1. **Lagrange's theorem:** If $\phi(z)$ is holomorphic at 
$z = x$ and $\phi(x) \neq 0$, and if $z = x + t \phi(z)$ then

\[
f(z) = f(x) + \sum_{n=1}^{\infty} \frac{t^n}{n!} D^n \left[ \phi(x)^n f(x) \right].
\]

This expansion in modified form, can be expressed as

\[
\frac{F(z)}{1 - t \phi'(z)} = \sum_{n=0}^{\infty} \frac{t^n}{n!} D^n \left[ \phi(x)^n f(x) \right].
\]

For details, see Whittaker and Watson [37].

2. **Extended Carlitz theorem:**

Let $A(z)$, $B(z)$ and $\gamma c(z)$ be arbitrary function which are analytic in a neighbourhood of the origin and assume that

\[
A(0) = B(0) = c'(0) = 1.
\]

Define the sequence of functions $\left\{ f_n(x) \right\}$ by means of

\[
A(z) \left[ B(z) \right]^{\alpha} \exp \left( \gamma c(z) \right) = \sum_{n=0}^{\infty} f_n(x) \frac{z^n}{n!},
\]

where $\alpha$ and $\gamma$ are arbitrary complex numbers, independent of $z$. Then, for arbitrary parameters $\lambda$ and $\gamma$ independent of $z$,

\[
\sum_{n=0}^{\infty} \frac{(\alpha + \alpha n)}{n!} f_n(x + \gamma y) \frac{t^n}{n!} = \frac{A(c \xi) \left[ B(c \xi) \right]^{\alpha} \exp \left( \gamma c(c \xi) \right)}{1 - \xi^{\lambda} B(c \xi) A(c \xi) + y c(c \xi)^{\gamma}},
\]

where $\xi = \frac{1}{t} \left[ \frac{B(c \xi)}{B(c \xi) - c(c \xi)} \right]^{\lambda} \exp(y c(c \xi)).$
3 Generalised Hypergeometric Function:

Define

\[
P F q \left[ \alpha_1, \alpha_2, \ldots, \alpha_p; \beta_1, \beta_2, \ldots, \beta_q; z \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n \cdots (\alpha_p)_n}{(\beta_1)_n (\beta_2)_n \cdots (\beta_q)_n} \frac{z^n}{n!},
\]

where \((\lambda)_n\) is Pachchamer symbol defined by

\[
(\lambda)_n = \lambda (\lambda + 1) \cdots (\lambda + n - 1), \quad n \geq 1,
\]

\[
(\lambda)_0 = 1, \quad \lambda \neq 0.
\]

The function defined by (6) is known as generalised hypergeometric series or function of \(z\), and parameters \(\alpha_1, \alpha_2, \ldots, \alpha_p; \beta_1, \beta_2, \ldots, \beta_q\) take the values in such a way that the infinite series either terminates or is convergent. In particular, denominator parameters, \(\beta_i\), are neither Zero nor a negative integer.

We can easily see that

\[
\frac{d}{dz} P F q \left[ \alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_q; z \right] = \frac{\alpha_1, \ldots, \alpha_p}{\beta_1, \beta_2, \ldots, \beta_q} P F q \left[ \alpha_1 + 1, \ldots, \alpha_p + 1; \beta_1 + 1, \ldots, \beta_q + 1; z \right].
\]

For more details one can refer to Rainville [25].
Generalised Lauricella Hypergeometric Function of many variables:

The generalised Lauricella hypergeometric function of \( n \) variables has been denoted and defined as follows \(^{[35]}\):

\[
F_{\mathbf{C}}: \mathbf{b}^{(1)}; \ldots, \mathbf{b}^{(n)} \left( \frac{Z_1}{z_2}; \ldots, \frac{Z_n}{z_n} \right) = F_{\mathbf{C}}: \mathbf{b}^{(1)}; \ldots, \mathbf{b}^{(n)} \left( \sum \left( \mathbf{d}^{(m)} \right) \left( \mathbf{s}^{(m)} \right) \right) = \sum_{m_1, \ldots, m_n = 0}^{\infty} \mathcal{J} (m_1, m_2, \ldots, m_n) \frac{Z_1^{m_1}}{m_1!} \ldots \frac{Z_n^{m_n}}{m_n!},
\]

where

\[
\mathcal{J} (m_1, m_2, \ldots, m_n) = \prod_{j=1}^{A} \left( c_j \right)^{m_1 \phi_j + m_2 \phi_j + \ldots + m_n \phi_j} \prod_{j=1}^{B} \left( b_j \right)^{m_1 \phi_j + \ldots + m_n \phi_j} \prod_{j=1}^{C} \left( d_j \right)^{m_1 \phi_j + \ldots + m_n \phi_j}.
\]