of positive membership, degree of negative membership, the degree of positive non-membership and degree of negative of non-membership respectively.

Definition: 1.3.2

Let $G$ be a group. $A$ be a bipolar valued intuitionistic fuzzy set (or) bipolar (IFS) $A$ of $G$ is called a bipolar intuitionistic fuzzy subgroup of $G$ if for all $x, y \in G$

i) $\mu_A^+(xy) \geq \min (\mu_A^+(x), \mu_A^+(y))$ and $\nu_A^+(xy) \leq \max (\nu_A^+(x), \nu_A^+(y))$.

ii) $\mu_A^-(xy) \leq \max (\mu_A^-(x), \mu_A^-(y))$ and $\nu_A^-(xy) \geq \min (\nu_A^-(x), \nu_A^-(y))$.

iii) $\mu_A^+(x^{-1}) = \mu_A^+(x)$, $\mu_A^-(x^{-1}) = \mu_A^-(x)$ and $\nu_A^+(x^{-1}) = \nu_A^+(x)$, $\nu_A^-(x^{-1}) = \nu_A^-(x)$.

Example 1.3.3

\[
\mu_A^+(x) = \begin{cases} 
0.7 & \text{if } x = 1 \\
0.6 & \text{if } x = -1 \\
0.4 & \text{if } x = i, -i
\end{cases} \quad \nu_A^+(x) = \begin{cases} 
0.2 & \text{if } x = 1 \\
0.3 & \text{if } x = -1 \\
0.5 & \text{if } x = i, -i
\end{cases}
\]
Definition: 1.3.4

Let $G$ be a group. $A$ be a bipolar valued IFS (or) bipolar IFS $A$ of $G$ is called a bipolar intuitionistic anti fuzzy subgroup of $G$ if for all $x, y \in G$,

i) $\mu_{A}(xy) \leq \max (\mu_{A}^{+}(x), \mu_{A}^{-}(y))$ and $\nu_{A}(xy) \geq \min (\nu_{A}^{+}(x), \nu_{A}^{-}(y))$.

ii) $\mu_{A}(xy) \geq \min (\mu_{A}^{+}(x), \mu_{A}^{-}(y))$ and $\nu_{A}(xy) \leq \max (\nu_{A}^{+}(x), \nu_{A}^{-}(y))$.

iii) $\mu_{A}^{+}(x^{-1}) = \mu_{A}^{+}(x)$, $\mu_{A}^{-}(x^{-1}) = \mu_{A}^{-}(x)$ and $\nu_{A}^{+}(x^{-1}) = \nu_{A}^{+}(x)$, $\nu_{A}^{-}(x^{-1}) = \nu_{A}^{-}(x)$.

Example: 1.3.5

$$\mu_{A}^{+}(x) = \begin{cases} 0.4 & \text{if } x = 1 \\ 0.6 & \text{if } x = -1 \\ 0.7 & \text{if } x = i, -i \end{cases}$$ \hspace{1cm} \nu_{A}^{+}(x) = \begin{cases} 0.5 & \text{if } x = 1 \\ 0.3 & \text{if } x = -1 \\ 0.2 & \text{if } x = i, -i \end{cases}$$

$$\mu_{A}^{-}(x) = \begin{cases} -0.3 & \text{if } x = 1 \\ -0.5 & \text{if } x = -1 \\ -0.8 & \text{if } x = i, -i \end{cases}$$ \hspace{1cm} \nu_{A}^{-}(x) = \begin{cases} -0.6 & \text{if } x = 1 \\ -0.4 & \text{if } x = -1 \\ -0.1 & \text{if } x = i, -i \end{cases}$$

Definition: 1.3.6

Let $f$ and $g$ be a mapping of a group $G_{1}$ to a group $G_{2}$. Let $\mu = (\mu^{+}, \mu^{-})$ and $\phi = (\phi^{+}, \phi^{-})$ and $\nu = (\nu^{+}, \nu^{-})$, $\psi = (\psi^{+}, \psi^{-})$ are bipolar intuitionistic fuzzy subset of $G_{1}$ and $G_{2}$ respectively, then the image $f(\mu)$
and \( g(\nu) \) is a bipolar intuitionistic fuzzy subset defined by
\[
f(\mu) = (f(\mu)^+, f(\mu)^-) \quad \text{and} \quad g(\nu) = (g(\nu)^+, g(\nu)^-) \quad \text{of} \quad G_2 \quad \text{for all} \quad u, v \in G_2.
\]

i) \( f(\mu)^+(u) = \max\{\mu^+(x); x \in f^{-1}(u)\} \) if \( f^{-1}(u) \neq \emptyset \), 0

ii) \( g(\nu)^+(v) = \min\{\nu^+(x); x \in g^{-1}(v)\} \) if \( g^{-1}(v) \neq \emptyset \), 0 otherwise

iii) \( f(\mu)^-(u) = \min\{\mu^-(x); x \in f^{-1}(u)\} \) if \( f^{-1}(u) \neq \emptyset \), 0

iv) \( g(\nu)^-(v) = \max\{\nu^-(x); x \in g^{-1}(v)\} \) if \( g^{-1}(v) \neq \emptyset \), 0 otherwise.

The preimage \( f^{-1}(\phi) \) under \( f \) and \( g^{-1}(\psi) \) under \( g \) is defined by the bipolar intuitionistic fuzzy subset of \( G_i \) defined by for \( x \in G_i \),

v) \( (f^{-1}(\phi^+))(x) = \phi^+(f(x)) \); \( (f^{-1}(\phi^-))(x) = \phi^-(f(x)) \) and

vi) \( (g^{-1}(\psi^+))(x) = \psi^+(g(x)) \); \( (g^{-1}(\psi^-))(x) = \psi^-(g(x)) \).

**Definition: 1.3.7**

Let \( G_1 \) and \( G_2 \) be any two bipolar intuitionistic \( M \)-groups then the function \( f: G_1 \to G_2 \) and \( g: G_1 \to G_2 \) is said to be an intuitionistic \( M \)-homomorphism if

i) \( f(xy) = f(x)f(y) \) for all \( x, y \in G_i \).

ii) \( f(mx) = mf(x) \) for all \( m \in M \) and \( x \in G_i \).

iii) \( g(xy) = g(x)g(y) \) for all \( x, y \in G_i \).

iv) \( g(mx) = mg(x) \) for all \( m \in M \) and \( x \in G_i \).
Definition: 1.3.8

Let $G_1$ and $G_2$ be any two bipolar intuitionistic $M$-groups (not necessarily commutative) then the function $f: G_1 \rightarrow G_2$ and $g: G_1 \rightarrow G_2$ is said to be an intuitionistic $M$-anti homomorphism if

i) $f(xy) = f(x)f(y)$ for all $x, y \in G_1$.

ii) $f(mx) = m f(x)$ for all $m \in M$ and $x \in G_1$.

iii) $g(xy) = g(x)g(y)$ for all $x, y \in G_1$.

iv) $g(mx) = m g(x)$ for all $m \in M$ and $x \in G_1$.

Definition: 1.3.9

Let $A$ be a fuzzy subgroup of $G$ of a group $G$ is called a fuzzy prime if for all $a, b \in G$ and $a, b \in A$ either $\mu_A(ab) = \mu_A(a)$ or else $\mu_A(ab) = \mu_A(b)$.

Definition: 1.3.10 [9]

Let $G$ be a group and $A$ be a fuzzy subgroup of $G$. The subgroups $A_t, t \in [0,1]$ and $t \leq A(e)$, are called level subgroup of $A$.

Definition: 1.3.11 [18]

Let $\mu$ be a fuzzy subset of $G$. For any $t$ in $I$, the set $G_t = \{x \in G / \mu(x) \geq t\}$ is called a level subset of the fuzzy subset $\mu$. 
Proposition: 1.3.12. [32]

A homomorphic image of a fuzzy sub groupoid which has the supremum property is a fuzzy sub groupoid.

Definition: 1.3.13. [24]

Let $\mu = (\mu^+, \mu^-)$ be a bipolar fuzzy subset of $G$, for any $\alpha, \beta \in [0, 1] \times [-1, 0]$, the bipolar set $L_{\mu_{\alpha, \beta}} = \left\{ x \in G / \mu^+(x) \leq \alpha \text{ and } \mu^-(x) \geq \beta \right\}$ is called a lower level subset of the bipolar fuzzy subset $\mu$.
1.4. Summary of Results

Properties of bipolar intuitionistic M- Fuzzy group and anti M- Fuzzy group.

- If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $A \cup B$ is a bipolar intuitionistic $M$-fuzzy group of $G$.

- Union of any two bipolar intuitionistic $M$-fuzzy group is also a bipolar intuitionistic $M$-fuzzy group if either is contained in the other.

- If $A$ is a bipolar intuitionistic $M$-fuzzy group of $G$, then $\overline{A} = A$ is also a bipolar intuitionistic $M$-fuzzy group of $G$.

- Let $\mu$ and $\nu$ be a bipolar intuitionistic fuzzy subset of an $M$-fuzzy group, then $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic $M$-fuzzy group of $G$ if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti $M$-fuzzy group of $G$.

- If $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic $M$-fuzzy group of $G_1$, then the image of $\mu$ under $f$ is a bipolar intuitionistic $M$-fuzzy group of $G_2$ if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti $M$-fuzzy group of $G_1$ then the image of $\nu$ under $g$ is a bipolar intuitionistic anti $M$-fuzzy group of $G_2$. 
The $M$-homomorphic preimage of a bipolar intuitionistic $M$-fuzzy group of $G_2$ is a bipolar intuitionistic $M$–fuzzy group of $G_1$ if and only if $M$-fuzzy group of $G_2$ is a bipolar intuitionistic anti $M$–fuzzy group of $G_1$.

All the above theorem are satisfied the bipolar intuitionistic anti $M$-fuzzy group.

Some operations on bipolar intuitionistic $M$-Fuzzy group and anti $M$-Fuzzy group.

- If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $\Box A$ and $\Diamond A$ is an bipolar intuitionistic $M$-fuzzy group of $G$ respectively.

- If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $\Box (A \cap B) = \Box A \cap \Box B$ is also an bipolar intuitionistic $M$-fuzzy group of $G$.

- If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $\Diamond (A \cap B) = \Diamond A \cap \Diamond B$ is also an bipolar intuitionistic $M$-fuzzy group of $G$.

- If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $\Box(\Box A) = \Box A$ is an bipolar intuitionistic $M$-fuzzy group of $G$. 
• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then 
$\Box(\Diamond A) = \Diamond A$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then 
$\Diamond(\Box A) = \Box A$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then 
$\Diamond(\Diamond A) = \Diamond A$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then 
$\Box(\Box A) = \Box A$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• All the above theorem are satisfied the bipolar intuitionistic anti $M$-fuzzy group.

**Level operators over bipolar intuitionistic $M$-Fuzzy group and Anti $M$-Fuzzy group.**

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then 
$!(A)$ and $?(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$ respectively.
• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $!(A \cap B) = !(A) \cap !(B)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $?((A \cup B)) = ?(A) \cup ?(B)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $?(\overline{A}) = !(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $!(?(A)) = ?(!!(A))$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $!(\square (A)) = \square (!!(A))$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $?(\diamond (A)) = \diamond (?!(A))$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $!(\hat{\diamond} (A)) = \hat{\diamond} (!!(A))$ is an bipolar intuitionistic $M$-fuzzy group of $G$. 

20
• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $\hat{\hat{\Diamond}}(\hat{\hat{\Diamond}}(A)) = \hat{\hat{\Diamond}}(\hat{\hat{\Diamond}}(A))$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• All the above theorem are satisfied the bipolar intuitionistic anti $M$-fuzzy group.

**Interior operators over bipolar intuitionistic $M$-Fuzzy prime group, anti $M$-Fuzzy prime group and Special operator over bipolar intuitionistic $M$-Fuzzy group and anti $M$-Fuzzy group.**

• If $A$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$, then $I(A)$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$, then $I(I(A)) = I(A)$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$.

• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy prime group of $G$, then $I(A \cap B) = I(A) \cap I(B)$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$, then $\Box(I(A)) = I(\Box(A))$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$, then $\Diamond(I(A)) = I(\Diamond(A))$ is an bipolar intuitionistic $M$-fuzzy prime group of $G$.
• All the above theorem are satisfied the bipolar intutionistic anti $M$-fuzzy Prime group.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $P_{\alpha,\beta}(A)$ and $Q_{\alpha,\beta}(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$ respectively.

• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $P_{\alpha,\beta}(A \cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$ is an bipolar intuitionistic $M$-fuzzy group of $G$ and for every $\alpha, \beta \in [0,1]$ and $\alpha, \beta \in [-1,0]$ implies $\alpha + \beta \leq 1$ and $\alpha + \beta \leq -1$.

• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $Q_{\alpha,\beta}(A \cap B) = Q_{\alpha,\beta}(A) \cap Q_{\alpha,\beta}(B)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $P_{\alpha,\beta}(A) = Q_{\beta,\alpha}(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $G_{\alpha,\beta}(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

• If $A$ and $B$ are bipolar intuitionistic $M$-fuzzy group of $G$, then $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.  

22
● If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $G_{\alpha,\beta}(IA) = IG_{\alpha,\beta}(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

● If $A$ is an bipolar intuitionistic $M$-fuzzy group of $G$, then $\overline{G_{\alpha,\beta}(A)} = G_{\beta,\alpha}(A)$ is an bipolar intuitionistic $M$-fuzzy group of $G$.

● All the above theorem are satisfied the bipolar intuitionistic anti $M$-fuzzy group.

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