Chapter 1. Introduction
1.1 Inventory Management

Inventory is any stock of goods or resources maintained by an organization for the smooth and efficient running of its business. An inventory system defines a set of policies and controls that monitor the level of inventory and determines how much to order/manufacture at a time and when to order/manufacture. The basic aim of any inventory analysis is to determine the optimum policy that would maximize the organization’s expected profit or minimize its expected loss over the period of its implementation.

Most businesses hold inventory for the following reasons:

a) To maintain independence of operations.
b) To meet variation in product demand.
c) To allow flexibility in production scheduling.
d) To provide a safeguard for variation in raw material or finished products delivery time.
e) To take advantage of economic purchase order size.
f) To hedge against price increase.
g) To take the advantages of price discount.
h) To minimize losses through deterioration, pilferage, wastages and damages.
i) To ensure right quality goods at reasonable prices.
j) To avoid both over-stocking and under-stocking of inventory.
k) To remove repetition in ordering or replenishing stocks.

An inventory evaluation allows a company to provide a monetary value for items that make up its inventory. Inventory is the main asset of a business, and suitable measurement of it is necessary to assure true financial statements. If the inventory is not appropriately measured, an organization could make poor business decisions.

The designing of inventory models depends heavily on the number of different items being stocked, the type of demand, the cost structure, inflation rate and physical characteristics of the items under consideration. Many items deteriorate or decay with time or have an expiry date, and one has to take this into account while developing the model.
In a real life situation, inventory control problems usually involve multiple products. However, it is often true that single product models are able to capture the essential elements of the problem, so that it is not necessary to explicitly comprise the relations of different items into the formulation. Further, multiple-product models are often too involved to be of much use when the number of products being stocked is very large. For this reason, single-product models govern the literature, and are used most commonly in practice. In this thesis, we have, therefore, restricted our attention to models involving a single product. The demand for the product may be deterministic or stochastic, it may be completely predictable or predictable under some probabilistic distribution only, its probability distribution may be even unknown. Further, demand may be stationary or non-stationary, and may depend on economic factors that vary randomly over time.

1.2 Effect of inflation on Inventory Management

As inventory represents an important part of the company’s financial assets, it is very much affected by the market’s response to various situations, especially inflation. Inflation is a global phenomenon in present days. Inflation means that the purchasing power of money decreases in an economy. It creates a number of uncertainties because of rising prices of raw materials, semi-finished and finished goods. The problem with predicting the effects of inflation on a business’s inventory is that there are always two distinct factors to consider. First is the actual effect of inflation, which decreases buying power and the second is the effect that the expectations of inflation can have on the spending of both consumers and businesses.

An inventory turnover ratio reveals how effectively an organization stocks and sells products. If a business anticipates inflation, it might improve its inventory while prices are low. Its inventory will then be larger than normal, which means its inventory ratio will decrease. A business might increase its inventory in this manner for two reasons. First, if it buys inventory while prevailing prices are low and then waits to sell the inventory when prices are high, it increases its profit margin. Second, buying a large amount of stock at once keeps ordering costs low.

On the other hand, inflation might increase inventory turnover ratios if customers expect an inflationary trend to continue. Theoretically, as prices increase due to inflation,
demand should decrease in response. There is always an effect of inflation on economic order quantity.

1.3 Different cost associated with inventory systems

The various costs in inventory models can be categorized as follows: ordering or setup cost, holding or carrying cost, procurement cost, shortage cost, and deterioration cost.

1.3.1 Ordering Cost

Ordering cost is the cost associated with placing an order to the supplier, while the set-up cost is associated with setting up the production process when units are procured through manufacturing. Apart from placing orders, the other costs related to ordering also form part of ordering cost. It includes postage, the expense to write up or phone in an order, the cost to check the order when it is received, and other fixed labor and transportation expenses. The ordering cost may or may not depend on the amount ordered. The ordering cost changes from time to time due to inflation in an economy.

1.3.2 Holding Cost

The cost of carrying inventory is an important component of the total expected profit or cost. This cost is what a business incurs to hold and store its inventory. It is often described as a percentage of the value of inventory. This percentage can include taxes, employee costs, depreciation, insurance, and the cost of insuring and replacing items. There are four main components to the holding cost of inventory - capital cost, storage space cost, inventory service cost, and inventory risk cost. In the retail industry the risk is much higher as finished items may be seasonally specific. The cost of carrying inventory goes up as a result of increase in the interest rates due to inflation.

1.3.3 Deteriorating Cost

An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Deterioration or decay of items is a common phenomenon that is observed for many products like volatile liquids, agricultural items, blood, drugs, films, electrical components, etc. These items undergo deterioration through evaporation, spoilage, dryness etc. during their normal storage period. As such, it is essential to take into account the loss due to deterioration while
developing inventory policies. For exponentially deteriorating items the deterioration rate is, however, constant. The deterioration cost is also affected by inflation in economy.

1.3.4 Procurement/Manufacturing Costs

Procurement or manufacturing costs represent the cost of the items placed in inventory. If the item is obtained from an outside supplier, the procurement cost is the purchase cost per unit together with any shipping costs paid on a per unit basis. In some cases, a vendor may offer quantity discounts that enable the buyer to pay a reduced cost per unit if the amount purchased is above certain thresholds. In these instances, the order quantity plays an important role in determining the procurement cost. If the item is manufactured in-house, the procurement cost represents the incremental production cost per unit. Note that the procurement cost in this case does not include the production setup cost. The purchase cost is highly dependent on inflation rate.

1.3.5 Shortage Cost

The occurrence of shortage is a common phenomenon faced by the inventory manager. When the stock runs out, the arriving customer may respond in one of following two ways. The customer may wait until the next replenishment arrives and fulfills his demand, or he may leave. In the former case, there is a cost associated with waiting that is proportional to the waiting time of the customer. This is called the backorder cost. In the latter case where a sale is lost, the corresponding loss is referred to as the lost sales cost. Generally, inventory models assume all customers to react in the same way. However, this is not the case in real life. During stock-out, a fraction of the arriving customers may choose to backlog their demands while the remaining customers may leave. The willingness of a customer to wait is likely to depend on the waiting time. In fact, it is prone to decline with the length of waiting time. To capture this situation, the backlogging rate should be taken as a function of time.

1.4 Inventory management Practice

A company carries out purchasing, production and marketing activities independently, and recent research shows that inventory cost accounts for almost 30% of the total capital cost in this regard. Thus, inventory can be considered as a capital investment of
the organization, and successful inventory management is an indication of competition victory and a well run organization.

There are two broad classes of inventory policies – continuous review policy and periodic review policy. In the former case, the inventory level is usually monitored, and whenever it reaches a certain low level, called re-order level, a fixed quantity is ordered. In periodic review model the inventory level is checked at fixed points of time on the planning period called the reorder points. The quantity ordered at any re-order point is just sufficient to bring the stock height to a certain maximum level. Another policy based on Japanese management philosophy is the just in time policy which focuses on providing customers with stocks at the right time and with the right stock quality and quantity. It aims at reducing in-process inventory and carrying costs and maximizing profit at the same time. It was developed as a way for the company to meet its customers’ demands on time and with minimum time, resource, and material wastes.

In this thesis, we have studied periodic review inventory models involving a single product. Inventory models are often distinguished by the assumptions made about different aspects of the timing and logistics of the model. Examples of these may be as follows:

a) The lead-time is often zero, but can also be of a fixed or random length.

b) Back-ordering assumptions may be specified about the way the customers react when demand exceeds supply. The most common assumption in this regard is that all excess demand is backlogged; the other intense assumption is that all excess demand is lost. Combination of both “backlogging” and the “lost sales” cases has been explored.

c) Stock levels are reviewed periodically, and are assumed to be known exactly or approximately.

d) The quality of stored items, usually constant, is allowed to change over time. Here one may distinguish between continuously deteriorating items and items with a fixed or random lifetime. Furthermore, the quality of incoming goods may be inconsistent due to the presence of random numbers of defective items.

e) Different forms of ordering, such as emergency orders, as well as limited capacities of the resources used in production, can be considered.
1.5 Inflation rate and Price

Inflation is a persistent rise over time in the average level of prices in the economy. Prices tend to go up when demand for goods and services exceeds the economy’s capacity to supply those goods and services. The most commonly used measure of inflation is the total consumer price index (CPI). It reflects changes in the average price of a representative “basket” of goods and services such as food, housing, transportation, furniture, clothing, recreation, and other items that Indians typically buy. The inflation rate is expressed as the year-over-year percentage increase in the CPI. Inflation reduces the purchasing power of money over time. High and unstable inflation can be costly. It undermines the economy’s ability to generate long-lasting gains in output, incomes, and employment. It creates uncertainty for consumers, businesses, and investors, and erodes the value of incomes and savings. People on fixed incomes, including many elderly and less well-off Indians, are particularly vulnerable to high inflation, since it erodes the value of their investment income or social benefits such as pensions. High inflation and expectations of high inflation also encourage speculative activities rather than investments that increase production capacity and enable firms to stay competitive at home and abroad.

1.6 Inflation and Interest rate

An interest rate is the cost of borrowing money. A borrower pays interest for the ability to spend money now, rather than wait until he's saved the same amount. Interest rates are expressed as an annual percentage of the total amount borrowed, also known as the principle.

The important thing is that interest rates work both ways. Banks, governments and other large financial institutions need cash for running their businesses. If someone put money into a savings account at a bank, the bank will pay the interest for the temporary use of that money. Governments sell bonds and other securities for the same reason. In the case of lending money the interest is the compensation for temporarily giving up the ability to spend the invested cash. Unfortunately, savings accounts and government-issued bonds pay comparatively low interest rates because the risk of failure to pay is close to zero. Long-term loans also carry higher interest rates than short-term loans, because the more time a borrower has to pay back a loan, the more time there is for
things to go rotten financially, causing the borrower to default. Another factor that makes long-term loans less attractive to lenders -- and therefore raises long-term interest rates -- is inflation. In a healthy economy, inflation almost always rises, meaning the same rupees amount today is worth less five years from now. Lenders know that the longer it takes the borrower to pay back a loan, the less that money is going to be worth. That's why interest rates are actually calculated as two different values: the nominal rate and the real rate. The nominal rate is the interest rate set by the lending institution.

An economic theory proposed by Irving Fisher describes the relationship between inflation and both real and nominal interest rates. The Fisher effect states that the real interest rate equals the nominal interest rate minus the expected inflation rate. Therefore, real interest rates fall as inflation increases, unless nominal rates increase at the same rate as inflation.

1.7 Permissible Delay in Payments

A permissible credit period is usually allowed to an inventory manager to pay back the dues without paying any interest to the supplier. The inventory manager can pay the supplier either by the end of the credit period or later, in which case he incurs interest charges on the unpaid balance for the overdue period. Clearly, it will be most profitable to the inventory manager to repay his dues at the end of the permitted period since he can sell his goods, invest his revenue and earn interest within that period.

Partial trade credit finance refers to paying partial amount for the purchased items as soon as the items are received and the remaining amount should be settled at the end of a trade credit period. In practice, this partial trade credit financing is more matched to real life supply chains. Partial trade credit is similar to payment in installments, which is a common scenario in real life dealings now-a-days.

1.8 Price –Dependent Demand Curve

Demand is the key factor in any inventory management. Though most studies in literature assume constant demand rate, this assumption is valid only during the matured phase of a product’s life cycle. Further, for items like fashion goods, electronic equipments, etc. the demand rate tends to vary.
Governments, business firms, supermarkets, consumers, and law courts require a way to measure how responsive demand is to price changes. Economists measure the responsiveness of quantity demanded to price changes via a concept called elasticity. Marketers sometimes use estimates of elasticity to decide how to price their products or whether to add new product models. If a small change in price is accompanied by a large change in quantity demanded, the product is said to be elastic (or responsive to price changes). Conversely, a product is inelastic if a large change in price is accompanied by a small amount of change in quantity demanded. Demand rate of finished product is marginally affected by price because most of the finished product has a maximum retail price but the demand rate for raw material is significantly affected by price fluctuation.

In this thesis we have used two main types of price dependent demand curves - linear demand curves, and iso-elastic demand curves because the demand rate is dependent on price and price is dependent on inflation rate. The linear demand curve is governed by the equation \( d(p) = a - bp \), where \( p \) denotes the price of the item. The elasticity is not constant along this curve. The iso-elastic demand curve is given by \( d(p) = ap^{-\gamma} \), and it has the property that the elasticities are constant for any price and quantity. Examples of commodities with linear demand rate and iso-elastic demand rate are as follows:

Linear demand rate: sugar, coffee, tea, vegetables, etc

Iso-elastic demand rate: Medicine, bread, books, pencils, semi luxurious goods, etc

Another type of price dependent demand curve is the hybrid demand curve which is a mixture of linear and iso-elastic demands, and is given by \( d(p) = \tau(a - bp) + (1 - \tau)ap^{-\gamma} \), \( \tau \in [0,1] \). Commodities like coffee, cotton, tin, copper, wristwatch, automobile man’s suit, snow skis,, sailboat, mattress, floor lamp, refrigerator, PC game, trash compactor, ice maker, etc exhibit hybrid demand.

1.9 Literature Review

Existing literature and survey of the developed inventory models related to our work are reviewed in this sub section.
1.9.1 Survey of Inventory Models with Inflation

In the classical inventory it is assumed that all the costs associated with the inventory system remains constant over time. But due to large scale of inflation the financial situation in almost all of the countries has changed to an extent during the last thirty years. Currently inflation has become a permanent feature in the inventory system. Inflation enters in the picture of inventory only because it may have an impact on the present value of the future inventory cost. Thus the inflation plays a vital role in the inventory model though the decision makers may face difficulties in arriving at answers related to decision making.

Buzacott (1975) has developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. Misra (1975, 1979) investigated inventory systems under the effects of inflation. Bierman and Thomas (1977) suggested the inventory decision policy under inflationary conditions. Economic analysis of dynamic inventory models with non-stationary costs and demand was presented by Hariga (1994). The effect of inflation was also considered in this analysis. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). Effects of inflation and time-value of money on an inventory model was discussed by Hariga (1995) with linearly increasing demand rate and shortages. Hariga and Ben-Daya (1996) then discussed the inventory replenishment problem over a fixed planning horizon for items with linearly time-varying demand under inflationary conditions. Ray and Chaudhuri (1997) developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on-hand inventory at that instant. A generalized dynamic programming model for inventory items with Weibull distributed deterioration was proposed by Chen (1998). The demand was assumed to be time-proportional, and the effects of inflation and time-value of money were taken into consideration.

The effects of inflation and time-value of money on an economic order quantity model had been discussed by Moon and Lee (2000). Chang (2004) has proposed an inventory model under a situation in which the supplier had provided a permissible delay in payments to the purchaser if the ordering quantity was greater than or equal to a
predetermined quantity. Shortage was not allowed and the effect of the inflation rate, deterioration rate and delay in payments were discussed as well. Models for ameliorating / deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005). The effects of inflation and time value of money were also taken into account.

An inventory model for deteriorating items with stock dependent consumption rate with shortages was produced by Hou (2006). Model was developed under the effects of inflation and time discounting over a finite planning horizon. Jolai et al. (2006) have presented an optimization framework to derive optimal production over a fixed planning horizon for items with a stock-dependent demand rate under inflationary conditions. Jaggi et al. (2007) have presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. Optimal solution for the proposed model was derived and the comprehensive sensitivity analysis has also been performed to observe the effects of deterioration and inflation on the optimal inventory replenishment policies.

Mirzazadeh et al. (2009) analyzes EOQ model for inflation rate dependent demand under uncertain inflationary conditions. Yang et al.(2010) presented the inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages for unequal cycle lengths. Jaggi et al. (2011) discussed EOQ model with linear trend in demand under inflationary conditions. Singh et al. (2010) have presented a replenishment Policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging with two-storage facilities under inflation.

1.9.2 Survey of Inventory Models with Permissible Delay in Payments

In today’s business transactions, it is very common to see the customers being allowed some grace period before they settle their accounts with the supplier. This is trade credit financing. It is advantageous to the customers as they do not have to pay the supplier immediately after receiving the product, but instead, they can delay their payment until the end of the allowed period. The customer pays no interest during the
fixed period they have to settle the account, but if the payment is delayed beyond that, interest is charged by the supplier. The permissible delay in payments is generally welcomed by the buyer, as he would try to earn some interest on the revenue accumulated during this period by investing his income.

Inventory model with trade credit financing was formulated by Haley and Higgins (1973). Goyal (1985) was the first to develop the economic order quantity under conditions of permissible delay in payments. Aggarwal and Jaggi (1995) have developed ordering policies of deteriorating items under permissible delay in payments. Retailer’s pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments were discussed by Hwang and Shinn (1997). An ordering policy for deteriorating items under permissible delay in payments was proposed by Jamal et al. (1997). In this policy shortage was allowed.

Jamal et al. (2000) have presented optimal payment time for a retailer under permitted delay of payment by the wholesaler. The inventory replenishment policy for deteriorating items under permissible delay in payments was presented by Chung (2000). The problem was illustrated numerically. An inventory model for initial-stock-dependent consumption rate was suggested by Liao et al. (2000). Shortages were not allowed. The effects of inflation rate, deterioration rate, initial stock-dependent consumption rate and delay in payments were discussed. Chang and Dye (2001) have developed and inventory model for deteriorating items with partial backlogging and permissible delay in payments. Chung et al. (2002) have discussed the inventory decision for EOQ inventory model under permissible delay in payments. Dye (2002) developed a deteriorating inventory model with stock-dependent demand and partial backlogging. The conditions of permissible delay in payments were also taken into consideration. The shortages were neither completely backlogged nor completely lost assuming the backlogging rate to be inversely proportional to the waiting time for the next replenishment. Numerical examples were also presented to illustrate the model numerically. An EOQ model for deteriorating items with credit policy was discussed by Chang et al. (2003). Lot sizing decision policy was presented by Chung and Liao (2004). In this policy trade credit was depending on the ordering quantity. The effect of trade credit policy with limited storage capacity within the economic order quantity was proposed by Chung and Huang (2004). Numerical examples were given to illustrate all the theorems obtained in the study. The optimal cycle time for deteriorating items under permissible delay in payments was
proposed by Chung and Huang (2005). Teng et al. (2005) have presented an optimal pricing and ordering policy under permissible delay in payments. The optimal ordering policy in a DCF analysis for deteriorating items was suggested by Chung and Liao (2006). In this policy trade credit was also depending on the ordering quantity. Chen and Kang (2007) developed the integrated models with the permissible delay in payments for determining the optimal replenishment time interval and replenishment frequency. Sana and Chaudhuri (2008) have formulated the retailer’s profit maximizing strategy. An inventory model under two levels of trade credit policy was proposed by Jaggi et al. (2008) with credit-linked demand. A mathematical model was developed by Soni and Shah (2008) to formulate optimal ordering policies for retailer when demand was partially constant and partially dependent on the stock. Progressive credit period was offered to settle the account by the supplier. Teng et al. (2009) gave a comprehensive note on: an inventory model under two levels of trade credit and limited storage space derived without derivatives.

Liao and Huang (2010) have suggested a deterministic inventory model for deteriorating items with trade credit financing and capacity constraints. Thangam and Uthayakumar (2010) have developed an economic order quantity based model with perishable items and two-storage facility as a profit maximization problem under retailer’s partial trade credit policy and price dependent demand. Mathematical theorems were developed to determine optimal inventory policy for the retailer and numerical examples were given to illustrate the theory Kumar et al. (2011) have developed a model where the demand is dependent on the present stock level.

1.10 Statement of the Present Problems:

Two notable problems that arise due to inflation are uncertainty and haphazard redistribution. Inflation, especially inflation that varies from month to month and year to year, makes long-term planning quite difficult. Prices, wages, taxes, interest rates, and other nominal values that enter into consumer, business, and government planning decisions can be significantly affected by inflation. It is, therefore, highly desired to study the effect of inflation on all money related matters.

There is a direct relation between prices of commodities and inflation. Costs associated with the inventory system such as ordering cost, holding cost, backlogging cost, lost
sales cost, deterioration cost, purchasing price of the item, interest rate, etc are also affected by inflation. Therefore, to find the optimum inventory policy over a finite planning period one should take into account the effect of inflation.

The main objective of the research is to develop inventory models for deteriorating items with shortages under inflation. This objective can be divided into following sub problems.

(i) To propose an optimal replenishment policy for constant deteriorating items under constant and stochastic inflation rates.

(ii) To propose an optimal replenishment policy for deteriorating items with shortages under constant and stochastic inflation rates.

(iii) To propose an inventory model for deteriorating items with linear and iso-elastic stock price dependent demand under constant and stochastic inflation.

(iv) To propose an inventory model for deteriorating items with permissible delay in payments under constant and stochastic inflation.

1.11 Preview of Current work

An inventory policy depends largely on the demand rate. In most of the previous works, demand rate has been assumed to be independent of inflation rate. Further, the inflation rate has been taken to be constant or normally distributed (Mirzazadeh, 2009). However, demand may change with nominal price of the item due to price inflation. Also, for a good study, it is necessary to model the inflation rate appropriately.

The thesis studies the effect of inflation on an inventory model over a finite planning horizon, where the inflation rate may be constant or random. It concentrates on a periodic review inventory policy for deteriorating items allowing shortages and under permissible delay in payment, where demand is price dependent, and due to the effect of inflation on price, changes over time.

The finite planning horizon \( H \) years) is divided into \( n \) replenishment cycles, where \( n \) is to be decided upon optimally. Models with both equal and unequal cycle lengths have been investigated. The optimal ordering policy has been determined by maximizing the total discounted profit over the planning period. A sensitivity analysis is also carried out numerically to study the effect of change in model parameters on the optimum policy,
and also to study the effect of the distribution of inflation rate when that is assumed to be random.

A brief preview of the work done in the thesis is as follows:

Chapter Two studies an inventory model for deteriorating items with constant inflation rate, allowing shortage. Demand is assumed to be linearly dependent on the current selling price of the item, and is given by $d(t) = a - bpe^{-rt}$, where $a, b \geq 0$ are constant, $p$ is the price of the inventory item at a certain point of time, $r$ is the inflation rate. Three types of permissible delay in payment are considered – (i) when the trade credit period is a constant independent of the order quantity, (ii) when the period is dependent on the order quantity, and (iii) when the payment has to be made in two fixed installments. In all cases, failure to pay within the stipulated time imposes an interest on the unpaid amount. The sensitivity of the model to change in the parameters has been carried out numerically.

In Chapter Three we consider inventory models for deteriorating items with (i) iso-elastic, and (ii) hybrid demands under permissible delay in payment and constant inflation rate, and given by

Iso-elastic: $d(t) = bp^{-\gamma}e^{-\gamma rt}$, and

Hybrid: $d(t) = a(a - bpe^{-rt}) + (1-\alpha)pe^{-\gamma rt}$,

where $\gamma \geq 1, a, b, c \geq 0$ are constant, $p$ is the price of the inventory item at a certain point of time, $r$ is the inflation rate, and $0 \leq \alpha \leq 1$ is the mixing parameter of two types of demand in hybrid demand. In both cases we follow the discounted cash flow approach to develop inventory models which permits replenishment cycles of varying lengths. In (ii) we also assume that during shortage both backlogging and lost sales are possible. We determine the optimal number of replenishments and optimum cycle lengths for the different inventory cycles. In (i), we also determine the optimum selling price of the item stocked in the different cycles. A sensitivity analysis is also carried out for both the cases.

Chapter Four investigates an inventory model with permissible delay in payment and stochastic inflation conditions. The item stocked deteriorates at a constant rate over time,
and the demand rate for the item is assumed to depend linearly on its selling price. Here we assume that the inflation rate is random variable or mixture of two random variables over a inflationary period in an economy. The optimum policy determined so as to maximize the total profit over the finite planning horizon. The effect of a change in the distribution parameter of the inflation rate on the optimal policy has also been studied.

In **Chapter Five** it has been attempted to model the inflation rate movement using one-factor Vasicek and Cox-Ingersoll-Ross (CIR) models. The models are as follows:

(i) Vasicek Inflation model: 
\[ dr_t = a(b - r_t)dt + \sigma dW_t, \]
where 
- \( b = \) long term mean level,
- \( a = \) speed of reversion,
- \( \sigma = \) instantaneous volatility,
- \( W_t \) is a Wiener process.

The solution to the stochastic differential equation is
\[ r_t = r_0 e^{-at} + b \left( 1 - e^{-at} \right) + \sigma e^{-at} \int_0^t e^{as} dW_s \quad \text{and} \quad \frac{r_t}{r_0} \sim N \left( r_0 e^{-at} + b \left( 1 - e^{-at} \right), \frac{\sigma^2}{2a} \left( 1 - e^{-2at} \right) \right), \]
where \( r_0 \) is the initial inflation rate.

(ii) CIR inflation model: 
\[ dr_t = a(b - r_t)dt + \sigma \sqrt{r_t} dW_t, \]
where 
- \( b = \) long term mean level,
- \( a = \) speed of reversion,
- \( \sigma = \) instantaneous volatility,
- \( W_t \) is a Wiener process.

The CIR is an ergodic process, and the stationary distribution of the inflation rate is non-central \( \chi^2 \) with
\[ E \left( \frac{r_t}{r_0} \right) = r_0 e^{-at} + b \left( 1 - e^{-at} \right) \quad \text{and} \quad Var \left( \frac{r_t}{r_0} \right) = r_0 \frac{\sigma^2}{a} \left( e^{-at} - e^{-2at} \right) + \frac{b \sigma^2}{2a} \left( 1 - e^{-at} \right)^2. \]

Vasicek model captures the mean reversion property of the inflation rate (ref. Jensen, 2009). The mean reversion property is also indicated in the CIR model. The advantage of the CIR model over the Vasicek model is that while the latter allows inflation rate to become negative, this is curtailed in the CIR model. The 5 years’ (January, 2010 – May, 2015) inflation rate (CPI) data from the Ministry of Statistics and Programme Implementation (MOSPI) has been used to estimate the parameters of the models by the maximum likelihood method. The estimated models have been used to get the optimal inventory policies. The optimal policies have been determined by carrying out a “mean-risk” analysis. To perform the mean-risk analysis, we take the long-run average profit as
the “mean,” and propose the variance of the on-hand inventory as “risk” of the model. We apply numerical analysis to demonstrate how to construct the efficient policy, in the mean-risk sense. The policies have been obtained under linear and iso-elastic demand rates. Further, as in the other chapters, we carry out sensitivity analysis of the policies for change in model parameters, and also investigate the relationship between inflation rate and optimal inventory policies.