Chapter 7

Conclusion
Chapter-7

* CONCLUSION

At the end of this concluding part of this thesis, the novel features, ideas, results, and proofs given in various chapters are summarised. All the results obtained in the form of theorems and related lemmas, truths, remarks and corollaries in a new setting, bring out their novelties.

It is interesting to note that the new results presented in this thesis have been basically motivated by the published and unpublished research works of the guide and the research scholar of this thesis. References to the results are consistently made to maintain a thread of logical continuity in the evaluation of the thesis.

Indeed, the works of Shafiq and Fedorner (1979) were taken into consideration to define the concept of N-ary partially balanced incomplete block designs developed from any binary partially balanced scheme, where the incidence matrix $\mathbf{n}$ contains the N values 0, 1, 2, ..., (N-1) is extended to generalized N-ary partially balanced block designs, where the new incidence matrix $\mathbf{n}'$ contains N values $m_a$, for $a = 0, 1, 2, ..., (N-1)$, for $m_a = a.m_1 - (a-1). m_0$, and for any $m_0$ and $m_1$, satisfying the relation, $0 \leq m_0 < m_1$. A-, D- and E-optimality criteria are developed to select the two associate class scheme, partially balanced n-ary block designs in the class with smallest variance of a contrast. Consideration of special types of partially balanced n-ary block designs and efficiency factor for generalized two associate n-ary designs are also presented.

The idea of efficiency and optimality criteria have been historically traced from the days of Smith (1918) to the recent times, particularly that of Chang et al. (1998), Das (1998), Dettie et al. (1998), Montepiedra et al. (1998), Jacroux (1998), Rao M.O. et al. (1998) and Srivastava et al. (1998). Some more works on optimality

published in 1999 and 2000 by reputed authors have also been taken into consideration.

Equivalence theorem, specific optimal criteria like (i) D-optimality, (ii) $D_A$-optimality, (iii) S-optimality, (iv) $D_S$-optimality, (v) A-optimality, (vi) G-optimality (vii) E-optimality, (viii) V-optimality, (ix) Linear-optimality, (x) M.S.-optimality, (xi) M.V-optimality, (xii) Schur-optimality, (xiii) Universal-optimality, and (xiv) T-optimality have been defined clearly relevant to the latest papers. The interested work on partial optimality for which literature were collected, are reserved for future works on different optimality of block designs. Concepts and recollection of n-ary block designs and analysis of balanced and partially balanced n-ary block designs which are presented here cannot be found anywhere except from the publication of the guide and the present author of this thesis. They are the latest and upto date recollection.

Recent developments on optimal block designs, A-optimal block designs and latest A-optimal block designs are important research collections in the field of optimal designs. Similarly the review collection of n-ary block designs from Tocber (1952) to recent times, particularly the work of Soundarapandian (1979a,b, 1980a, b,c,d, 1981a,b,c,d) and Soundarapandian et.al. (1995a,b,c,d) is a landmark in incomplete block designs which have been utilized to find new A-optimal design in our thesis.

In chapter three, we discuss in detail the construction of Balanced Treatment Incomplete Block (BTIB) designs proposed by Bechhofer and Tamhane (1981). Robson (1961) suggested the use of balanced incomplete block design (BIBD) between all of the treatments, including the control. Cox (1958) noted BIBD may not be very appropriate for the multiple comparisions with the control because of the special role played by the control. For a brief exposition to the history of this problem and a list of references the reader will be referred to the article by Bechhofer and Tamhane (1981). Infact this article is basic to the developments in the research. From now on it will be referred to be BT.
In many industrial, agricultural, and biological experiments, the following problem is encountered frequently: p (new) varieties (or treatments) become available; an efficient, experiment is required to compare the p new treatments (labeled 1, 2, ..., p) with the old one (hereafter called the control and labeled as 0) and to compare the new treatments among themselves. But the emphasis will be on the comparison of the p-treatments with the control, hence higher precision is desired for these estimates.

In this research the necessary tools for determining that a certain set of BTIB designs constitutes the MCCGD are given. The minimal complete classes are provided for p=4(1)10, k=4, and p=5(1)10, k=5 and also p=6, k=6. The use of the tools are illustrated for p=4, k=4 and p=5, k=5. Also two methods of constructing BTIB designs are described. The first method is an effective "Ad hoc" method, i.e., it involves a systematic trial and error that allows one to construct MCCGD at least for practical values of (K, p). The second method is a computer algorithm that makes it possible to construct all binary BTIB designs for given (K, p). This method, too, involves little trial and error.

Now, the important questions, "How many units should be assigned to control?" and "How should they be allocated to blocks?" are answered. The guidelines for constructing the A-optimal or nearly A-optimal designs are provided and a computer algorithm that searches for A-optimal designs can be described.

In the last part of the thesis (chapter 4), we present the definitions of partially Balanced Treatment Incomplete Block design and partially Balanced Treatment n-ary Block design as a natural extension of the guide's definition of partially Balanced n-ary Block design. The information matrix and its combinatorial properties were discussed in detail. Finally some examples were presented without detailed discussion of the construction of these designs from MCCGD, due to non-availability of space and reserving these works for separate publication.
Since, space and other considerations do not permit, a new work will not be included in this thesis which will study in detail the BTIB designs and PBTIB designs for homosedastic as well as heterosedastic set up models. The C-matrix of BTIBDs under heterosedastic settings will be used to our treatment comparisons by using generalized least squares which holds good for homosedastic settings also. Many more theorems and results can be worked out for treatment comparision of our present designs, but for space and time, results will not be included in our present thesis. They will be published separately as research papers in due course.

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