In this chapter, we extend the concept of economic equilibrium to cover the intuitionistic fuzzy sense. Here we deal mainly with demand function and supply function as linear functions and also the case when they are exponential functions.

Here we consider linear demand function in which quantity demanded is a triangular intuitionistic fuzzy number. Using extension principle we find the membership function and non membership function of the IFN price corresponding to the intuitionistic fuzzy demand and then find its centroid. We also consider the linear supply function in which quantity supplied is a triangular intuitionistic fuzzy number. By extension principle, we find the membership function and non membership function of its corresponding price and its centroid. From these centroids we find the point of economic equilibrium. Thus we have extended the concept of economic equilibrium to the intuitionistic fuzzy sense. Also we have extended the concept of consumer surplus and producer surplus to cover the intuitionistic fuzzy sense.
5.1 Introduction

The problem of economic equilibrium, consumer surplus and producer surplus in a fuzzy sense have been discussed by Yao and Wu [88]. Here we shall extend the concepts of economic equilibrium, consumer surplus and producer surplus to the intuitionistic fuzzy context.

Let \( p = f(x) = a - bx \) be the linear demand function and \( p = h(x) = e + gx \) be the linear supply function. In the real situation, for a fixed price \( p \), the demand quantity is not necessarily fixed as \( x = \frac{a - p}{b} \). Corresponding to \( p = a - bx \), we consider the demand function \( p = a - bd \), and make \( d \) vague to \( \tilde{D} \).

Assume \( \tilde{D} \) is an intuitionistic fuzzy number \((x - \Delta_{11}, x, x + \Delta_{12}; x - \Delta'_{11}, x, x + \Delta'_{12})\). That is, the membership grade of the quantity demanded \( d \) when price is \( p \) varies around \( x \) \((= \frac{a - p}{b})\) in the interval \((x - \Delta_{11}, x + \Delta_{12})\), \(0 < \Delta_{11} < x\), \(\Delta_{12} > 0\) and the non membership grade of \( d \) varies around this \( x \) in the interval \((x - \Delta'_{11}, x + \Delta'_{12})\), \(\Delta'_{11} \geq \Delta_{11}, \Delta'_{12} > \Delta_{12}\). Thus we have an intuitionistic fuzzy set \( \tilde{P} = f(\tilde{D}) \), called intuitionistic fuzzy price corresponding to demand \( x \).

Similarly the supply quantity \( x \) is not necessarily equal to \( \frac{p - e}{g} \). Corresponding to \( p = e + gx \), we consider the supply function \( p = e + gs \) and make \( s \) vague to \( \tilde{S} \). Assume \( \tilde{S} \) is an intuitionistic fuzzy number \((x - \Delta_{21}, x, x + \Delta_{22}; x - \Delta'_{21}, x, x + \Delta'_{22})\). Thus, we have and intuitionistic fuzzy set \( \tilde{P} = h(\tilde{S}) \), called the intuitionistic fuzzy price corresponding to supply \( x \).

By the extension principle, we have the membership function and non membership function of \( \tilde{P} = f(\tilde{D}) \) and \( \tilde{P} = h(\tilde{S}) \) and their centroids \( E_1(x) \) and \( E_2(x) \) respectively. \( E_1(x) \) estimates the price in the intuitionistic fuzzy sense when the demand quantity is \( x \) and \( E_2(x) \) estimates the price in the intuitionistic
fuzzy sense when the supply quantity is $x$. From $E_1(x)$ and $E_2(x)$ we can find
the point of economic equilibrium, consumer’s surplus and producer’s surplus.
These are explained in section 5.2. An example to find point of economic equi-
librium, consumer surplus and producer surplus is given in section 5.3. Other
cases are discussed in section 5.4.

5.2 Economic equilibrium in intuitionistic fuzzy

sense

Let the demand function be $p = f(x) = a - bx$, $0 \leq x \leq \frac{a}{b}$ and the supply
function be $p = h(x) = e + gx$, $x \geq 0$, where $a, b, e, g > 0$ are known and
$e < a$.

In reality, for the same price $p$, the daily demand quantity $x$ may not be
always equal to $x (= \frac{a - p}{b})$, the membership grade of the quantity demanded
when price is $p$ may vary around $x (= \frac{a - p}{b})$ and the non membership grade may
also vary around this $x$. Similarly, the daily supply quantity $s$ may not be always
equal to $x (= \frac{p - e}{g})$, the membership grade of the quantity supplied when price
is $p$ may vary around $x (= \frac{p - e}{g})$ and the non membership grade may also vary
around this $x$. Therefore, we are considering them in the intuitionistic fuzzy
sense.

Corresponding to $p = f(x) = a - bx$, we consider the demand function
$p = a - bd$ and for price $p$, let membership grade of the demand quantity $d$
varies around $x (= \frac{a - p}{b})$ in the interval $(x - \Delta_1, x + \Delta_2)$, $0 < \Delta_1 < x$,
$\Delta_2 > 0$ and the non membership grade of $d$ varies around this $x$ in the interval
$(x - \Delta_1', x + \Delta_2')$, $\Delta_1' \geq \Delta_1$, $\Delta_2' \geq \Delta_1$. We make $d$ vague into the IFN $\tilde{D}$
For the point \((x, p)\) on \(p = a - bx, 0 \leq x \leq \frac{a}{b}\), let

\[
\mu_{\tilde{D}}(d) = \begin{cases} 
\frac{d - (x - \Delta_{11})}{\Delta_{11}}, & d \in [x - \Delta_{11}, x] \\
\frac{(x + \Delta_{12}) - d}{\Delta_{12}}, & d \in [x, x + \Delta_{12}] \\
0 & \text{otherwise}
\end{cases}
\]

and

\[
\nu_{\tilde{D}}(d) = \begin{cases} 
x - d, & d \in [x - \Delta'_{11}, x] \\
d - x, & d \in [x, x + \Delta'_{12}] \\
1 & \text{otherwise}
\end{cases}
\]

That is, \(\tilde{D}\) is a TIFN \((x - \Delta_{11}, x, x + \Delta_{12}); (x - \Delta'_{11}, x, x + \Delta'_{12})\).

By extension principle, demand function \(f : X \rightarrow P, p = f(d) = a - bd\) induces a function, \(f : \mathcal{I}\mathcal{F}(X) \rightarrow \mathcal{I}\mathcal{F}(P)\) such that

\[
\mu_{f(\tilde{D})}(p) = \sup_{d/p = a - bd} \mu_{\tilde{D}}(d)
\]

where \(\tilde{D}\) is an intuitionistic fuzzy set defined on \(X\)

\[
\nu_{f(\tilde{D})}(p) = \inf_{d/p = a - bd} \nu_{\tilde{D}}(d).
\]

\[
\mu_{f(\tilde{D})}(p) = \sup_{d/p = a - bd} \mu_{\tilde{D}}(d) = \sup_{d/p = a - bd} \mu_{\tilde{D}}\left(\frac{a - p}{b}\right)
\]
\[
\nu_f(D_\delta)(p) = \inf_{d/p=a-bd} \nu_D(d)
\]

\[
\nu_D\left(\frac{a-p}{b}\right) = \inf_{d/p=a-bd} \nu_D\left(\frac{a-p}{b}\right)
\]

\[
\nu_D\left(\frac{a-p}{b}\right) = \begin{cases} 
\frac{x-[(a-p)/b]}{x-(a-\Delta_{11})}, & x - \Delta_{11} \leq \frac{a-p}{b} \leq x \\
\frac{(x+\Delta_{12})-[(a-p)/b]}{x+\Delta_{12}-x}, & x \leq \frac{a-p}{b} \leq x + \Delta_{12} \\
1, & \text{otherwise}
\end{cases}
\]

\[
\nu_D\left(\frac{a-p}{b}\right) = \begin{cases} 
\frac{a-p-bx+b\Delta_{11}}{b\Delta_{11}}, & -a + bx - b\Delta_{11} \leq -p \leq -a + bx \\
\frac{bx+b\Delta_{12}-a+p}{b\Delta_{12}}, & -a + bx \leq -p \leq -a + bx + b\Delta_{12} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\nu_D\left(\frac{a-p}{b}\right) = \begin{cases} 
\frac{-(a-bx+b\Delta_{12})}{b\Delta_{12}}, & a - bx - b\Delta_{12} \leq p \leq a - bx + b\Delta_{12} \\
0, & \text{otherwise}
\end{cases}
\]
\[ \begin{cases} \frac{p-(a-bx)}{b\Delta_{11}'}, & a - bx + b\Delta_{11}' \geq p \geq a - bx \\ \frac{(a-bx)-p}{b\Delta_{12}'}, & a - bx \geq p \geq a - bx - b\Delta_{12}' \\ 1, & \text{otherwise} \end{cases} \]

\[ \begin{cases} \frac{(a-bx)-p}{b\Delta_{12}'}, & a - bx - b\Delta_{12}' \leq p \leq a - bx \\ \frac{p-(a-bx)}{b\Delta_{11}'}, & a - bx \leq p \leq a - bx + b\Delta_{11}' \end{cases} \]  \hspace{1cm} (5.5)

From (5.3) and (5.5), we have the following proposition.

**Proposition 5.2.1.** The intuitionistic fuzzy price for demand \( x \) where \( 0 \leq x \leq \frac{a}{b} \) is the TIFN

\[ f(\tilde{D}) = (a-bx-b\Delta_{12}, a-bx, a-bx+b\Delta_{11}); (a-bx-b\Delta_{12}', a-bx, a-bx+b\Delta_{11}') \]

**Proposition 5.2.2.** Centroid of the TIFN \( f(\tilde{D}) \) is

\[ E_1(x) = a - bx + \frac{b}{3} \left[ \frac{\Delta_{11}'^2 + \Delta_{12}'^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta_{11}' + \Delta_{12}' + \Delta_{11} - \Delta_{12}} \right] . \]

**Proof.** From Chapter 4, we know that centroid of IFN \((k, l, m); (n, l, p)\) is

\[ \frac{1}{3} \left[ \frac{(p-n) ((l-2p-2n) + (m-k)(k+l+m) + 3(p^2 - n^2))}{p-n+m-k} \right] \]  \hspace{1cm} (5.6)

\[ f(\tilde{D}) = (a-bx-b\Delta_{12}, a-bx, a-bx+b\Delta_{11}); (a-bx-b\Delta_{12}', a-bx, a-bx+b\Delta_{11}') \]
Here let

\[ k = a - bx - b\Delta_{12} \]
\[ l = a - bx \]
\[ m = a - bx + b\Delta_{11} \]
\[ n = a - bx - b\Delta'_{12} \]
\[ p = a - bx + b\Delta'_{11} \]

Then

\[ p - n = b(\Delta'_1 + \Delta'_{12}) \]
\[ 2(p + n) = 4(a - bx) + 2b(\Delta'_1 - \Delta'_{12}) \]
\[ l - 2(p + n) = -3(a - bx) - 2b(\Delta'_1 - \Delta'_{12}) \]
\[ m - k = b(\Delta_{11} + \Delta_{12}) \]
\[ k + l + m = 3(a - bx) + b(\Delta_{11} - \Delta_{12}) \]
\[ p + n = 2(a - bx) + b(\Delta'_1 - \Delta'_{12}) \]

Centroid \( E_1(x) \)

\[
= \frac{1}{3b(\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12})} \times \\
[ b(\Delta'_{11} + \Delta'_{12})[-3(a - bx) - 2b(\Delta'_1 - \Delta'_{12})] \\
+ b(\Delta_{11} + \Delta_{12})[3(a - bx) + b(\Delta_{11} - \Delta_{12})] \\
+ 3b(\Delta'_{11} + \Delta'_{12})[2(a - bx) + b(\Delta'_1 - \Delta'_{12})] ] \\
= \frac{1}{3(\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12})} \times 
\]
\[
\begin{align*}
&\left[ -3(a - bx)(\Delta'_{11} + \Delta'_{12}) - 2b(\Delta'_{11} - \Delta'_{12})(\Delta_{11} + \Delta_{12}) \\
&\quad + 3(a - bx)(\Delta_{11} + \Delta_{12}) + b(\Delta_{11} - \Delta_{12})(\Delta_{11} + \Delta_{12}) \\
&\quad + 6(a - bx)(\Delta'_{11} + \Delta'_{12}) + 3b(\Delta'_{11} - \Delta'_{12})(\Delta'_{11} + \Delta'_{12}) \right] \\
&= \frac{1}{3(\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12})} \times \\
&\left[ 3(a - bx)(\Delta'_{11} + \Delta'_{12}) + 3(a - bx)(\Delta_{11} + \Delta_{12}) \\
&\quad + b(\Delta'_{11}^2 - \Delta'_{12}^2) + b(\Delta_{11}^2 - \Delta_{12}^2) \right] \\
&= a - bx + \frac{b}{3} \left[ \Delta_{11}^2 - \Delta_{12}^2 + \frac{2(\Delta_{11}^2 - \Delta_{12}^2)}{2(\Delta_{11} + \Delta_{12})} \right] \\
&\quad \frac{2(\Delta_{11}^2 - \Delta_{12}^2)}{2(\Delta_{11} + \Delta_{12})}
\end{align*}
\]

(5.7)

\(E_1(x)\) estimates the price in the intuitionistic fuzzy sense, when demand quantity is \(x\).

\[\square\]

**Remark.** Price in intuitionistic fuzzy sense is a generalization of price in fuzzy sense.

**Proof.** If \(\nu_{D}(d) = 1 - \mu_{D}(d)\), then \(\Delta'_{11} = \Delta_{11}\) and \(\Delta'_{12} = \Delta_{12}\), then by (5.7)

\[E_1(x) = a - bx + \frac{b}{3} \left[ \Delta_{11}^2 - \Delta_{12}^2 + \frac{2(\Delta_{11}^2 - \Delta_{12}^2)}{2(\Delta_{11} + \Delta_{12})} \right]
\]

which estimates the price in the fuzzy sense.

**Remark.** If \(\Delta_{11} = \Delta_{12}\), the estimate \(E_1(x) = a - bx\) coincides with the price in the crisp case.

Corresponding to \(p = h(x) = e + gx\), we consider the supply function
\( p = e + gs \) and for price \( p \), let the membership grade of the supply quantity \( s \) vary around \( x = \frac{p - e}{g} \) in the interval \( (x - \Delta_{21}, x + \Delta_{22}) \), \( 0 < \Delta_{21} < x, \Delta_{22} > 0 \) and the non membership grade of \( s \) vary around \( x \) in the interval \( (x - \Delta'_{21}, x + \Delta'_{22}) \), \( \Delta'_{21} \geq \Delta_{21}, \Delta'_{22} \geq \Delta_{22} \). We make \( s \) vague into the IFS \( \tilde{S} \).

For the point \( (x, p) \) on \( p = e + gx, x > 0 \), let

\[
\mu_{\tilde{S}}(s) = \begin{cases} 
\frac{s-(x-\Delta_{21})}{\Delta_{21}}, & s \in [x - \Delta_{21}, x] \\
\frac{x+\Delta_{22}-s}{\Delta_{22}}, & s \in [x, x + \Delta_{22}] \\
0, & \text{elsewhere}
\end{cases}
\]

and

\[
\nu_{\tilde{S}}(s) = \begin{cases} 
\frac{x-s}{\Delta'_{21}}, & s \in [x - \Delta'_{21}, x] \\
\frac{s-x}{\Delta'_{22}}, & s \in [x, x + \Delta'_{22}] \\
1, & \text{elsewhere}
\end{cases}
\]

That is, \( \tilde{S} \) is the TIFN \((x - \Delta_{21}, x, x + \Delta_{22}); (x - \Delta'_{21}, x, x + \Delta'_{22})\).

By extension principle, supply function \( h : X \rightarrow P, p = h(s) = e + gs \) induces a function \( h : \mathcal{IF}(X) \rightarrow \mathcal{IF}(P) \) such that

\[
\mu_{h(\tilde{S})}(p) = \sup_{s/p=e+gs} \mu_{\tilde{S}}(s)
\]

where \( \tilde{S} \) is an IFS defined on \( X \) and

\[
\nu_{h(\tilde{S})}(p) = \inf_{s/p=e+gs} \nu_{\tilde{S}}(s)
\]
\[
\mu_{h(S)}(p) = \sup_{s/p = e + gs} \mu_S(s) \\
= \sup_{s/p = e + gs} \mu_S\left(\frac{p-e}{g}\right) \\
= (e + gx - g\Delta_{21}, e + gx, e + gx + g\Delta_{22}) \quad (5.8)
\]

\[
\nu_{h(S)}(p) = \inf_{s/p = e + gs} \nu_S(s) \\
= \inf_{s/p = e + gs} \nu_S\left(\frac{p-e}{g}\right) \\
= \begin{cases} 
\frac{e+gx-p}{g\Delta_{21}}, & e + gx - g\Delta'_{21} \leq p \leq e + gx \\
\frac{p-(e+gx)}{g\Delta_{22}}, & e + gx \leq p \leq e + gx + g\Delta'_{22} \\
1 & \text{elsewhere}
\end{cases} \quad (5.9)
\]

From (5.8) and (5.9), we have the following proposition.

**Proposition 5.2.3.** The intuitionistic fuzzy price for supply \( x \) where \( x > 0 \) is the TIFN

\[
h(\tilde{S}) = (e + gx - g\Delta_{21}, e + gx, e + gx + g\Delta_{22});
\]

\[
(e + gx - g\Delta'_{21}, e + gx, e + gx + g\Delta'_{22})
\]

**Proposition 5.2.4.** Centroid of TIFN \( h(\tilde{S}) \) is

\[
E_2(x) = e + gx + \frac{g}{3} \left[ \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right] \quad x > 0
\]
Proof.

\[ h(\tilde{S}) = (e + gx - g\Delta_{21}, e + gx, e + gx + g\Delta_{22}); \]

\[ (e + gx - g\Delta'_{21}, e + gx, e + gx + g\Delta'_{22}). \]

Here let \( k = e + gx - g\Delta_{21}, l = e + yx, \)

\[ m = e + gx + g\Delta_{22}, \quad n = e + gx + g\Delta'_{21} \quad \text{and} \quad p = e + gx + g\Delta'_{22} \]

Using (5.6), we get centroid \( E_2(x) \)

\[
= \frac{1}{3(\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21})} \times \\
\left[ -3(e - gx)(\Delta'_{22} + \Delta'_{21}) - 2g(\Delta'_{22} - \Delta'_{21})(\Delta'_{22} + \Delta'_{21}) \\
+ 3(e + gx)(\Delta_{22} + \Delta_{21}) + g(\Delta_{22} - \Delta_{21})(\Delta_{22} + \Delta_{21}) \\
+ 6(e + gx)(\Delta'_{22} + \Delta'_{21}) + 3g(\Delta'_{22} - \Delta'_{21})(\Delta'_{22} + \Delta'_{21}) \right] \\
= 3(e + gx)(\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}) \frac{g(\Delta'_{22} - \Delta'_{21})(\Delta'_{22} + \Delta'_{21})}{3(\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21})} \\
= e + gx + \frac{g}{3} \left( \frac{\Delta'_{22} - \Delta'_{21} \Delta_{22} - \Delta_{21}}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \\
= e + gx + \frac{g}{3} \left( \frac{\Delta_{22}^2 - \Delta_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22}^2 + \Delta_{21}^2 + \Delta_{22}^2 + \Delta_{21}^2} \right). \quad (5.10)
\]

\( E_2(x) \) estimates the price in the intuitionistic fuzzy sense when the supply quantity is \( x \).

Remark. If \( \nu_S(s) = 1 - \mu_S(s) \), then \( \Delta'_{21} = \Delta_{21} \) and \( \Delta'_{22} = \Delta_{22} \).
Then

\[ E_2(x) = e + gx + \frac{g}{3} \left( \frac{\Delta_{22}^2 - \Delta_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22} + \Delta_{21} + \Delta_{22} + \Delta_{21}} \right) \]

\[ = e + gx + \frac{g}{3}(\Delta_{22} - \Delta_{21}) \]

which estimates the price in the fuzzy sense.

**Remark.** If \( \Delta_{21} = \Delta_{22} \), the estimate \( E_2(x) = e + gx \) coincides with the price \( p = e + gx \) in crisp case.

Now we proceed to find the point of equilibrium in the intuitionistic fuzzy sense.

**To find the point of equilibrium \((x_*, p_*)\) in the intuitionistic fuzzy sense**

\[ E_1(x) = E_2(x) \]

\[ a - bx + \frac{b}{3} \left( \frac{\Delta'_{12}^2 - \Delta'_{11}^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta'_{12} + \Delta'_{11} + \Delta_{12} + \Delta_{11}} \right) \]

\[ = e + gx + \frac{g}{3} \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \]

Solving for \( x \), we get

\[ x_* = \frac{1}{b + g} \left[ a - e - \frac{b}{3} \left( \frac{\Delta'_{12}^2 - \Delta'_{11}^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta'_{12} + \Delta'_{11} + \Delta_{12} + \Delta_{11}} \right) \right] \]

\[ - \frac{g}{3} \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \]

if \( 0 < x_* \leq \frac{a}{b} \) \hspace{1cm} (5.11)

and \( p_* = E_1(x_*) \)

\[ = a - bx_* - \frac{b}{3} \left( \frac{\Delta'_{12}^2 - \Delta'_{11}^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta'_{12} + \Delta'_{11} + \Delta_{12} + \Delta_{11}} \right) \]

(5.12)
\[ E_2(x_*) = e + gx_* + \frac{g}{3} \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \] (5.13)

Thus we have the following proposition.

**Proposition 5.2.5.** The point of equilibrium \((x_*, p_*)\) in the intuitionistic fuzzy sense is given by

\[
x_* = \frac{1}{b + g} \left[ a - e - b \left( \frac{\Delta_{12}^2 - \Delta_{11}^2}{3} \right) - \frac{g}{3} \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \right]
\]

if \(0 < x_* \leq \frac{a}{b}\)

and \(p_* = E_1(x_*) = E_2(x_*)\).

**Remark.** If \(\Delta'_{12} = \Delta_{12}, \Delta'_{11} = \Delta_{11}, \Delta'_{22} = \Delta_{22}, \Delta'_{21} = \Delta_{21}\) in (5.11), (5.12) and (5.13), we get

\[
x_* = \frac{1}{b + g} \left[ a - e - b \left( \frac{\Delta_{12}^2 - \Delta_{11}^2}{3} - \frac{g}{3} (\Delta_{22} - \Delta_{21}) \right) \right] \quad (5.14)
\]

\[
p_* = E_1(x_*) = a - bx_* - \frac{b}{3} (\Delta_{12} - \Delta_{11}) \quad (5.15)
\]

\[
p_* = E_2(x_*) = e + gx_* + \frac{g}{3} (\Delta_{22} - \Delta_{21}) \text{ if } 0 < x_* \leq \frac{a}{b}, \quad (5.16)
\]

which are same as \((x_*, p_*)\) in the fuzzy sense.

**Remark.** If, in addition, we take \(\Delta_{12} = \Delta_{11}\) and \(\Delta_{22} = \Delta_{21}\), then \(x_* = \frac{a - e}{b + g}\) and \(p_* = a - bx_* = e + gx_*\). \((x_*, p_*)\) is the point of equilibrium in the crisp case.

Hence point of equilibrium \((x_*, p_*)\) in the IF sense is a generalization of \((x_*, p_*)\) in the fuzzy sense.
To find consumer surplus and producer surplus in the intuitionistic fuzzy sense

\[ \text{Producer surplus} \]

\[ \text{Consumer surplus} \]

Figure 5.1: Consumer surplus and producer surplus in intuitionistic fuzzy sense

Consumer surplus in the intuitionistic fuzzy sense

\[
\begin{align*}
\text{Consumer surplus} &= \int_0^{x^*} (E_1(x) - p_*) \, dx \\
&= \int_0^{x^*} \left[ a - bx + \frac{b}{3} \left( \frac{\Delta'_{11}^2 - \Delta'_{12}^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12}} \right) - p_* \right] \, dx \\
&= \left[ ax - \frac{bx^2}{2} + \frac{bx}{3} \left( \frac{\Delta'_{11}^2 - \Delta'_{12}^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12}} \right) - p_* x^* \right]_0^{x^*} \\
&= ax_* - \frac{bx_*^2}{2} + \frac{bx_*}{3} \left( \frac{\Delta'_{11}^2 - \Delta'_{12}^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12}} \right) - p_* x_* \\
&= ax_* - \frac{bx_*^2}{2} + \frac{bx_*}{3} \left( \frac{\Delta'_{11}^2 - \Delta'_{12}^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12}} \right) \\
&\quad - \left[ ax_* - bx_*^2 + \frac{b}{3} x_* \left( \frac{\Delta'_{11}^2 - \Delta'_{12}^2 + \Delta_{11}^2 - \Delta_{12}^2}{\Delta'_{11} + \Delta'_{12} + \Delta_{11} + \Delta_{12}} \right) \right] \\
&= \frac{1}{2} bx_*^2
\end{align*}
\]

(5.17)

where \( x_* \) and \( p_* \) are same as in (5.11) and (5.12).
Producer surplus in the intuitionistic fuzzy sense

\[
= \int_0^{x_*} [p_* - E_2(x)] dx \\
= p_* x_* - e x_* - \frac{g x_*^2}{2} - \frac{g}{3} x_* \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22} + \Delta_{21} + \Delta_{22} + \Delta_{21}} \right) \\
= e x_* + \frac{g x_*^2}{2} + \frac{g}{3} x_* \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22} + \Delta_{21} + \Delta_{22} + \Delta_{21}} \right) \\
- e x_* - \frac{g x_*^2}{2} - \frac{g}{3} x_* \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22} + \Delta_{21} + \Delta_{22} + \Delta_{21}} \right) \\
= \frac{1}{2} g x_*^2
\] (5.18)

where \( x_* \) and \( p_* \) are as in (5.11) and (5.13).

**Remark.** Consumer surplus in the fuzzy sense

\[
= \int_0^{x_*} [E_1(x) - p_*] dx \\
= \int_0^{x_*} [a - b x - \frac{1}{3} b (\Delta_{12} - \Delta_{11}) - p_*] dx \\
= \frac{1}{2} b x_*^2
\] (5.19)

where \( x_* \) is given by (5.14).

Producer surplus in fuzzy sense

\[
= \int_0^{x_*} [p_* - E_2(x)] dx \\
= p_* x_* - e x_* - \frac{1}{2} g x_*^2 - \frac{1}{3} g (\Delta_{22} - \Delta_{21}) x_* \\
= \frac{1}{2} g x_*^2
\] (5.20)

where \( x_* \) is given by (5.14).
Remark. By putting $\Delta'_{12} = \Delta_{12}$, $\Delta'_{11} = \Delta_{11}$, $\Delta'_{22} = \Delta_{22}$ and $\Delta'_{21} = \Delta_{21}$ in (5.17) and (5.18), we get consumer surplus and producer surplus in the fuzzy sense. Hence consumer surplus and producer surplus in the intuitionistic fuzzy sense are generalizations of that in the fuzzy sense. If, in addition, we put $\Delta_{12} = \Delta_{11}$, $\Delta_{22} = \Delta_{21}$, then we get consumer surplus and producer surplus in the crisp sense.

5.3 Example

Let the demand function be $p = 22 - 2x, 0 \leq x < 11$ and supply function be $p = 10 + 2x, x \geq 0$.

$a = 22, b = 2, e = 10, g = 2, e < a$ is true.

Crisp sense

$22 - 2x_0 = 10 + 2x_0$.

$x_0 = 3 \in (0, 11)$ and $p_0 = 16$.

Consumer surplus in the crisp sense $= \frac{1}{2}bx_0^2 = 9$

Producer surplus in the crisp sense $= \frac{1}{2}gx_0^2 = 9$

Fuzzy sense

Case 1. If $\Delta_{11} = \Delta_{12}$ and $\Delta_{21} = \Delta_{22}$, $x_* = \frac{a-x}{b+g} = 3$ by (5.14).

$p_* = 16$ by (5.15).

Hence, $x_*, p_*$, consumer surplus and producer surplus are same as in crisp sense.
Case 2. Let \( \Delta_{12} - \Delta_{11} = 1, \Delta_{22} - \Delta_{21} = -1 \)

\[
x_* = \frac{1}{b + g} \left( a - e + \frac{1}{3} (g - b) \right) \text{ by (5.14)}
\]

\[
= 3
\]

\[
p_* = E_1(x_*) = 15.3333 \text{ by (5.15)}
\]

Consumer surplus in the crisp sense = \( \frac{1}{2}bx_0^2 = 9 \)

Producer surplus in the crisp sense = \( \frac{1}{2}gx_0^2 = 9 \)

Case 3. Let \( \Delta_{12} - \Delta_{11} = 1, \Delta_{22} - \Delta_{21} = 1 \).

\[
x_* = \frac{1}{4} \left[ 12 - \frac{2}{3} (\Delta_{12} - \Delta_{11}) - \frac{2}{3} (\Delta_{22} - \Delta_{21}) \right] = 2.667 \in (0, 11) \text{ by (5.14).}
\]

\[
p_* = E_1(x_*) = 22 - 2 \times 2.667 - \frac{1}{3} \times 2 \times 1 = 15.999.
\]

Hence by (5.19) and (5.20).

Consumer surplus in the fuzzy sense = \( \frac{1}{2} \times 2 \times 2.667^2 = 7.113 \)

Producer surplus in the fuzzy sense = \( \frac{1}{2} \times 2 \times 2.667^2 = 7.113 \)

Case 4. Let \( \Delta_{12} - \Delta_{11} = -1, \Delta_{22} - \Delta_{21} = -1 \).

By (5.14) and (5.15),

\[
x_* = \frac{1}{4} \left[ 12 - \frac{2}{3} \times -1 - \frac{2}{3} \times -1 \right]
\]

\[
= 3.333 \in (0, 11)
\]

\[
p_* = 22 - 2 \times 3.333 - \frac{1}{3} \times 2 \times -1 = 16.001
\]

By (5.19) and (5.20).

Consumer surplus in the fuzzy sense = \( \frac{1}{2} \times 3 \times 3.333^2 = 11.109 \)

Producer surplus in the fuzzy sense = \( \frac{1}{2} \times 3 \times 3.333^2 = 11.109 \)
Intuitionistic fuzzy sense

Let $\Delta_{11} = 0.3, \Delta_{12} = 1.3, \Delta'_{11} = 0.5, \Delta'_{12} = 1.8$
$\Delta_{21} = 0.2, \Delta_{22} = 1.2, \Delta'_{21} = 0.5, \Delta'_{22} = 1.6$
Therefore $\Delta_{12} - \Delta_{11} = 1, \Delta_{22} - \Delta_{21} = 1, \Delta'_{12} - \Delta'_{11} = 1.3, \Delta'_{22} - \Delta'_{21} = 1.1,$
$\Delta_{12} + \Delta_{11} = 1.6, \Delta_{22} + \Delta_{21} = 1.4, \Delta'_{12} + \Delta'_{11} = 1.3, \Delta'_{22} + \Delta'_{21} = 2.1$
$x_* = 2.62718 \in (0, 11)$ by (5.11)
$p_* = E_1(x_*) = 15.961025$ by (5.12)
Consumer surplus in intuitionistic fuzzy sense $= \frac{1}{2}bx_*^2 = \frac{1}{2} \times 2 \times 2.62718^2 = 6.902075$
Producer surplus in intuitionistic fuzzy sense $= \frac{1}{2}gx_*^2 = \frac{1}{2} \times 2 \times 2.62718^2 = 6.902075$
Here is a comparison of the proposed method with the method already in the literature.

<table>
<thead>
<tr>
<th></th>
<th>Crisp sense</th>
<th>Fuzzy sense</th>
<th>IF sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_*$</td>
<td>3</td>
<td>2.667</td>
<td>2.62718</td>
</tr>
<tr>
<td>$p_*$</td>
<td>16</td>
<td>15.999</td>
<td>15.961025</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>9</td>
<td>7.113</td>
<td>6.902075</td>
</tr>
<tr>
<td>Producer surplus</td>
<td>9</td>
<td>7.113</td>
<td>6.902075</td>
</tr>
</tbody>
</table>

5.4 Discussion on other cases

Case 1

Demand function is $p = a_0^{-bx}, x > 0, a_0 > 1, b_0 > 0$ and supply function is $p = c_0e^{dx}, x > 0, c_0 > 1, d_0 > 0$.

Consider the demand function $p = a_0e^{-bx}, x > 0, a_0 > 1, b_0 > 0$. 

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Then \( \ln p = \ln a_0 - b_0 x \).

Let \( \ln p = p_1 \) and \( \ln a_0 = a_1 \).

Then the above equation becomes

\[
p_1 = a_1 - b_0 x
\]

which is a linear demand function where \( x > 0, a_1 > 0, b_0 > 0 \).

Also if the supply function is \( p = c_0 e^{d_0 x}, x > 0, c_0 > 1, d_0 > 0 \) then

\[
\ln p = \ln c_0 + d_0 x.
\]

Let \( \ln p = p_1 \) and \( \ln c_0 = c_1 \), then the equation becomes \( p_1 = c_1 + d_0 x \) which is a linear supply function, \( x > 0, c_1 > 0, d_0 > 0 \).

Then by (5.11), (5.17) and (5.18) in section 5.2,

\[
x_\ast = \frac{1}{b_0 + d_0} \left\{ (a_1 - c_1) - \frac{b_0}{3} \left( \frac{\Delta'_{12}^2 - \Delta'_{11}^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta'_{12} + \Delta'_{11} + \Delta_{12} + \Delta_{11}} \right) 
    - \frac{d_0}{3} \left( \frac{\Delta'_{22}^2 - \Delta'_{21}^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta'_{22} + \Delta'_{21} + \Delta_{22} + \Delta_{21}} \right) \right\}
\]

Consumer surplus in intuitionistic fuzzy sense

\[
= \frac{1}{2} b_0 x_\ast^2
\]

and producer surplus in intuitionistic fuzzy sense

\[
= \frac{1}{2} d_0 x_\ast^2.
\]

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The point of equilibrium is \((x_*, p_*)\) where \(p_* = E_1(x_*) = E_2(x_*)\).

If \(\Delta'_1 = \Delta_{12}, \Delta'_1 = \Delta_{11}, \Delta'_2 = \Delta_{22}, \Delta'_2 = \Delta_{21}\), then

\[
x_* = \frac{1}{b_0 + d_0} \{\ln a_0 - \ln c_0 - \frac{b_0}{3} (\Delta_{12} - \Delta_{11}) - \frac{d_0}{3} (\Delta_{22} - \Delta_{21})\}
\]

which is the same as \(x_*\) in the fuzzy sense. If \(\Delta_{11} = \Delta_{12}\) and \(\Delta_{22} = \Delta_{21}\), we get the results in the crisp sense.

**Case 2**

Demand function is \(p = a_0b_0^{-x}, x > 0, a_0 > 1, b_0 > 1\).

Supply function is \(p = c_0d_0^x, x > 0, c_0 > 1, d_0 > 1\).

Consider the demand function \(p = a_0b_0^{-x}, x > 0, a_0 > 1, b_0 > 1\).

Then \(\ln p = \ln a_0 - x \ln b_0\).

Let \(\ln p = p_2, \ln a_0 = a_2, \ln b_0 = b_2\). Then we have a linear demand function.

\(p_2 = a_2 - b_2x, x > 0, a_2 > 0, b_2 > 0\). If the supply function is \(p = c_0d_0^x, c_0 > 1, d_0 > 1\) then \(\ln p = \ln c_0 + x \ln d_0\).

Let \(\ln p = p_2, \ln c_0 = c_2, \ln d_0 = d_2\).

Then we have a linear supply function

\(p_2 = c_2 + d_2x, c_2 > 0, d_2 > 0\).
By (5.11), (5.17) and (5.18) in section 5.2, we have

\[ x^{**} = \frac{1}{b_2 + d_2} \left\{ (a_2 - c_2) - \frac{b_2}{3} \left( \frac{\Delta_{12}'^2 - \Delta_{11}'^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta_{12}' + \Delta_{11}' + \Delta_{12} + \Delta_{11}} \right) \\
- \frac{d_2}{3} \left( \frac{\Delta_{22}'^2 - \Delta_{21}'^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22}' + \Delta_{21}' + \Delta_{22} + \Delta_{21}} \right) \right\} \]

\[ = \frac{1}{\ln b_0 d_0} \left\{ \ln a_0 - \ln c_0 - \frac{\ln b_0}{3} \left( \frac{\Delta_{12}'^2 - \Delta_{11}'^2 + \Delta_{12}^2 - \Delta_{11}^2}{\Delta_{12}' + \Delta_{11}' + \Delta_{12} + \Delta_{11}} \right) \\
- \frac{\ln d_0}{3} \left( \frac{\Delta_{22}'^2 - \Delta_{21}'^2 + \Delta_{22}^2 - \Delta_{21}^2}{\Delta_{22}' + \Delta_{21}' + \Delta_{22} + \Delta_{21}} \right) \right\}. \]

Consumer surplus in intuitionistic fuzzy sense \(= \frac{1}{2} b_2 x^{**} = \frac{1}{2} \ln b_0 \times x^{**} \)

Producer surplus in intuitionistic fuzzy sense \(= \frac{1}{2} d_2 x^{**} = \frac{1}{2} \ln d_0 \times x^{**} \).

Point of equilibrium is \((x^{**}, P^{**})\) where

\[ P^{**} = E_1(x^{**}) = E_2(x^{**}). \]

If \(\Delta_{12}' = \Delta_{12}, \Delta_{11}' = \Delta_{11}, \Delta_{22}' = \Delta_{22}, \Delta_{21}' = \Delta_{21},\) then

\[ x^{**} = \frac{1}{\ln b_0 d_0} \left\{ \ln a_0 - \ln c_0 - \frac{\ln b_0}{3} (\Delta_{12} - \Delta_{11}) - \frac{\ln d_0}{3} (\Delta_{22} - \Delta_{21}) \right\} \]

which is \(x^{**}\) in the fuzzy sense and if in addition, \(\Delta_{12} = \Delta_{11}\) and \(\Delta_{22} = \Delta_{21},\) we get \(x^{**}\) in the crisp sense.

Hence we have extended the concepts of economic equilibrium, consumer surplus and producer surplus in intuitionistic fuzzy sense.