CHAPTER IV

FRAGMENTATION OF RELATIVISTIC CARBON NUCLEI IN EMULSION*

* Results to be published in Nuclear Physics A (1989).
4.1 Introduction

The first experimental information about the fragmentation of nuclei was obtained in experiments with cosmic rays (1,2). Acceleration of nuclei to relativistic energies at Berkeley and Dubna provided an opportunity to study this phenomenon in a systematic manner. Study of the fragmentation of nuclei permits qualitative information to be obtained on the internal structure of nuclei under conditions of small transfer of energy-momentum. Further knowledge of fragmentation parameters is required for estimating the abundances of cosmic ray nuclei at their sources from their abundances at the top of the atmosphere.

The geometrical aspects of the fragmentation of a nucleus can be understood in terms of the participant-spectator model (3-5). According to this model, at finite impact parameter three regions are produced after a collision between two nuclei. The participant region, the projectile spectator region and the target spectator region. The projectile spectator decays mainly into nuclear clusters since very little momentum transfer is required to form these fragments. The projectile fragments may thus be useful in determining the momentum distribution of a nuclear cluster inside the projectile nucleus. At relativistic energies, the separation in rapidity between projectile and target fragments is large, \( \geq 1 \) unit of rapidity. Consequently, no correlation exists between the projectile and target fragments and therefore the modes of fragmentation are independent of the target mass (6-8). The fragmentation cross-section can thus be factorized into projectile and target related parts.
In inclusive experiments in which fragments ($\Theta < 0.7^\circ$) of light nuclei were detected with a spectrometer and there was no restriction on the degree of target excitation, the factorization has been found to be valid (7). Exceptions to strict factorization have however been observed for fragmentation reactions in hydrogen (8) and for heavy target where single nucleon stripping is increased by the dissociation of carbon and oxygen projectiles in the virtual photon field of the target nucleus (9).

Another important feature of the projectile fragmentation is that it exhibits features of limiting fragmentation. This means that a distribution of products with finite energies in the rest frame of the projectile or the target approaches a limiting form as the projectile energy increases. Greiner et al (7) have studied the momentum distribution of carbon fragments in the projectile rest frame at energies 1.05 and 2.1 A GeV and found that the widths of the distributions were independent of energy. In some emulsion experiments, the angular distributions of projectile fragments for events exhibiting either no or very small target excitation exhibited features of limiting fragmentation (10,11).

In this chapter we study the fragmentation of relativistic carbon nuclei in emulsion at 4.5 A GeV/c. With a view to testing the validity of factorization and limiting fragmentation observed in inclusive experiments, the projected angular distributions and momentum distributions of $^{12}$C-fragments have been studied and compared with similar distributions at different energies. Azimuthal correlations
have also been studied for $Z = 2$ and $Z \geq 3$ fragments. Special attention has been paid to helium fragments. For this propose events have been divided into different reaction channels, according to the number of helium fragments emitted and various characteristics of helium fragments have been studied in each channel.

4.2 Multiplicities of Projectile Fragments

Berkeley group (6,7,12), using a spectrometer, studied the single particle inclusive spectrum of fragments of $^4\text{He}$, $^{12}\text{C}$, $^{14}\text{N}$ and $^{16}\text{O}$ at 2.1 A GeV/c, emitted near $0^\circ$. They found that the fragmentation cross-sections could be factorized into projectile and target related parts. In other words, the charge composition of projectile fragments does not depend on the mass of the target.

Data on multiplicities of fragments of different charges in different target groups of $^{12}\text{C}$-emulsion collisions are presented in Table 4.1 and 4.2. It can be seen from these tables that the multiplicity of fragments decreases as the charge of the fragments increases. Also for a fragment of a given charge, the multiplicity decreases with increasing mass of the target. Thus the composition of fragments depends considerably on the mass of the target. This is an explicit violation of the principle of factorization. It was also observed by the Berkeley group that the ratios of different cross-sections for the production of fragments near $0^\circ$ in different targets are constant and approximately equal to the ratios of the geometrical cross-sections. The results presented in Table 4.3 contradict this statement also. In fact the results indicate that there is a considerable dependence of the cross-section on the mass of the target.
TABLE 4.1

Average multiplicities of fragments produced in $^{12}$C-emulsion collisions at 4.5 A GeV/c.

<table>
<thead>
<tr>
<th>Charge of fragments</th>
<th>$^{12}$C-H</th>
<th>$^{12}$C-CNO</th>
<th>$^{12}$C-AgBr</th>
<th>$^{12}$C-Em</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z = 1</td>
<td>0.63 ± 0.06</td>
<td>0.75 ± 0.04</td>
<td>0.56 ± 0.04</td>
<td>0.65 ± 0.03</td>
</tr>
<tr>
<td>Z = 2</td>
<td>1.32 ± 0.11</td>
<td>0.74 ± 0.05</td>
<td>0.29 ± 0.03</td>
<td>0.68 ± 0.03</td>
</tr>
<tr>
<td>Z = 3</td>
<td>0.17 ± 0.03</td>
<td>0.08 ± 0.01</td>
<td>0.04 ± 0.01</td>
<td>0.08 ± 0.01</td>
</tr>
<tr>
<td>Z = 4</td>
<td>0.06 ± 0.01</td>
<td>0.04 ± 0.01</td>
<td>0.02 ± 0.01</td>
<td>0.03 ± 0.01</td>
</tr>
<tr>
<td>Z = 5</td>
<td>0.03 ± 0.01</td>
<td>0.06 ± 0.01</td>
<td>0.03 ± 0.01</td>
<td>0.04 ± 0.01</td>
</tr>
<tr>
<td>Z = 6</td>
<td>0.04 ± 0.01</td>
<td>0.07 ± 0.01</td>
<td>0.03 ± 0.01</td>
<td>0.04 ± 0.01</td>
</tr>
</tbody>
</table>
### TABLE 4.2

Charge composition of projectile fragments in $^{12}\text{C}$-emulsion collisions at 4.5 A GeV/c.

<table>
<thead>
<tr>
<th>Multiplicity ratio</th>
<th>Target nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td>$\langle N_z = 2 \rangle$</td>
<td>2.18 ± 0.28</td>
</tr>
<tr>
<td>$\langle N_z = 1 \rangle$</td>
<td></td>
</tr>
<tr>
<td>$\langle N_z \geq 3 \rangle$</td>
<td>0.47 ± 0.07</td>
</tr>
<tr>
<td>$\langle N_z = 1 \rangle$</td>
<td></td>
</tr>
<tr>
<td>$\langle N_z \geq 3 \rangle$</td>
<td>0.22 ± 0.03</td>
</tr>
<tr>
<td>$\langle N_z = 2 \rangle$</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 4.3

The ratio of the average multiplicities of projectile fragments for different target combinations.

<table>
<thead>
<tr>
<th>Charge of fragments</th>
<th>$\langle N_z \rangle_{\text{CNO}}$</th>
<th>$\langle N_z \rangle_{\text{AgBr}}$</th>
<th>$\langle N_z \rangle_{\text{AgBr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle N_z \rangle_{\text{H}}$</td>
<td>$\langle N_z \rangle_{\text{H}}$</td>
<td>$\langle N_z \rangle_{\text{CNO}}$</td>
<td></td>
</tr>
<tr>
<td>$Z = 1$</td>
<td>1.20 ± 0.13</td>
<td>0.90 ± 0.11</td>
<td>0.75 ± 0.07</td>
</tr>
<tr>
<td>$Z = 2$</td>
<td>0.56 ± 0.06</td>
<td>0.22 ± 0.03</td>
<td>0.40 ± 0.05</td>
</tr>
<tr>
<td>$Z \geq 3$</td>
<td>0.80 ± 0.13</td>
<td>0.36 ± 0.07</td>
<td>0.45 ± 0.08</td>
</tr>
</tbody>
</table>
Thus our results on the multiplicities of fragments from the fragmentation of carbon nuclei in emulsion at 4.5 A GeV/c show that the factorization observed in electronic experiments for fragments emitted near 0° has a restricted region of applicability. It is broken in an emulsion experiment wherein the total cross-sections are measured. Similar results have also been obtained in case of \(^{14}\text{N}\) and \(^{56}\text{Fe}\) projectiles (11,15,17).

Figure 4.1 shows the multiplicity distributions of \(Z = 1,2,3\) and \(\geq 4\) fragments. Multiplicities \(n > 1\) for \(Z = 3\) fragments are very rare. Out of 1250 collisions we found only 17 events in which two \(Z = 3\) fragments were emitted. The breakup in different target groups was: \(10(^{12}\text{C} - \text{H})\) group, \(7(^{12}\text{C} - \text{CNO})\) group and nil(\(^{12}\text{C} - \text{AgBr}\)) group. The cross-section for the process in which carbon nuclei breakup into two \(Z = 3\) fragments comes about 1.36\(\times 10^{-2}\) of the total inelastic cross-section. This value contradicts the results of Jakobsson et al (13) and Judek et al (14) who did not find even a single event emitting two fragments with \(Z \geq 3\). The corresponding values obtained by Jakobsson et al (13) and Judek et al (14) are respectively 3.7\(\times 10^{-3}\) and 1.3\(\times 10^{-3}\). It is worth mentioning here that in the present experiment the charge of the fragments has been measured by doing ionization measurement and by \(\gamma\)-ray count method. The charge calibration has been done with the help of events in which projectile carbon nucleus is dissociated into three \(Z = 2\) fragments. Thus in our experiment charge measurement is very reliable.

It would be interesting to investigate the dependence of the average multiplicities of fragments, \(<N_z>\) on the mass of the projectile.
Fig. 4.1 The multiplicity distributions of projectile fragments of charge $Z = 1, 2, 3$ and $\geq 4$. 
In fig. 4.2, $\langle N_z \rangle$ is plotted against $A$, the mass number of the projectile, for fragments with charge $Z = 1, 2,$ and $\geq 3$. Data of other projectiles have been taken from references 15-18. An expression of the type $\langle N_z \rangle = \text{Const.} \, A^\alpha$ can well describe the dependence. The best fit values of $\alpha$ are $(0.76 \pm 0.08)$, $(0.50 \pm 0.26)$ and $(1.22 \pm 0.23)$ for fragments with charge $Z = 1, 2$ and $\geq 3$ respectively.

Figure 4.3 shows the dependence of the average multiplicity of $Z = 2$ and multicharged ($Z \geq 3$) fragments on $\langle N_h \rangle$, which is a measure of target excitation and also a characteristic of impact parameter. It can be seen from the figure that the multiplicity of $Z = 2$ fragments decreases as $N_h$ increases. The large multiplicity of $Z = 2$ fragments at $N_h = 0, 1$ is due to extreme peripheral collisions with very small target excitation and no emission of $Z = 2$ fragments at $N_h \geq 28$ is due to central collisions of the projectile with AgBr nuclei in which all nuclei of the projectile participate. Like $Z = 2$ fragments, the multiplicity of multicharge fragments ($Z \geq 3$) also decreases with $N_h$. This result is in agreement with the picture of smooth variation of projectile spectators with changing impact parameter (19).

4.3 Angular Distribution of Projectile Fragments

Lepore and Riddell (20) have studied the fragmentation of relativistic nuclei using the sudden approximation and shell model functions. They have shown that the momentum distribution of fragments in projectile rest frame is approximately Gaussian and the width of the distribution is given by

$$\sigma^2(P) = \left[\mu \omega P (A_P - A_p)/2A_p\right] (\text{MeV})^2,$$  \hspace{1cm} (4.1)
The dependence of the average multiplicity of projectile fragments of charge $Z = 1$, $2$ and $\geq 3$ on the mass of the projectile, $A$. The solid line represents the relation $\langle N_Z \rangle = \text{Const.} \, A^\alpha$ (see text).
Fig. 4.3 The average multiplicity of projectile fragments \((Z = 2\) and \(Z \geq 3\)) as a function of the average target particle multiplicity \(<N_h>\).
where $m$ is the mass of the proton in MeV, $A_p$ is the mass number of the projectile, $A_F$ is the mass number of the fragment and $\omega = 45 A_p^{-1} - 25 A_F^{-2/3}$. The width of the corresponding projected angular distribution in the laboratory frame can be obtained using the following relation

$$\sin\sigma(\theta_p) = \frac{\sigma(p) \cdot A_p}{P_0 \cdot A_F} \quad (4.2)$$

where $P_0$ is the projectile momentum. From equations (4.1) and (4.2), it is clear that the width of the momentum or projected angular distribution is independent of target mass.

Figures 4.4 - 4.6 show the projected angular distributions of fragments with $Z = (1-5)$ for different target groups produced in inelastic collisions of carbon nuclei in emulsion at 4.5 A GeV/c. A Gaussian curve of the form

$$N(\theta_p) = A \exp \left(-\frac{\theta_p^2}{2\sigma^2}\right), \quad (4.3)$$

was fitted to each of these distributions (excepting $Z = 1$ fragments) and the standard deviation $\sigma$ in each case was found. It is evident from figures 4.4 - 4.6 that, excepting $Z = 1,2$ fragments, the widths of the distributions of fragments of charge $Z = 3-5$ are the same within statistical errors. This observation is not in agreement with equation (4.2) which predicts that the angular distribution becomes narrower as the charge of the fragment increases. However, it should be mentioned that statistics of fragments of charge $Z = 3-5$ are not very good.
The projected angular distributions of $Z = 1$ fragments in different target groups with double Gaussian curves fitted to the data.
Fig. 4.5 The projected angular distributions of $Z = 2$ fragments in different target groups with Gaussian curves fitted to the data for $\theta_p \leq 2^\circ$. 

$^{12}\text{C} - \text{Em}$

$\sigma = 0.51 \pm 0.11$

$\chi^2 / \text{DOF} = 1.21 / 8$

$^{12}\text{C} - \text{H}$

$\sigma = 0.57 \pm 0.08$

$\chi^2 / \text{DOF} = 1.28 / 8$

$^{12}\text{C} - \text{AgBr}$

$\sigma = 0.61 \pm 0.10$

$\chi^2 / \text{DOF} = 1.28 / 8$

$^{12}\text{C} - \text{CNO}$

$\sigma = 0.65 \pm 0.17$

$\chi^2 / \text{DOF} = 1.34 / 8$
Fig. 4-6 The projected angular distributions of multicharged fragments with Gaussian curves fitted to the data; for Z = 3 fragments, \( \Theta_p \leq 1.5^\circ \); Z = 4 fragments, \( \Theta_p \leq 1.0^\circ \); Z = 5 fragments, \( \Theta_p \leq 1.0^\circ \).
The angular distribution of $Z = 1$ fragments (Fig. 4.4) for different target groups consists of a narrow forward peak superimposed upon a large angle broad distribution. The composition of $Z = 1$ fragments includes all singly charged particles, protons, deuterons, pions etc., having ionization $g < 1.4 g_0$. The particles lying within the forward narrow peak are predominantly singly charged fragments, although a small number of pions may also be present. The particles lying within the large angle broad distribution are primarily pions, although nucleons with high transverse momentum, as suggested by the fire-ball model (21), may also be present. A double Gaussian distribution of the form

$$N(\theta_p) = A \exp \left( - \frac{\theta_p^2}{2\sigma_1^2} \right) + B \exp \left( - \frac{\theta_p^2}{2\sigma_2^2} \right)$$

was fitted. The fitted values of $\sigma_1$ and $\sigma_2$ for different target groups are also shown in fig. 4.4. The values of $\sigma_1$ for different target are approximately constant and compare well with the theoretical values calculated from equation (4.2). It should be mentioned here that the values of $\sigma_1$ are not very reliable because of the presence of the broad distribution.

The projected angular distributions for $Z = 2$ fragments for $^{12}\text{C} - \text{H}$, $^{12}\text{C} - \text{CNO}$, $^{12}\text{C} - \text{AgBr}$ and $^{12}\text{C}$-emulsion collisions are shown in fig. 4.5 along with the fitted Gaussian curves (equation 4.3) for $\theta_p \leq 2^\circ$. The values of $\sigma$ are also shown in the figure. Figure 4.6 shows the projected angular distributions of $Z = (3-5)$ fragments. Due to low statistics of these fragments, distributions for $^{12}\text{C}-(\text{H} + \text{CN})$...
and $^{12}\text{C}$-emulsion only have been plotted. The values of standard deviations of the fitted Gaussian curves (equation 4.3) are also shown in the figures.

Table 4.4 presents the summary of results on the projected angular distributions of projectile fragments. Also presented in the table are $\sigma$ values calculated using equation (4.2). For the calculation of $\sigma$ for $Z = 1$ fragments we used weight factors for p,d and t equal to their fraction obtained in reference (22). For $Z = 2$ fragments, the weights of $^3\text{He}$ and $^4\text{He}$ were calculated using the isotropic production cross-section data of Lindstrom et al (8). It can be noted from the table that $\sigma$ values are higher than those predicted by equation (4.2). The deviation from the theoretical values increases with the charge of the fragment. Further, our results on the projected angular distributions are not in agreement with those of Heckman et al (10) and Bhanja et al (11). Heckman et al (10) studied the projected angular distributions of $Z = 2$ fragments from collision of $^{12}\text{C}$, $^{14}\text{N}$ and $^{16}\text{O}$ nuclei in emulsion at an energy of 2.1 A GeV. For $N_h = 0$ events, the values of standard deviation are in agreement with those predicted by equation (4.2). Bhanja et al (11) studied the projected angular distributions of fragments with charge $Z = 2,3,4$ and 5 from collisions of $^{14}\text{N}$ with emulsion nuclei at an energy of 2.1 A GeV. For events with $N_h \leq 8$, the values of standard deviation are in agreement with those expected from equation (4.2).

In order to test the validity of the limiting fragmentation for $^{12}\text{C}$-emulsion collisions at energy 3.7 A GeV (momentum 4.5 A GeV/c),
TABLE 4.4

Standard deviations of projected angular distributions of projectile fragments from $^{12}\text{C}$-emulsion and $^{14}\text{N}$-emulsion collisions.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Ensemble</th>
<th>P</th>
<th>He</th>
<th>Li</th>
<th>Be</th>
<th>B</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$-H</td>
<td>1.26 ± 0.23</td>
<td>0.57 ± 0.08</td>
<td></td>
<td>0.37 ± 0.06</td>
<td>0.29 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$-CNO</td>
<td>1.71 ± 1.2</td>
<td>0.65 ± 0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>1.31 ± 0.18</td>
<td>0.61 ± 0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$-AgBr</td>
<td>1.53 ± 0.17</td>
<td>0.61 ± 0.11</td>
<td>0.35 ± 0.05</td>
<td>0.29 ± 0.03</td>
<td>0.30 ± 0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$-Em</td>
<td>Theory</td>
<td>0.96</td>
<td>0.46</td>
<td>0.31</td>
<td>0.22</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>N$_h$ ≥ 0</td>
<td>-</td>
<td>0.71 ± 0.03</td>
<td>0.71 ± 0.05</td>
<td>0.53 ± 0.05</td>
<td>0.35 ± 0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>0.68 ± 0.03</td>
<td>0.53 ± 0.06</td>
<td>0.43 ± 0.04</td>
<td>0.25 ± 0.03</td>
<td></td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>N$_h$ ≤ 8</td>
<td>Theory</td>
<td>0.73</td>
<td>0.47</td>
<td>0.39</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
we present in Table 4.5 $\sigma(p)$ values calculated, using equation (4.2), from the observed $\sigma(0p)$ values for fragments of different charges. Also presented in the table are $\sigma(p)$ values obtained by Greiner et al (7) for fragments with $Z = 2, 3, 4$ and 5 from the fragmentation of $^{12}\text{C}$ nuclei with energies 1.05 and 2.1 A GeV. The $\sigma(p)$ values at 3.7 A GeV obtained in the present experiment are higher than those at 1.05 and 2.1 A GeV obtained by Greiner et al (7). This is a clear violation of the limiting fragmentation which requires the width of momentum distribution of fragments to be independent of energy.

4.4 Emission of $Z = 2$ Fragments in Different Reaction Channels

In this section we study the production of $Z = 2$ fragments in detail by studying their features in different reaction channels.

4.4.1 General Characteristics of the Reaction Channels

The sample of events emitting $^4\text{He}$ fragments were divided into different types of reaction channels. The different reaction channels are defined as follows:

(a) 3 x He channel - the projectile breaks up into three $Z = 2$ fragments;

(b) 2 x He channel - the projectile breaks up into two $Z = 2$ fragments and singly charged particles;

(c) 1 x He channel - the projectile breaks up into one $Z = 2$ fragment and singly charged particles;

(d) 1 x He + F$_{z \geq 3}$ channel - the projectile breaks up into one $Z = 2$ and one $Z = 3$ or 4 fragment.
### TABLE 4.5

σ(P) values of fragments of different charges

<table>
<thead>
<tr>
<th>Projectile</th>
<th>σ(P) (Experimental)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>He</td>
<td>Li</td>
</tr>
<tr>
<td>$^{12}\text{C}$ (1.05 A GeV)</td>
<td>125 ± 3</td>
<td>122 ± 10</td>
</tr>
<tr>
<td>$^{12}\text{C}$ (2.1 A GeV)</td>
<td>129 ± 1</td>
<td>127 ± 7</td>
</tr>
<tr>
<td>$^{12}\text{C}$ (3.7 A GeV)</td>
<td>192 ± 5</td>
<td>165 ± 9</td>
</tr>
</tbody>
</table>
Table 4.6 shows the percentage of occurrence of different reaction channels for different target groups and similar data for $^{14}\text{N}(17)$ and $^{16}\text{O}(13)$ projectiles.

4.4.2 Correlations Between the Projectile and Target Breakup

To study the correlation between the projectile and target breakup, we present in Table 4.7 the average target particle multiplicity, $\langle N_{h} \rangle$ in all channels for various target groups. We observe that the mode of projectile breakup is almost independent of the target breakup for CNO and AgBr targets, although a weak dependence of $\langle N_{h} \rangle$ on the degree of projectile breakup cannot be ruled out. We also notice that $\langle N_{h} \rangle$ for $3 \times \text{He}$ is $1.22 \pm 0.14$ while it is $2.94 \pm 0.49$ for $1 \times \text{He} + F_{z \geq 3}$ channel for the combination of all targets, although both channels represent peripheral collisions. The difference in $\langle N_{h} \rangle$ values is due to the presence of diffraction dissociation events in $3 \times \text{He}$ channel in which the projectile $^{12}\text{C}$ breaks up into three $Z = 2$ fragments without any target excitation. If we exclude these events, $\langle N_{h} \rangle$ for $3 \times \text{He}$ channel becomes $2.89 \pm 0.44$, which is comparable to the corresponding value $2.94 \pm 0.49$ for $1 \times \text{He} + F_{z \geq 3}$ channel.

Figure 4.7 shows the dependence of the average multiplicity of $Z = 2$ fragments with the degree of target breakup $\langle N_{h} \rangle$ for different reaction channels. We observe that while $3 \times \text{He}$ and $1 \times \text{He} + F_{z \geq 3}$ channels extend up to $N_{h} = 10$ only, $1 \times \text{He}$ and $2 \times \text{He}$ channels extend up to $N_{h} = 24$. Moreover, the average multiplicity of $Z = 2$ fragments in all channels decreases with $\langle N_{h} \rangle$, excepting $1 \times \text{He}$ channel for which it remains almost constant over a wide range of $\langle N_{h} \rangle$. These
TABLE 4.6

Percentage of occurrence of different reaction channels

<table>
<thead>
<tr>
<th>Reaction channel</th>
<th>$^{12}\text{C}-\text{H}$</th>
<th>$^{12}\text{C}-\text{CNO}$</th>
<th>$^{12}\text{C}-\text{AgBr}$</th>
<th>$^{12}\text{C}-\text{Em}$ (4.5 A GeV/c)</th>
<th>$^{14}\text{N}-\text{Em}$ (2.8 A GeV/c)</th>
<th>$^{16}\text{O}-\text{Em}$ (2.8 A GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x He</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3 x He</td>
<td>$2.64 \pm 0.47$</td>
<td>$0.96 \pm 0.28$</td>
<td>-</td>
<td>$3.60 \pm 0.54$</td>
<td>$6 \pm 2$</td>
<td>$10 \pm 2$</td>
</tr>
<tr>
<td>2 x He</td>
<td>$6.48 \pm 0.74$</td>
<td>$7.52 \pm 0.80$</td>
<td>$2.64 \pm 0.47$</td>
<td>$16.64 \pm 1.25$</td>
<td>$10 \pm 1$</td>
<td>$10 \pm 2$</td>
</tr>
<tr>
<td>1 x He</td>
<td>$4.16 \pm 0.59$</td>
<td>$9.12 \pm 0.89$</td>
<td>$6.32 \pm 0.73$</td>
<td>$19.60 \pm 1.37$</td>
<td>$18 \pm 2$</td>
<td>$15 \pm 2$</td>
</tr>
<tr>
<td>1 x He $+ F_{Z\geq 3}$</td>
<td>$2.16 \pm 0.42$</td>
<td>$1.12 \pm 0.30$</td>
<td>$0.56 \pm 0.21$</td>
<td>$3.84 \pm 0.56$</td>
<td>$3 \pm 2$</td>
<td>$3 \pm 2$</td>
</tr>
<tr>
<td>Total no. of events</td>
<td>1250</td>
<td>923</td>
<td>269</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Present work</td>
<td>17</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4.7

Average target particle multiplicity in various reaction channels.

<table>
<thead>
<tr>
<th>Target</th>
<th>Reaction channel</th>
<th>1 x He</th>
<th>2 x He</th>
<th>3 x He</th>
<th>1 x He + F_z&gt;3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNO</td>
<td>1 x He</td>
<td>4.44 ± 0.46</td>
<td>4.31 ± 0.35</td>
<td>4.00 ± 0.74</td>
<td>3.21 ± 0.98</td>
</tr>
<tr>
<td></td>
<td>2 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 x He + F_z&gt;3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AgBr</td>
<td>1 x He</td>
<td>12.97 ± 1.51</td>
<td>12.24 ± 1.57</td>
<td>12.71 ± 2.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 x He + F_z&gt;3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Em</td>
<td>1 x He</td>
<td>6.33 ± 0.43</td>
<td>4.01 ± 0.22</td>
<td>1.22 ± 0.14</td>
<td>2.94 ± 0.49</td>
</tr>
<tr>
<td></td>
<td>2 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 x He</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 x He + F_z&gt;3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* <N_h> after excluding carbon dissociation events.
Fig. 4.7 The average multiplicity of \( Z = 2 \) fragments as a function of the average target particle multiplicity in different reaction channels.
results indicate that $3 \times \text{He}$ and $1 \times \text{He} + F_{z \geq 3}$ channels represent peripheral collisions while $1 \times \text{He}$ channel represents more violent collisions of the projectile with the target.

Figure 4.8 shows the dependence of the average angle of emission of $Z = 2$ fragments of $\langle N_h \rangle$ for different channels. An examination of the figure shows that the average angle of emission is almost independent of $N_h$.

4.4.3 Angular Distribution of Fragments in Different Reaction Channels

Figure 4.9 shows the projected angular distributions of $Z = 2$ fragments in different reaction channels for different target groups along with the fitted Gaussian curves. The values of the standard deviation $\sigma$ of the fitted curves with $\chi^2/DOF$ are given in the figures. The notable features of these distributions are as follows:

In each channel, the values of $\sigma$ for different target groups are comparable within statistical errors. The values of $\sigma$ for $3 \times \text{He}$ and $1 \times \text{He} + F_{z \geq 3}$ channels are almost the same, thus confirming our earlier observation that both these channels represent peripheral collisions. The values of $\sigma$ exhibit a weak dependence on the degree of projectile breakup. The angular distribution of He fragments in all the channels combined together ($N_h \geq 0$; Fig. 4.5) also has a Gaussian shape having comparable $\sigma$ values which implies no correlations between the nucleons in the projectile as predicted by Goldhaber fragmentation process. Any correlation among nucleons in the projectile would have otherwise been reflected in the distribution of fragments in
Fig. 4.9 The projected angular distributions of $Z = 2$ fragments in different target groups and in different reaction channels with Gaussian curves fitted to the data.
individual channels. The slight difference in $\sigma$ values of $3 \times \text{He}$ and $1 \times \text{He} + \text{F}_{z \geq 3}$ with other channels may be reflection of such outgoing He fragments as have been imparted a large transverse momentum.

4.5 Transverse Momentum Distribution

Much information about the dynamics of the fragmentation process can be obtained from the analysis of momentum distribution of fragments. A very important result found in the inclusive experiments (7) is that the momentum distributions of the fragments have approximately a Gaussian shape in the rest frame of the fragmenting nucleus. The momentum distribution has also a parabolic dependence on the mass of the fragment. These regularities follow naturally from the statistical approach to the fragmentation process with minimal correlations between momenta of intranuclear nucleons.

In an emulsion experiment it is not possible to make direct measurement of momentum of high energy projectile fragments. However, the transverse momentum can indirectly be estimated using the fact that the fragments have nearly the same momentum per nucleon as that of the projectile. Thus the transverse momentum of a fragment of charge $Z$ can be calculated using the relation (15)

$$P_t = A_F P_o \sin \Theta, \quad (4.5)$$

where $P_o$ is the momentum per nucleon of the projectile, $A_F$ is the mass number of the fragment and $\Theta$ is the angle of emission of the
fragment. For fragments with \( Z \leq 2 \) the above relation gives a reliable estimate of the transverse momentum. However, due to the general excess of neutron-rich isotopes, it gives a lower limit for the transverse momentum of fragments with \( Z \geq 3 \).

Table 4.8 gives the average transverse momentum \( \langle P_t \rangle \) of fragment with different charges in collision of \(^{12}\text{C}\) with different target groups. We note that \( \langle P_t \rangle \) increases with the mass of the target. Figures 4.10 and 4.11 show the transverse momentum distributions of fragments with charge \( Z \geq 2 \). For \( Z = 2 \) fragments, these distributions are plotted for different target groups. Each of these distributions is fitted with a curve of the type

\[
N(P_t) = A P_t \exp\left(-\frac{P_t^2}{2\sigma_t^2}\right),
\]  

with \( \sigma = (2/\pi)^{1/2} \langle P_t \rangle \) for \( P_t \leq 500 \text{ MeV}/c \). The curve is expected if each component of transverse momentum, \( P_x \) and \( P_y \), follows a Gaussian distribution \( (23,24) \)

\[
N(P) = A \exp\left(-\frac{P^2}{2\sigma^2}\right).
\]  

It should be mentioned here that if we include fragments with \( P_t > 500 \text{ MeV}/c \), the \( P_t \) distribution cannot be fitted with the curve given by equation (4.6).

The observed values of \( \sigma(P) \) can be related to the nuclear Fermi momentum, \( P_f \) \( (1/5 \langle P_f^2 \rangle = 1/3 \langle P^2 \rangle) \), assuming the sudden emission of \( \alpha \)-clusters \( (23,24) \). The relation comes out to be
<table>
<thead>
<tr>
<th>Charge of fragments</th>
<th>$^{12}\text{C-H}$</th>
<th>$^{12}\text{C-CNO}$</th>
<th>$^{12}\text{C-AgBr}$</th>
<th>$^{12}\text{C-Em}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = 2$</td>
<td>$0.22 \pm 0.03$</td>
<td>$0.24 \pm 0.03$</td>
<td>$0.27 \pm 0.05$</td>
<td>$0.24 \pm 0.02$</td>
</tr>
<tr>
<td>$Z = 3$</td>
<td>$0.20 \pm 0.08$</td>
<td>$0.37 \pm 0.12$</td>
<td>$0.31 \pm 0.15$</td>
<td>$0.28 \pm 0.06$</td>
</tr>
<tr>
<td>$Z = 4$</td>
<td>$0.21 \pm 0.12$</td>
<td>$0.23 \pm 0.13$</td>
<td>$0.37 \pm 0.27$</td>
<td>$0.25 \pm 0.09$</td>
</tr>
<tr>
<td>$Z = 5$</td>
<td>$0.22 \pm 0.17$</td>
<td>$0.26 \pm 0.12$</td>
<td>$0.31 \pm 0.15$</td>
<td>$0.27 \pm 0.08$</td>
</tr>
<tr>
<td>$Z = 6$</td>
<td>$0.22 \pm 0.19$</td>
<td>$0.27 \pm 0.10$</td>
<td>$0.29 \pm 0.17$</td>
<td>$0.27 \pm 0.08$</td>
</tr>
</tbody>
</table>
Fig. 4.10 The transverse momentum distributions of $Z = 2$ fragments in different target groups with Gaussian curves fitted to the data for $P_t \leq 500$ MeV/c.
Fig. 4.11  The transverse momentum distributions of $Z = 3$, 4 and 5 fragments with Gaussian curves fitted to the data for $P_t \leq 500$ MeV/c.
\[ \sigma^2(p) = \frac{P_f^2 A_F (A_p - A_F)}{5 (A_p - 1)}, \quad (4.8) \]

and if it is assumed that the nucleus comes to thermal equilibrium, \( \sigma(p) \) can also be related to the excitation energy, \( K_T \), through the relation

\[ \sigma^2(p) = K_T \frac{m (A_p - A_F) A_F}{A_p}, \quad (4.9) \]

where \( A_p \) and \( A_F \) are respectively the masses of the projectile and the fragment and \( m \) is the mass of the proton.

Table 4.9 presents values of \( \sigma(p) \) for fragments of different charges along with the values of \( P_f \) and \( K_T \), calculated using relations (4.8) and (4.9). The values of nuclear Fermi momentum obtained in the present experiment are comparable to those obtained in electron scattering experiment of Moniz et al (25). We further notice that the values of the excitation energy \( K_T \) are also comparable to the binding energy per nucleon, indicating that very small energy transfer takes place between the target and fragments during the fragmentation process.

4.6 Azimuthal Correlations

In the last section we studied the transverse momentum distributions of fragments and it was observed that the presence of high \( P_t \) tail distorts the distribution and increases \( \langle P_t \rangle \) of fragments. This may be due to the transverse motion and/or the angular momentum of the fragmenting projectile spectator. These features of the projectile spectator can be detected by studying the azimuthal correlations among
<table>
<thead>
<tr>
<th>Projectile</th>
<th>Fragments</th>
<th>$\sigma(P)$ (Experimental) MeV/c</th>
<th>$\sigma(P)$ (Theoretical) MeV/c</th>
<th>Fermi momentum ($p_f$) MeV/c</th>
<th>Temperature (KT) MeV</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$ (1 A GeV/c)</td>
<td>He</td>
<td>125 ± 3</td>
<td>134</td>
<td>176(221)*</td>
<td>6.2</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Li</td>
<td>122 ± 10</td>
<td>146</td>
<td>151</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Be</td>
<td>131 ± 9</td>
<td>125</td>
<td>187</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>135 ± 9</td>
<td>108</td>
<td>224</td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$ (2.85 A GeV/c)</td>
<td>He</td>
<td>129 ± 1</td>
<td>134</td>
<td>169(221)*</td>
<td>6.7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Li</td>
<td>127 ± 7</td>
<td>146</td>
<td>157</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Be</td>
<td>133 ± 3</td>
<td>125</td>
<td>190</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>134 ± 3</td>
<td>108</td>
<td>222</td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$ (4.5 A GeV/c)</td>
<td>He</td>
<td>179 ± 3</td>
<td>134</td>
<td>235(221)*</td>
<td>12.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Li</td>
<td>192 ± 4</td>
<td>146</td>
<td>237</td>
<td>13.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Be</td>
<td>193 ± 3</td>
<td>125</td>
<td>275</td>
<td>17.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>183 ± 5</td>
<td>108</td>
<td>303</td>
<td>21.42</td>
<td>Present work</td>
</tr>
<tr>
<td>$^{14}\text{N}$ (2.8 A GeV/c)</td>
<td>He</td>
<td>141 ± 6</td>
<td>134</td>
<td>180(226)*</td>
<td>7.4</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Li</td>
<td>212 ± 15</td>
<td>153</td>
<td>247</td>
<td>13.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Be</td>
<td>237 ± 22</td>
<td>152</td>
<td>285</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>174 ± 20</td>
<td>131</td>
<td>221</td>
<td>11.3</td>
<td></td>
</tr>
</tbody>
</table>

* Fermi momentum obtained in electron scattering experiment of Moniz et al (25).
the fragments. For example, a transverse motion/angular momentum of the projectile spectator may lead to non-zero value of coefficient of asymmetry (A)/Collinearity (B). These coefficients are defined as

\[ A = \frac{\int_{\pi/2}^{\pi} \frac{d\sigma}{d\epsilon} d\epsilon}{\int_{0}^{\pi/2} \frac{d\sigma}{d\epsilon} d\epsilon} \]

and

\[ B = \frac{\int_{0}^{\pi/4} \frac{d\sigma}{d\epsilon} d\epsilon - \int_{\pi/4}^{3\pi/4} \frac{d\sigma}{d\epsilon} d\epsilon + \int_{3\pi/4}^{\pi} \frac{d\sigma}{d\epsilon} d\epsilon}{\int_{0}^{\pi} \frac{d\sigma}{d\epsilon} d\epsilon} , \]

where \( \epsilon \) is the angle between the transverse momenta of the fragments. \( \epsilon \) can be calculated using the relation

\[ \epsilon_{ij} = \text{Cosh} \left( \frac{\vec{P}_{ti} \cdot \vec{P}_{tj}}{|\vec{P}_{ti}| \cdot |\vec{P}_{tj}|} \right) , \]

where \( \vec{P}_{ti} \) and \( \vec{P}_{tj} \) are the transverse momenta of the ith and jth particles respectively. In Table 4.10 we present the values of A and B for fragments with \( Z = 2 \) produced in collisions of \( ^{12}\text{C} \) with different target groups and for \( Z \geq 3 \). An examination of the table shows that there are significant, if not large, correlations among the projectile fragments. This indicates that the fragmenting nucleus gets a transverse momentum during the collision.

4.7 Diffraction Dissociation of \( ^{12}\text{C} \) in Nuclear Emulsion

In this section we discuss the diffraction dissociation of relativistic carbon nucleus into three \( ^{4}\text{He} \) fragments \( (^{12}\text{C} \rightarrow 3 \; ^{4}\text{He}) \).
### TABLE 4.10

Values of $A$ and $B$ in various subensembles of events

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C-H}$</td>
<td>$-0.089 \pm 0.017$</td>
<td>$-0.020 \pm 0.008$</td>
</tr>
<tr>
<td>$^{12}\text{C-CNO}$</td>
<td>$-0.066 \pm 0.014$</td>
<td>$-0.066 \pm 0.014$</td>
</tr>
<tr>
<td>$^{12}\text{C-AgBr}$</td>
<td>$-0.102 \pm 0.028$</td>
<td>$-0.068 \pm 0.022$</td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>$-0.085 \pm 0.010$</td>
<td>$-0.047 \pm 0.008$</td>
</tr>
<tr>
<td>$Z \geq 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C-Em}$</td>
<td>$-0.147 \pm 0.030$</td>
<td>$-0.021 \pm 0.011$</td>
</tr>
</tbody>
</table>
at 4.5 A GeV/c. In our sample of 1250 events, 26 belong to this category which is 2.08% of the total inelastic collisions.

We assume that the momentum of each $^4\text{He}$ fragment emitted in a dissociated event is equal to one third of the projectile nucleus momentum $P_o$. Consequently, the transverse momentum of each $^4\text{He}$ fragment is given by

$$q_i = \frac{1}{3} P_o \sin \Theta_i,$$  \hspace{1cm} (4.13)

where $\Theta_i$ is the angle of emission of the fragment. The vector sum of $q_i$ in each event is equal to the transverse momentum transferred to the $^{12}\text{C}$ nucleus in the diffraction dissociation process. Therefore, to a first approximation, the transverse momentum of each $^4\text{He}$ particle inside the $^{12}\text{C}$ nucleus is represented by

$$\vec{q}^* = \vec{q} - \frac{1}{3} \sum_{i=1}^{3} q_i.$$ \hspace{1cm} (4.14)

Figure 4.12 shows the transverse momentum distribution of $^4\text{He}$ particles inside the carbon projectile nucleus, $q^*$. In the case of the diffraction dissociation events, it has been observed that in most of the events, two $^4\text{He}$ fragments are emitted within a very narrow angle about the beam direction, i.e., having low transverse momentum. The third $^4\text{He}$ fragment is emitted at relatively large angle, i.e., it has a relatively large transverse momentum, probably to compensate for the sum of transverse momenta of other two $^4\text{He}$ particles. This indicates that the dissociation of $^{12}\text{C}$ to $^{3}\text{He}$ goes through an intermediate $^8\text{Be}$ state.
Fig. 4.12 The transverse momentum distribution of $^4$He particles inside the carbon projectile nucleus as deduced from $^{12}\text{C} \rightarrow 3^{^4}\text{He}$ dissociation events.
4.8 Summary and Conclusions

From the detailed investigation of the fragmentation of $^{12}\text{C}$-nuclei in nuclear emulsion, the following conclusions can be drawn.

(i) The factorization of cross-section observed in $^0\text{O}$ electronic experiments does not hold in the present emulsion experiment wherein the total cross-sections have been measured.

(ii) The cross-section of the reaction in which the projectile $^{12}\text{C}$-nucleus breaks up into two Li fragments is $1.36\times10^{-2}$ of the total inelastic cross-section.

(iii) The average multiplicities of fragments of all charges is found to increase with the mass of the projectile and the dependence can be well described by the relation of type: $\langle N_z \rangle = \text{Const.} \ A^\alpha$.

(iv) Analysis of angular distribution of projectile fragments indicates that for a given charge, the distribution is independent of the target mass, excepting $Z = 2$ fragments for which a weak target mass dependence is observed. The observed $\sigma$ values are higher than those predicted by Lepore and Reddell (20). Further, the analysis also indicates that the limiting fragmentation observed by Greiner et al (7) for fragments from the fragmentation of $^{12}\text{C}$ nuclei with energies 1.05 and 2.1 A GeV is not valid for fragments from the fragmentations of $^{12}\text{C}$ nuclei with energy 3.7 A GeV.

(v) Analysis of angular distributions of $Z = 2$ fragments in different
channels indicates that no correlation exists between nucleons in $^{12}\text{C}$ projectile. This result is in agreement with Goldhaber model of nuclear fragmentation.

(vi) Presence of large $P_t$ particles distorts the transverse momentum distributions. However, for $P_t \leq 500 \text{ MeV/c}$, the distributions agree with the predictions of the fragmentation model.

(vii) The value of nuclear Fermi momentum calculated from the observed values of $\sigma(P)$ is in agreement with that obtained in electron scattering experiment. Further, the observed excitation energy is of the order of binding energy per nucleon, indicating that little energy transfer takes place during the fragmentation of $^{12}\text{C}$-nuclei.

(viii) The emission of fragments is asymmetrical in the azimuthal plane, which could mean that the fragmenting residual nucleus gets a transverse momentum during the collision process.

(ix) The diffraction dissociation events of $^{12}\text{C} \rightarrow 3\ ^4\text{He}$ are 2.08% of the total inelastic collisions. The study of such events indicates that this reaction goes through an intermediate $^8\text{Be}$ state.
References


2. V.S. Barashenkov and V.D. Toneev: Interactions of high energy particles and atomic nuclei with nuclei; Atonizdat, Moscow, 1972.


