CHAPTER 4

OPTIMAL DESIGN OF PROPOSED TOPOLOGY

4.1 INTRODUCTION

In this chapter, expressions for voltage and current stress experienced by the resonant tank elements are derived theoretically and validated using simulation results. The size of the converter depends on the energy stored (E) and kVA/kW ratio of the resonant tank network as mentioned by Borage et al (2005) and Borage et al (2009 a, 2009 b and 2009 c). The value of the resonant inductor that optimises E and kVA/kW is found out graphically. The resonant capacitors are designed from the optimal inductor value and resonant frequency. Simulation results of the proposed converter that was optimally designed are presented.

4.2 VOLTAGE AND CURRENT STRESS OF PASSIVE ELEMENTS

The resonant tank elements of topology 1 have to be optimally designed since they meet the load requirements. The stresses experienced by the passive elements have to be determined to ensure safe operation of the resonant tank. The expressions for voltage stress across the resonant capacitors and the current stress experienced by the resonant inductor are derived from Figure 4.1.
The normalized value of voltage stress across resonant inductor is given by

\[
V_{L,N} = \frac{V_{L,RMS}}{V_b} \times \frac{I_{L,RMS} \times sL}{V_b} \times \frac{V_{in} \times sL}{Z_{in} \times V_d} \tag{4.1}
\]

But,

\[
\frac{V_{in}}{V_d} = \frac{\pi}{2\sqrt{2}} \therefore V_{L,N} = \frac{\pi}{2\sqrt{2}} \times \frac{sL}{Z_{in}} \tag{4.2}
\]

The input impedance is given by Equation (3.10) and is repeated here in Equation (4.3).

\[
Z_{in} = \frac{1 + sC_2R_{ac} + s^2LC_1 + s^2LC_2 + s^3LC_1C_2R_{ac}}{sC_1 + sC_2 + s^2LC_1C_2R_{ac}} \tag{4.3}
\]

Substituting Equation (4.3) in Equation (4.2),

\[
V_{L,N} = \frac{\pi}{2\sqrt{2}} \times \frac{sL\left(sC_1 + sC_2 + s^2LC_1C_2R_{ac}\right)}{1 + sC_2R_{ac} + s^2LC_1 + s^2LC_2 + s^3LC_1C_2R_{ac}} \tag{4.4}
\]

Substituting \( s = j\omega \) in Equation (4.4), the normalised voltage stress across inductor is given by

\[
V_{L,N} = \frac{\pi}{2\sqrt{2}} \times \left[ \frac{-\omega_n^2 - \omega_n^2x - j\omega_n^3 \frac{\pi^2 x}{8Q}}{1 + j\omega_n \frac{\pi^2 x}{8Q} - \omega_n^2 - \omega_n^2x - j\omega_n^3 \frac{\pi^2 x}{8Q}} \right] \tag{4.5}
\]
Simplifying Equation (4.5),

\[ V_{L,N} = \frac{-\omega_n^2 (1 + x) - j\omega_n^3 \frac{\pi^2}{8} \frac{x}{Q}}{A} \]  

(4.6)

where \( A = \frac{2\sqrt{2}}{\pi} \left( 1 + j\omega_n^3 \frac{\pi^2}{8} \frac{x}{Q} - \omega_n^2 (1 + x) - j\omega_n^3 \frac{\pi^2}{8} \frac{x}{Q} \right) \)  

(4.7)

The current stress experienced by the resonant inductor can be found out from the normalised value of current through the resonant inductor. The normalized value of inductor current stress is given by

\[ I_{L,N} = \frac{I_{L,RMS}}{I_b} \frac{V_{in}}{V_d} \frac{Z_{in}}{Z_n} \]  

(4.8)

But, \( \frac{V_{in}}{V_d} = \frac{\pi}{2\sqrt{2}} \) and \( Z_n = Q \times R_L \)  

(4.9)

Substituting Equations (4.9) and (4.3) in Equation (4.8),

\[ I_{L,N} = \frac{\pi}{2\sqrt{2}} Q \times R_L \frac{sC_1 + sC_2 + s^2C_1C_2R_{ac}}{1 + sC_2R_{ac} + s^2LC_1 + s^2LC_2 + s^3LC_1C_2R_{ac}} \]  

(4.10)

Substituting \( s = j\omega \) and \( R_{ac} \) in Equation (4.10),

\[ I_{L,N} = \frac{\pi}{2\sqrt{2}} \times \frac{j\omega C_1 R_L + j\omega C_2 R_L - \omega^2 C_1 C_2 Q \frac{\pi^2}{8} R_L^2}{1 + j\omega C_2 \frac{\pi^2}{8} R_L - \omega^2 LC_1 - \omega^2 LC_2 + j\omega^3 LC_1 C_2 \frac{\pi^2}{8} R_L} \]  

(4.11)
Simplifying Equation (4.11),

\[
I_{L,N} = \frac{j\omega_n(1 + x) - \frac{\pi^2 \omega_n^2 x}{8} Q}{A}
\]  

(4.12)

The voltage and current stresses experienced by the resonant capacitors also need to be found out for proper design of resonant tank elements. By using circuit theory basics and Figure 4.1, the normalized value of current stress experienced by capacitor \(C_1\) is given by

\[
I_{C_1,N} = I_{L,N} - I_{C_2,N}
\]  

(4.13)

Now, the normalized value of current stress experienced by the capacitor \(C_2\) \((I_{C_2,N})\) is given by

\[
I_{C_2,N} = \frac{I_o}{I_b} \left[ \frac{V_o}{R_{ac}} - \frac{MV_{in}Z_n}{R_{ac}V_d} \right. \left. + \frac{M \times V_{in} \times Z_n}{R_{ac}} \right]
\]  

(4.14)

Simplifying Equation (4.14) by using Equation (4.9) and \(R_{ac}\),

\[
I_{C_2,N} = M \frac{2\sqrt{2}}{\pi} Q
\]  

(4.15)

Substituting \(s = j\omega\) and \(M\) (from Equation (3.14)),

\[
I_{C_2,N} = \frac{2\sqrt{2}}{\pi} Q \times \frac{j\pi^2 \omega_n x}{8Q} \frac{1 - \omega_n^2 (1 + x) + j \frac{\pi^2 \omega_n x}{8Q} \left(1 - \omega_n^2 \right)}{1 - \omega_n^2 (1 + x) + j \frac{\pi^2 \omega_n x}{8Q} \left(1 - \omega_n^2 \right)}
\]  

(4.16)
Simplifying Equation (4.16),

\[
I_{C_{1,N}} = j \frac{\omega_n x}{A} \tag{4.17}
\]

Substituting Equations (4.12) and (4.17) in Equation (4.13), we get

\[
I_{C_{1,N}} = \frac{j \omega_n (1 + x) - \frac{\pi^2 \omega_n^2 x}{8} Q}{A} - j \frac{\omega_n x}{A} \tag{4.18}
\]

Simplifying (4.18),

\[
I_{C_{1,N}} = \frac{j \omega_n - \frac{\pi^2 \omega_n^2 x}{8} Q}{A} \tag{4.19}
\]

The normalized voltage stress across capacitor \( C_1 \) is given by

\[
V_{C_{1,N}} = \frac{V_{C_{1,RMS}}}{V_b} = \frac{I_{C_{1,RMS}} \times \frac{1}{sC_1}}{V_b} \tag{4.20}
\]

But,

\[
I_{C_{1,RMS}} = I_{L,RMS} - I_{C_{2,RMS}} \tag{4.21}
\]

Substituting \( I_{L,RMS} \) and \( I_{C_{2,RMS}} \) in Equation (4.21),

\[
I_{C_{1,RMS}} = \frac{V_m}{Z_{in}} - \frac{MV_{in}}{R_{ac}} V_m \left[ \frac{1}{Z_{in}} - \frac{M}{R_{ac}} \right] \tag{4.22}
\]

Substituting Equation (4.22) in Equation (4.20),

\[
V_{C_{1,N}} = \frac{V_m}{V_d} \left[ \frac{1}{Z_{in}} - \frac{M}{R_{ac}} \right] \times \frac{1}{sC_1} \tag{4.23}
\]
Substituting for \( R_{ac} \), Equations (3.11) and (4.9) in Equation (4.23),

\[
V_{C_{1,N}} = \frac{\pi}{2\sqrt{2}} \left[ \frac{sC_1 + sC_2 + s^2C_1C_2R_{ac}}{B} - \frac{sC_2R_{ac}}{B} \times \frac{1}{R_{ac}} \right] \times \frac{1}{sC_1} \tag{4.24}
\]

where \( B = 1 + sC_2R_{ac} + s^2LC_1 + s^2LC_2 + s^3LC_1C_2R_{ac} \) \tag{4.25}

Simplifying Equation (4.25), normalised voltage stress across \( C_1 \) is given by

\[
V_{C_{1,N}} = \frac{\pi}{2\sqrt{2}} \left[ \frac{sC_1 + s^2C_1C_2R_{ac}}{B} \right] \times \frac{1}{sC_1} \tag{4.26}
\]

Substituting \( s = j\omega \) in Equation (4.26),

\[
V_{C_{1,N}} = \frac{1 + j \omega X}{8Q} \tag{4.27}
\]

The normalized voltage stress across capacitor \( C_2 \) is given by

\[
V_{C_{2,N}} = \frac{V_{C2,RMS}}{V_b} = \frac{I_{C2,RMS} \times \frac{1}{sC_2}}{V_b} \tag{4.28}
\]

Substituting Equations (4.9) and (4.22) in Equation (4.28),

\[
V_{C_{2,N}} = \frac{V_m}{V_b} \times \frac{M}{R_{ac}} \times \frac{1}{sC_2} = \frac{\pi}{2\sqrt{2}} \times \frac{M}{sC_2R_{ac}} \tag{4.29}
\]

Substituting for \( M \) from Equation (3.11),

\[
V_{C_{1,N}} = \frac{\pi}{2\sqrt{2}} \times \frac{sC_2R_{ac}}{1 + sC_2R_{ac} + s^2LC_1 + s^2LC_2 + s^3LC_1C_2R_{ac}} \times \frac{1}{sC_2R_{ac}} \tag{4.30}
\]
Substituting \( s = j\omega \) in Equation (4.30) and simplifying,

\[
V_{C_{2,N}} = \frac{1}{\mathcal{A}}
\]

(4.31)

**Table 4.1 Summary of voltage and current stress of resonant tank**

<table>
<thead>
<tr>
<th>Resonant Element</th>
<th>Voltage stress</th>
<th>Current stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>(-\omega_n^2 (1 + x) - j\omega_n^3 \frac{\pi^2 x}{8 \mathcal{Q}})</td>
<td>( j\omega_n (1 + x) - \frac{\pi^2 \omega_n^2 x}{8 \mathcal{Q}})</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>( 1 + j\frac{\pi^2 \omega_n x}{8\mathcal{Q}} )</td>
<td>( j\omega_n - \frac{\pi^2 \omega_n^2 x}{8 \mathcal{Q}} )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( \frac{1}{\mathcal{A}} )</td>
<td>( j\frac{\omega_n x}{\mathcal{A}} )</td>
</tr>
</tbody>
</table>

\[
\mathcal{A} = \frac{2\sqrt{2}}{\pi} \left( 1 + j\omega_n \frac{\pi^2 x}{8 \mathcal{Q}} - \omega_n^2 (1 + x) - j\omega_n^3 \frac{\pi^2 x}{8 \mathcal{Q}} \right)
\]

Table 4.1 gives the summary of voltage and current stress expressions of the resonant tank elements present in topology 1. To check the validity of the expressions derived, the theoretical and the simulated plots are compared as shown next. The expressions for resonant inductor current and voltage across the resonant capacitors are validated initially. The voltage stress across the resonant inductor and the current stress of resonant capacitors can safely be assumed to be correct, once the voltage stress of the capacitors and the current stress of the inductor are found to be correct. Figure 4.2 shows the theoretical and simulated plot of resonant inductor current stress. It is observed that the theoretical and simulated values closely match and confirms the validity of the expression derived.
Figure 4.2 Theoretical and simulated plots of inductor current stress

Figure 4.3 Theoretical and simulated plots of voltage stress across $C_1$

Figure 4.3 shows that the theoretical and simulated plots of voltage across capacitor $C_1$ closely match with each other. Thus, the validity of the voltage stress across capacitor $C_1$ is proved. Figure 4.4 shows the closeness between the theoretically obtained voltage stress across $C_2$ and the simulated values.
4.3 OPTIMAL DESIGN OF RESONANT TANK

The size of the resonant tank has to be minimal to obtain a compact converter. To obtain a minimum sized resonant tank, the size of the resonant inductor has to be minimised. This is because the resonant capacitor is generally smaller in size compared to the inductor and does not affect the overall size of the converter.

The function of the resonant tank is to transfer the energy from the input to the output side. While performing this function, due to the energy storage nature of the resonant tank elements, some amount of energy will be stored in the resonant tank and not be transferred from the input to the output. The amount of energy stored in the resonant tank (E) should be minimal so that the size of the resonant inductor becomes small. This is justified because the physical size of an inductor is indicated by the area, which is directly proportional to the energy handled by the inductor. Hence, lower value of inductance results in smaller sized inductor.

Also, kVA/kW rating of the resonant network is considered as an index for physical size of the resonant network. The kVA/kW capacity
demanded by the resonant tank for efficient energy transfer from input to output must be met from the power transferred from the supply to the load. For minimum sized resonant tank, the kVA/kW ratio has to be minimal. In other words, minimum amount of reactive power would be consumed by the resonant tank elements if kVA/kW ratio is minimal. Therefore, expressions for $E$ and kVA/kW ratio are derived and optimised graphically as shown later.

Using individual current stresses ($I_{C1}$, $I_{C2}$, $I_L$) and voltage stresses ($V_{C1}$, $V_{C2}$, $V_L$) across resonant elements, the expressions for energy stored in the resonant tank and kVA/kW are obtained as shown below.

Normalised energy is given by

$$E = \frac{1}{2} \frac{L I_{L \text{RMS}}^2}{\frac{V_d^2}{\omega_0 R_L}}$$

(4.32)

where $R_L$ is the equivalent load resistance and $V_d$ is the input DC voltage.

The total kVA demanded by the resonant tank is given by

$$\text{kVA} = \left[ V_{C1} I_{C1} + V_{C2} I_{C2} + V_L I_L \right] \times 10^{-3}$$

(4.33)

Since $V_{C,N} = \frac{V_C}{V_d}$ and $I_{C,N} = \frac{I_C}{V_d/Z_n}$ and so on for other variables,

$$\text{kVA} = \left[ V_{C1,N} I_{C1,N} + V_{C2,N} I_{C2,N} + V_{L,N} I_{L,N} \right] \times V_d \times \frac{V_d}{Z_n} \times 10^{-3}$$

(4.34)

The output power in kW is given by

$$\text{kW} = V_d I_0 \times 10^{-3}$$

(4.35)
From Equations (4.34) and (4.35),
\[
\frac{kVA}{kW} = \left[ \frac{V_{c_1,n}I_{c_1,n} + V_{c_2,n}I_{c_2,n} + V_{l,n}I_{l,n}}{V_0I_0} \right] \times \frac{V_d}{Z_n} \tag{4.36}
\]

Rearranging Equation (4.36),
\[
\frac{kVA}{kW} = \left[ \frac{V_{c_1,n}I_{c_1,n} + V_{c_2,n}I_{c_2,n} + V_{l,n}I_{l,n}}{\left( \frac{V_0}{V_d} \right) I_0 / \left( \frac{V_d}{Z_n} \right)} \right] \tag{4.37}
\]

Simplifying Equation (4.37), the kVA/kW ratio is given by
\[
\frac{kVA}{kW} = \frac{V_{c_1,n}I_{c_1,n} + V_{c_2,n}I_{c_2,n} + V_{l,n}I_{l,n}}{M \times H} \tag{4.38}
\]

where H is the current gain of the topology and is given by
\[
H = \frac{M}{Q} \tag{4.39}
\]

When the parameters from Table 4.1 and Equation (4.39) are substituted in Equation (4.38) and simplified, the expression for kVA/kW ratio will be in terms of Q. From the voltage gain plot of topology 1 shown in Figure 3.3, it is observed that the desired operating point for all load conditions occurs at the normalised frequency \( \omega_n = 0.7 \). Therefore, E and kVA/kW ratio are evaluated at \( \omega_n = 0.7 \). The expressions for E and kVA/kW are given by Equations (4.40) and (4.41) respectively.

\[
E = \frac{\pi^2}{16Q} \times \frac{0.00375 \frac{\pi^4}{Q^2} + 0.0218 \frac{\pi^2}{Q^2} + 0.028}{0.00199 \frac{\pi^4}{Q^2} + 0.0004} \tag{4.40}
\]
In order to obtain a compact resonant tank, the optimum value of inductance that would result in minimizing both \( E \) and \( \text{kVA/kW} \) has to be found out. Equations (4.40) and (4.41) give the relationship between \( Q \) and these parameters. To obtain the required voltage gain for wide load variations, the value of \( Q \) is set between 1 and 2 – lower value of \( Q \) corresponds to light load condition and higher value for full load condition.

The load resistance value is experimentally determined to be 5.76\( \Omega \). This value corresponds to full load condition. The value of \( L \) for \( Q \leq 2 \) is found out by using Equation (3.7). For a resonant frequency of 100kHz, the value of \( L \) is computed and found to be \( L \geq 18.335\mu\text{H} \). To verify the converter’s performance under light load condition, its load must be reduced. This means that the converter should supply less current. Hence, a larger resistance of 100\( \Omega \) is connected across the converter’s load terminals. Using Equation (3.7), the computed value of \( L \) is \( L \leq 318.31\mu\text{H} \).

When both the loads are connected in parallel, the net resistance value is 5.45\( \Omega \). This means the converter is slightly overloaded. To make the design suitable under slightly over load condition, the value of \( Q \) is chosen to be \( Q \geq 1 \). This gives the minimum inductance value. Using Equation (3.7), this minimum value of inductance is found to be \( L \geq 8.6739\mu\text{H} \). For the same slightly overload condition, when \( Q \leq 2 \) the maximum value of \( L \) is found to be \( L \leq 17.35\mu\text{H} \). Thus, the range over which \( L \) can be varied is given by \( 8.6739 \leq L \leq 17.35\mu\text{H} \).

\[
\frac{\text{kVA}}{\text{kW}} = \frac{\pi^2}{8Q} \left[ \frac{0.127 \frac{\pi^2}{8Q} - j\left(0.03 + 0.072 \left(\frac{\pi^2}{8Q}\right)^2\right)}{j0.7 \frac{\pi^2}{8Q}^2} \right] \tag{4.41}
\]
Equations (4.40) and (4.41) are rewritten in terms of $L$ and plotted with $L$ as the variable parameter over the range $8.6739 \leq L \leq 17.35\mu H$. Figure 4.5 shows the plot of $E$ and kVA/kW variation with respect to $L$.

![Figure 4.5 E and kVA/kW plot](image)

From Figure 4.5, it is observed that both $E$ and kVA/kW curves intersect at one particular value of $L$ which is the optimum value. Thus, the optimum value of $L$ is found out to be $11.35\mu H$. The resonant tank capacitors have to be designed based on this optimum value of $L$ and resonant frequency. For a resonant frequency of $100kHz$ and $L = 11.35\mu H$, from Equation (3.6) the values of resonant capacitors are calculated and found to be $C_1 = C_2 = 0.22\mu F$. Based on these values, the circuit diagram of proposed topology is shown in Figure 4.6 and is simulated using PSpice 9.2. The simulation results are discussed in the next section.

![Figure 4.6 Circuit diagram of proposed LCC topology](image)
4.4 SIMULATION RESULTS

The proposed topology is simulated using PSpice 9.2. The detailed specifications are given in Appendix 1. IRF540N power MOSFETs were chosen as the main power switches present in the inverter. Fast recovery diodes MUR8100E were chosen to perform high frequency rectification. The details of simulation output are shown in Figure 4.7. It is observed from Figure 4.7 that the output voltage is around 24V, which meets the load voltage requirement. The inverter output voltage oscillates between +12V and -12V as expected. The resonant tank makes the resonant inductor current sinusoidal besides providing the required voltage gain of 2. Further, since the switching frequency is less than the resonant frequency, the inductor current leads the inverter output voltage. This provides soft switching of the power switches.

![Figure 4.7 Simulation results](image)

Figure 4.7 Simulation results

Figure 4.8 shows the switching loss waveforms for the main power switch $S_1$ present in the proposed converter. It is observed that the turn on losses is very much reduced due to the presence of resonant tank. Also, the voltage stress across the device is also within safe limits. Since all the power
switches present in the full bridge inverter are identical, the switching losses of the other switches $S_2$ to $S_4$ will also be reduced.

![Figure 4.8 Switching loss waveform for switch $S_1$](image)

4.5 CONCLUSION

In this chapter, the size of the resonant tank and thereby the proposed converter has been optimally designed. The optimal design value of the resonant inductor was obtained graphically by optimising the energy stored in the resonant tank and kVA/kW ratio. The expressions for $E$ and kVA/kW were derived from the voltage and current stress experienced by the proposed resonant tank topology. Simulation results show that the optimally designed resonant tank elements provide the required voltage gain and soft switching of the power switches. The performance of the converter under normal temperature is found to be satisfactory. However, the converter’s behaviour when temperature varies should be tested. Chapter 5 examines the temperature sensitive nature of the converter.