

**DECLARATION BY THE CANDIDATE**

I declare that the thesis entitled **OSCILLATORY AND ASYMPTOTIC BEHAVIOR OF SECOND ORDER NEUTRAL TYPE DIFFERENTIAL EQUATIONS** submitted by me for the award of the Degree of **Doctor of Philosophy in Mathematics** is a bonafide record of work carried out by me under the guidance of **Major Dr.R.Arul**, Principal (Retd), Kandaswami Kandar's College, Velur - 638182, Namakkal District, Tamilnadu, India and has not formed the basis for the award of any degree, diploma, associateship, fellowship, titles in this or any other University or other similar Institution of higher learning.

Velur - 638182

July 2016

  
**Signature of the Candidate**

(V.S.Shobha)

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## ABSTRACT

The research reported in this thesis is concerned with the problem of oscillatory and asymptotic behavior of second order neutral type differential equations. Chapter 1 provides necessary introduction and motivation for the present work. In Chapter 2, we investigate the oscillatory behavior of solutions of a second order quasilinear differential equation with several neutral terms. In Chapters 3 and 4, we consider a second order delay differential equation with nonpositive neutral term and establish some sufficient conditions for the oscillation of all solutions of this equation. In Chapter 5 and 6, we consider a second order nonlinear differential equation with mixed neutral term and study the oscillatory behavior of solutions of this equation. Examples are provided to illustrate the main results. The results presented in this thesis improve, generalize and extend some of the known results.

# Contents

Certificate	i
Declaration by the Candidate	ii
Acknowledgements	iii
Abstract	1
<b>1 Introduction</b>	<b>4</b>
1.1 Differential Equations . . . . .	5
1.2 Motivation . . . . .	6
1.3 Plan of the Thesis . . . . .	7
<b>2 Quasilinear Differential Equation with Several Neutral Terms</b>	<b>11</b>
2.1 Introduction . . . . .	12
2.2 Oscillation Results . . . . .	13
2.3 Examples . . . . .	20
<b>3 Delay Differential Equation with Nonpositive Neutral Term - I</b>	<b>22</b>
3.1 Introduction . . . . .	23
3.2 Oscillation Results . . . . .	24
3.3 Examples . . . . .	33

	3
<b>4 Delay Differential Equation with Nonpositive Neutral Term - II</b>	<b>35</b>
4.1 Introduction . . . . .	36
4.2 Oscillation Results . . . . .	37
4.3 Examples . . . . .	43
<b>5 Neutral Differential Equation with Mixed Neutral Term - I</b>	<b>45</b>
5.1 Introduction . . . . .	46
5.2 Oscillation Results . . . . .	47
5.3 Examples . . . . .	53
<b>6 Neutral Differential Equation with Mixed Neutral Term - II</b>	<b>56</b>
6.1 Introduction . . . . .	57
6.2 Oscillation Results . . . . .	58
6.3 Examples . . . . .	66
<b>Bibliography</b>	<b>69</b>
<b>Papers Covering the Content of the Thesis</b>	<b>81</b>