Chapter -3

Plane waves in a rotating monoclinic magneto thermo elastic medium

Many researchers have done impressive research work on propagation of plane wave in anisotropic media and eminent of them are Keith and Crampin [33]; Chattopadhyay and Choudhury [39]; Chattopadhyya et al. [40]; Singh and Khurana [44]; Ting [48]; Singh and Tomar [49]; Singh [54]; Sharma and Sidhu [162]; Singh [177]; Singh [184]; and Singh et al. [185]. No study is reported till date on plane wave propagation in monoclinic magneto thermo elastic medium rotating uniformly.

In this chapter, the governing differential equations of a monoclinic magneto-thermo-elastic rotating medium are formulated in the framework of Lord-Shulman theory. The solutions of these partial differential equations showed the presence of three quasi plane waves. Velocity equations for some special cases without anisotropy, rotation parameter, magnetic and thermal fields are derived. The wave speeds are calculated for particular material modelling the half-space. The procedure of computing reflected angles is derived for a given incident wave. Some particular cases are also obtained. The dependence of wave speeds on incident angle, reflected angle, rotation parameter and magnetic field is shown by a numerical example and depicted graphically.

3.1 Notations

\( \mathbf{B} \): vector of magnetic induction  \( \mathbf{E} \): electric field vector

\( \mathbf{H} \): total magnetic field vector  \( \mathbf{J} \): vector of electric current density

\( \sigma_{ij} \): components of the stress tensor  \( e_{ij} \): components of the strain

\( \Omega \): rotation vector  \( \delta_{ij} \): Kronecker delta

\( V \): phase velocity  \( \omega \): circular frequency

\( \lambda_1 \): Lame’s constant  \( \rho \): density of the medium

\( \tau_0 \): thermal relaxation time  \( k \): wave number

\( \mu \): Lame’s constant  \( T_0 \): reference uniform temperature
\( u_2, u_3 \): displacement components.  \( C_E \): specific heat.

\( \alpha_0 \): coefficient of thermal expansion  \( \sigma \): electric conductivity of medium

\( \beta_2, \beta_3 \): thermal coefficients  \( C_{ij} \): elastic constants

\( K_2, K_3 \): thermal conductivities  \( \mu_e \): magnetic permeability

\( T \): change in temperature from  \( t \): time

Reference temperature \( T_0 \)

### 3.2 Derivation of basic equations and Solution

Consider a homogenous monoclinic thermoelastic half-space in yz-plane which is rotating about x–axis with angular velocity \( \Omega = (\Omega, 0, 0) \) in the presence of magnetic field \( H = (H, 0, 0) \) at reference temperature \( T_0 \). Following Dhaliwal and Sherief [116] and Singh [177] the equation governing various fields in yz-plane becomes

\[
C_{22} \frac{\partial^2 u_2}{\partial y^2} + C_{44} \frac{\partial^2 u_2}{\partial z^2} + C_{24} \frac{\partial^2 u_2}{\partial y^2} + C_{34} \frac{\partial^2 u_3}{\partial y^2} + 2C_{24} \frac{\partial^2 u_2}{\partial y \partial z} \\
+ (C_{23} + C_{44}) \frac{\partial u_0}{\partial y \partial z} - \beta_2 \frac{\partial T}{\partial y} + (J \times B)_2 = \rho \left( \frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 - 2\Omega \frac{\partial u_3}{\partial t} \right), \tag{3.1}
\]

\[
C_{24} \frac{\partial^2 u_2}{\partial y^2} + C_{34} \frac{\partial^2 u_2}{\partial z^2} + C_{41} \frac{\partial^2 u_3}{\partial y^2} + C_{33} \frac{\partial^2 u_3}{\partial z^2} + 2C_{34} \frac{\partial^2 u_2}{\partial y \partial z} \\
+ (C_{23} + C_{44}) \frac{\partial u_0}{\partial y \partial z} - \beta_3 \frac{\partial T}{\partial z} + (J \times B)_3 = \rho \left( \frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 + 2\Omega \frac{\partial u_2}{\partial t} \right), \tag{3.2}
\]

Following Roy Chaudhuri and Debnath [160], Maxwell’s equations which govern electromagnetic field equations without the charge density and displacement current are

\[
\text{Curl} \ H = J, \text{Curl} \ E = -\frac{\partial B}{\partial t}, \ B = \mu_e H, \ \text{div} \ B = 0, \tag{3.3}
\]

Ohm’s Law in generalized form is

\[
J = \sigma \left[ E + (u \times B) \right], \tag{3.4}
\]
where, the magnetic field is taken as \( \mathbf{H} = \mathbf{H}_0 + \mathbf{h} \), \( \mathbf{H}_0 = (H_0, 0, 0) \) and the effect on the conduction current \( \mathbf{J} \) due to thermal gradient is neglected. Linearizing the basic equations by neglecting the product terms of \( \mathbf{h}, \mathbf{u} \) and their derivatives as induced magnetic field \( \mathbf{h} \) is very small.

Making use of equations (3.3) and (3.4), equations (3.1) and (3.2) become

\[
C_{22} \frac{\partial^2 u_2}{\partial y^2} + C_{44} \frac{\partial^2 u_2}{\partial z^2} + C_{24} \frac{\partial^2 u_3}{\partial y^2} + C_{34} \frac{\partial^2 u_3}{\partial z^2} + 2C_{24} \frac{\partial^2 u_2}{\partial y \partial z} + (C_{23} + C_{44}) \frac{\partial^2 u_3}{\partial y \partial z}
\]

\[= -\beta_2 \frac{\partial T}{\partial y} + \mu H_0^2 \left( \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_3}{\partial y \partial z} \right) = \rho \left( \frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 - 2\Omega \frac{\partial u_2}{\partial t} \right), \tag{3.5} \]

\[
C_{24} \frac{\partial^2 u_2}{\partial y^2} + C_{34} \frac{\partial^2 u_3}{\partial z^2} + C_{24} \frac{\partial^2 u_3}{\partial y^2} + C_{33} \frac{\partial^2 u_3}{\partial z^2} + 2C_{34} \frac{\partial^2 u_2}{\partial y \partial z} + (C_{23} + C_{44}) \frac{\partial^2 u_3}{\partial y \partial z}
\]

\[= -\beta_3 \frac{\partial T}{\partial z} + \mu H_0^2 \left( \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2} \right) = \rho \left( \frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 + 2\Omega \frac{\partial u_3}{\partial t} \right). \tag{3.6} \]

The heat equation is

\[
K_2 \frac{\partial^2 T}{\partial y^2} + K_3 \frac{\partial^2 T}{\partial z^2} - T_0 \left[ \beta_2 \left( \frac{\partial^2 u_2}{\partial y \partial t} + \tau_0 \frac{\partial^2 u_2}{\partial y^2} \right) \right] + \beta_3 \left( \frac{\partial^2 u_3}{\partial z \partial t} + \tau_0 \frac{\partial^2 u_3}{\partial z^2} \right) = \rho C_e \left( \frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right), \tag{3.7} \]

The plane wave solutions for incident waves are taken as

\[
\begin{align*}
    u_2 &= A e^{ik \left( \sin \theta y + \cos \theta z - \nu \tau \right)} \\
    u_3 &= B e^{ik \left( \sin \theta y + \cos \theta z - \nu \tau \right)} \\
    T &= C e^{ik \left( \sin \theta y + \cos \theta z - \nu \tau \right)}
\end{align*} \tag{3.8} \]

and for reflected waves as

\[
\begin{align*}
    u_2 &= A e^{ik \left( \sin \theta y - \cos \theta z - \nu \tau \right)} \\
    u_3 &= B e^{ik \left( \sin \theta y - \cos \theta z - \nu \tau \right)} \\
    T &= C e^{ik \left( \sin \theta y - \cos \theta z - \nu \tau \right)}
\end{align*} \tag{3.9} \]

where, A,B,C are constants and \( (\sin \theta, \cos \theta) \) represents the projection of wave normal onto \( yz \) plane.

Using equations (3.8) and (3.9) in equations (3.5) to (3.7), it is obtained
(D_1 - \Omega^* \zeta) A + (D_2 + 2i \frac{\Omega}{\omega} \zeta) B + i \frac{\beta_c^*}{k} \sin \theta . C = 0, \quad (3.10)

(D_2 - 2i \frac{\Omega}{\omega} \zeta) A + (D_3 - \Omega^* \zeta) B + \pm \frac{i \beta_r^*}{k} \cos \theta . C = 0, \quad (3.11)

\varepsilon_0 A \pm \beta_0 \varepsilon \cos \theta \cos \theta + B + i \frac{\beta_c^*}{k} (D_5 - \zeta) C = 0, \quad (3.12)

where, \( \zeta = \rho V^2 \),

\[ D_1 = (C_{22} + \mu_c H_0^2) \sin^2 \theta + (C_{44} \cos \theta \pm 2C_{24} \sin \theta) \cos \theta, \]

\[ D_2 = (C_{24} \sin \theta \pm \mu_c H_0^2 \cos \theta) \sin \theta + (C_{34} \cos \theta \pm (C_{23} + C_{44}) \sin \theta) \cos \theta, \]

\[ D_3 = (C_{44} \sin \theta \pm 2C_{34} \cos \theta) \sin \theta + (C_{33} + \mu_c H_0^2) \cos^2 \theta, \]

\[ D_5 = \frac{D_4}{\tau^* E}, \quad D_4 = K_2 \sin^2 \theta + K_3 \cos^2 \theta, \]

\[ \tau^* = \frac{i}{\omega} + \tau_0, \quad \Omega^* = 1 + \left( \frac{\Omega}{\omega} \right)^2, \quad \varepsilon = \frac{\beta_c^* T_0}{\rho C_E}, \quad \beta_0 = \frac{\beta_r^*}{\beta_c^*}, \quad \omega = kV, \]

where upper sign (+) represent for incident wave and the lower sign (-) represent for reflected waves respectively. The non-trivial solution of equations (3.10) to (3.12) require

\[
\begin{vmatrix}
D_1 - \Omega^* \zeta & D_2 + 2i \frac{\Omega}{\omega} \zeta & i \frac{\beta_c^*}{k} \sin \theta \\
D_2 - 2i \frac{\Omega}{\omega} \zeta & D_3 - \Omega^* \zeta & \pm \frac{i \beta_r^*}{k} \cos \theta \\
\varepsilon_0 \sin \theta & \pm \beta_0 \varepsilon \cos \theta & i \frac{\beta_c^*}{k} (D_5 - \zeta)
\end{vmatrix} = 0, \quad (3.13)
\]

which can be written as

\[ Q_0 \zeta^3 + Q_1 \zeta^2 + Q_2 \zeta + Q_3 = 0, \quad (3.14)\]

where, \( Q_0 = 4 \left( \frac{\Omega}{\omega} \right)^2 - \Omega^* \),

\[ Q_1 = \Omega^* (D_1 + D_3 + \varepsilon \sin^2 \theta + \beta_0^2 \varepsilon \cos^2 \theta) + (\Omega^* + 4 \left( \frac{\Omega}{\omega} \right)^2) D_3, \]

\[ Q_2 = \Omega^* (D_2 + D_3 + \varepsilon \sin^2 \theta + \beta_0^2 \varepsilon \cos^2 \theta) + (\Omega^* + 4 \left( \frac{\Omega}{\omega} \right)^2) D_3, \]

\[ Q_3 = \Omega^* (D_3 + \varepsilon \sin^2 \theta + \beta_0^2 \varepsilon \cos^2 \theta) + (\Omega^* + 4 \left( \frac{\Omega}{\omega} \right)^2) D_3, \]
\[Q_2 = D_2^2 - D_1 D_3 - \Omega^*(D_1 D_5 + D_3 D_5)
- \varepsilon \left( \bar{\beta}_0^2 D_1 \cos^2 \theta + D_5 \sin^2 \theta \mp 2 \bar{\beta}_0 D_2 \sin \theta \cos \theta \right),\]

\[Q_3 = (D_1 D_3 - D_2^2) D_5.\]

The three roots \( \zeta_j = \rho V_j^2 \), \((j = 1, 2, 3)\) of equation (3.14) relates to \( V_j \) \((j = 1, 2, 3)\) complex values of phase velocities of three waves, namely, qP, qSV and qT waves, respectively. Expressing \( V_j^{-1} = V_j^{* -1} - i \omega q_j \), \((j = 1, 2, 3)\), where clearly \( q_j \) is the coefficients of attenuation and \( V_j^* \) is the propagation speeds of qP, qSV and qT waves.

### 3.3 Particular cases of velocity equation (3.14)

#### 3.3.1 Rotating transversely isotropic magneto-thermoelastic case:

If we put \( C_{24} = C_{34} = 0 \), \( C_{23} = C_{33} - 2C_{44} \), then the equation (3.14) reduces for rotating transversely isotropic magneto-thermoelastic case as

\[Q_0^* \zeta^3 + Q_1^* \zeta^2 + Q_2^* \zeta + Q_3^* = 0, \quad (3.15)\]

where, \( Q_0^* = 4 \left( \frac{\Omega}{\omega} \right)^2 - \Omega^{*2}, \)

\[Q_1^* = \Omega^* (D_1^{\omega} + D_3^{\omega} + \varepsilon \sin^2 \theta + \bar{\beta}_0^3 \varepsilon \cos^2 \theta) + (\Omega^{*2} - 4 \left( \frac{\Omega}{\omega} \right)^2) D_5,\]

\[Q_2^* = D_2^{b2} - D_1^{b2} D_3^{\omega} - \Omega^* \left( D_1^{\omega} D_5 + D_3^{\omega} D_5 \right)
- \varepsilon \left( \bar{\beta}_0^2 D_1^{\omega} \cos^2 \theta + D_5^{\omega} \sin^2 \theta - 2 \bar{\beta}_0 D_2^{b2} \sin \theta \cos \theta \right),\]

\[Q_3^* = (D_1^{\omega} D_3^{\omega} - D_2^{b2}) D_5.\]

\[D_1^{\omega} = C_{22} \sin^2 \theta + C_{44} \cos^2 \theta + \mu_{e} H_0^2 \sin^2 \theta,\]

\[D_2^{b2} = (C_{33} - C_{44} + \mu_{e} H_0^2) \sin \theta \cos \theta,\]

\[D_3^{\omega} = C_{44} \sin^2 \theta + C_{33} \cos^2 \theta + \mu_{e} H_0^2 \cos^2 \theta,\]

The three roots \( \zeta_p = \rho V_p^2 \), \((p = 1, 2, 3)\) of equation (3.15) relate to complex value of phase velocities \( V_p \) \((p = 1, 2, 3)\) of qP, qSV and qT waves, respectively.
3.3.2 Rotating isotropic magneto-thermoelastic case: If we take

\[ \beta_0 = 1, \ K_2 = K_3 = K, \ C_{22} = C_{33} = \lambda + 2\mu, \ \beta_2^\prime = \beta_3^\prime = \beta_0^\prime, \ C_{13} = C_{23} = C_{12} = \lambda, \]

\[ C_{44} = C_{55} = C_{66} = \mu, \ C_{34} = 0, \ C_{24} = 0, \ C_{23} = C_{33} - 2C_{44}, \]

then the equation (3.14) reduces for rotating isotropic magneto-thermoelastic case as

\[ B_0 \zeta^3 + B_1 \zeta^2 + B_2 \zeta + B_3 = 0, \quad (3.16) \]

where, \( B_0 = 4 \left( \frac{\Omega^2}{\omega} \right)^2 - \Omega^*^2, \)

\[ B_1 = \Omega^* (D_1^\prime + D_3^\prime + \epsilon^c) + (\Omega^*^2 - 4 \left( \frac{\Omega^2}{\omega} \right)^2)D_5^\prime, \]

\[ B_2 = D_2^\epsilon^2 - D_1^\prime D_3^\prime - \Omega^* \left( D_1^\prime + D_3^\prime \right)D_5^\prime \]

\[- \epsilon^c \left( D_1^\prime \cos^2 \theta + D_3^\prime \sin^2 \theta - 2 D_5^\prime \sin \theta \cos \theta \right), \]

\[ B_3 = \left( D_1^\prime D_3^\prime - D_2^\epsilon^2 \right)D_5^\prime. \]

\[ D_1^\prime = (\lambda + 2\mu + \mu_\epsilon H_0^2) \sin^2 \theta + \mu \cos^2 \theta, \]

\[ D_3^\prime = (\lambda + \mu + \mu_\epsilon H_0^2) \sin \theta \cos \theta, \]

\[ D_5^\prime = \mu \sin^2 \theta + (\lambda + 2\mu + \mu_\epsilon H_0^2) \cos^2 \theta, \]

\[ D_5^\prime = \frac{D_0}{\tau^* C_E}, \quad D_3 = K, \quad \epsilon^c = \frac{\beta_0^2 T_0}{\rho C_E}, \]

and the terms \( D_1^\prime + D_3^\prime \) and \( D_1^\prime D_3^\prime - D_2^\epsilon^2 \) are independent of \( \theta \)

\[ D_1^\prime + D_3^\prime = [(\lambda + 2\mu + \mu_\epsilon H_0^2) \sin^2 \theta + \mu \cos^2 \theta] + [\mu \sin^2 \theta + (\lambda + \mu_\epsilon H_0^2 + 2\mu) \cos^2 \theta], \]

\[ = (\sin^2 \theta + \cos^2 \theta)(\lambda + \mu_\epsilon H_0^2 + 2\mu) + \mu(\cos^2 \theta + \sin^2 \theta), \]

\[ = (\lambda + 2\mu + \mu_\epsilon H_0^2) + \mu_\epsilon = \lambda + 3\mu + \mu_\epsilon H_0^2, \]

which is independent of \( \theta \).

\[ D_1^\prime D_3^\prime - D_2^\epsilon^2 = [(\lambda + 2\mu + \mu_\epsilon H_0^2) \sin^2 \theta + \mu \cos^2 \theta][\mu \sin^2 \theta + (\lambda + 2\mu + \mu_\epsilon H_0^2) \cos^2 \theta] \]

\[- [(\lambda + \mu + \mu_\epsilon H_0^2) \sin \theta \cos \theta]^2, \]
\[ D' \cos^2 \theta + D' \sin^2 \theta - 2 D' \sin \theta \cos \theta = \cos^2 \theta [(\lambda + 2\mu + \mu_r H_0^2) \sin^2 \theta + \mu \cos^2 \theta] + \sin^2 \theta [\mu \sin^2 \theta + (\lambda + 2\mu + \mu_r H_0^2) \cos^2 \theta] - 2(\lambda + \mu + \mu_r H_0^2) \sin \theta \cos \theta, \]
\[
= (\lambda + 2\mu + \mu_r H_0^2) \sin^2 \theta \cos^2 \theta + \mu \cos^4 \theta + \mu \sin^4 \theta + \cos^2 \theta(\lambda + 2\mu + \mu_r H_0^2) \sin^2 \theta - 2 \cos^2 \theta (\lambda + \mu + \mu_r H_0^2) \sin^2 \theta,
\]
\[
= \mu \cos^4 \theta + \mu \sin^4 \theta + \cos^2 \theta[(\lambda + 2\mu + \mu_r H_0^2) + (\lambda + 2\mu + \mu_r H_0^2)] - 2(\lambda + \mu + \mu_r H_0^2) \sin^2 \theta,
\]
\[
= \mu (\cos^4 \theta + \sin^4 \theta) + \cos^2 \theta[2(\lambda + \mu + \mu_r H_0^2) + 2\mu - 2(\lambda + \mu + \mu_r H_0^2)] \sin^2 \theta,
\]
\[
= \mu (\cos^4 \theta + \sin^4 \theta) + 2\mu \sin^2 \theta \cos^2 \theta = 2\mu \sin^2 \theta \cos^2 \theta + \mu (1 - 2\sin^2 \theta \cos^2 \theta),
\]

which is independent of \( \theta \). So in isotropic case all terms are independent of \( \theta \).

The three roots of equation (3.16), \( \zeta_p = \rho V_p^2 \), \((p = 1,2,3)\) relate to complex values of phase velocities \( V_p \) \((p = 1,2,3)\) of P, SV and T waves, respectively.

**3.3.3 Monoclinic magneto-thermoelastic case:** If rotation is neglected \((i.e. \Omega = 0, \Omega^* = 1)\), then the equation (3.14) reduces for monoclinic magneto-thermoelastic case as

\[ C_0 \zeta^3 + C_1 \zeta^2 + C_2 \zeta + C_3 = 0, \]  
(3.17)

where, \( C_0 = 1, C_1 = -(D_0 + D_5 + D_7 + \varepsilon \sin^2 \theta + \bar{R}_0^2 \varepsilon \cos^2 \theta) \),
\[ C_2 = D_1 D_3 - D_2^2 + (D_1 + D_3) D_5 + \varepsilon \left( \bar{\rho}^2 D_1 \cos^2 \theta + D_3 \sin^2 \theta \mp 2 \bar{\rho} D_2 \sin \theta \cos \theta \right), \]

\[ C_3 = (D_2^2 - D_1 D_3) D_5. \]

The three roots of equation (3.17), \( \zeta_p = \rho V_p^2, \) \((p = 1, 2, 3)\) relate to complex values of phase velocities \( V_p \) \((p = 1, 2, 3)\) of qP, qSV and qT waves, respectively.

### 3.3.4 Rotating Monoclinic thermoelastic case

If magnetic parameter is neglected \( (H_0 = 0,) \), then equation (3.14) reduces for rotating monoclinic thermoelastic case as

\[ R_0 \zeta^3 + R_1 \zeta^2 + R_2 \zeta + R_3 = 0, \quad (3.18) \]

where, \( R_0 = 4 \left( \frac{\Omega}{\omega} \right)^2 - \Omega^*^2, \)

\[ R_1 = \Omega^* (D_1^d + D_3^d + \varepsilon \sin^2 \theta + \bar{\rho}^2 \varepsilon \cos^2 \theta) + (\Omega^*^2 - 4 \left( \frac{\Omega}{\omega} \right)^2) D_5, \]

\[ R_2 = D_2^d - D_1^d D_3^d - \Omega^* \left( D_1^d + D_3^d \right) D_5 \]

\[ - \varepsilon \left( \bar{\rho}^2 D_1^d \cos^2 \theta + D_3^d \sin^2 \theta \mp 2 \bar{\rho} D_2^d \sin \theta \cos \theta \right), \]

\[ R_3 = (D_1^d D_3^d - D_2^d D_2^d) D_5. \]

\[ D_1^d = C_{22} \sin^2 \theta + (C_{44} \cos \theta \pm 2C_{24} \sin \theta) \cos \theta, \]

\[ D_2^d = C_{24} \sin^2 \theta + (C_{34} \cos \theta \pm (C_{23} + C_{44}) \sin \theta) \cos \theta, \]

\[ D_3^d = C_{44} \sin^2 \theta + (C_{35} \cos \theta \pm 2C_{34} \sin \theta) \cos \theta, \]

The three roots of equation (3.18), \( \zeta_p = \rho V_p^2, \) \((p = 1, 2, 3)\) relate to complex values of phase velocities \( V_p \) \((p = 1, 2, 3)\) of qP, qSV and qT waves, respectively.

### 3.4 Procedure to compute reflected angles

The ratios of amplitude depends on the velocities \( V_s (e_s), \) \((s = 1, 2, \ldots, 6)\) which are the functions of incident as well as reflected angles. For given incident qP wave, incident angle \( e_1 \) and therefore \( V_1 (e_1) \) is assumed to be known. One has to determine reflected angles \( e_4, e_5 \) and
e_6 for given incident angle e_1. The velocities V_4(e_4), V_5(e_5) and V_6(e_6) are determined by algebraic formulae explicitly. The technique is given below for finding out reflected angles e_4, e_5, e_6 from known values of incident angles. For known incident angle e_1, e_2, e_3 for the case of incident qP waves, incident qT waves and incident qSV wave respectively.

Putting \( \zeta = \rho V^2 \) in equation (3.14), it can be written as

\[
A_0 \left( \rho V^2 \right)^3 + A_4 \left( \rho V^2 \right)^2 + A_5 \rho V^2 + A_3 = 0, 
\]

(3.19)

Defining dimensionless apparent velocity \( \bar{V} \) as

\[
\bar{V} = \frac{V}{\beta_0} = \frac{V}{P_2 \beta_0}, 
\]

(3.20)

where, \( \beta_0 = \sqrt{\frac{C_{44}}{\rho}} \).

From equation (3.20), it is obtained as \( \rho V^2 = P_2^2 C_{44} \bar{V}^2 \) and putting this value in equation (3.19), then equation (3.19) becomes

\[
A_0 P_2^6 C_{44}^3 \bar{V}^6 + A_4 P_2^4 C_{44}^2 \bar{V}^4 + A_5 P_2^2 C_{44} \bar{V}^2 + A_3 = 0, 
\]

(3.21)

Dividing equation (3.21) by \( P_2^6 C_{44}^3 \) and putting

\[
\bar{C}_g = \frac{C_g}{C_{44}}, \quad \bar{K}_2 = \frac{K_2}{C_{44}}, \quad \bar{K}_3 = \frac{K_3}{C_{44}}, \quad \bar{\varepsilon} = \frac{\varepsilon}{C_{44}}, \quad \bar{\mu}_e = \frac{\mu_e}{C_{44}}, \quad V = \bar{V} P_2 \beta_0, 
\]

it is obtained

\[
A_0 \bar{V}^6 + A_4 \bar{V}^4 + A_5 \bar{V}^2 + A_3 = 0, 
\]

(3.22)

where, \( A_0 = 4 \left( \frac{\Omega}{\omega} \right)^2 - \Omega^2 \),

\[
\bar{A}_4 = \Omega^2 \left( \bar{B}_1 \bar{B}_3 + \bar{\varepsilon} + \bar{\varepsilon} \bar{\mu}_e \bar{p}^2 \right) + \bar{B}_4 \left( \Omega^2 - 4 \left( \frac{\Omega}{\omega} \right)^2 \right),
\]

\[
\bar{A}_5 = \bar{B}_2 - \bar{B}_1 \bar{B}_3 - \Omega^2 \left( \bar{B}_1 + \bar{B}_3 \right) \bar{B}_5 - \bar{\varepsilon} \left( \bar{\beta}_0 \bar{B}_1 \bar{p}_1^2 + \bar{B}_3 - 2 \bar{\beta}_0 \bar{D}_2 \bar{p} \right),
\]

\[
\bar{A}_3 = \left( \bar{B}_1 \bar{B}_3 - \bar{B}_2^2 \right) \bar{B}_5, \quad \bar{B}_1 = \bar{C}_{22} + \bar{p}^2 + 2 \bar{C}_{24} \bar{p} + \bar{p}_e \bar{H}_0^2,
\]
\[ \bar{D}_2 = \bar{C}_{24} + \bar{C}_{34} p^2 + \left(1 + \bar{C}_{23}\right) p + \tilde{\mu}_e H_0^2 p, \quad \bar{D}_3 = 1 + \bar{C}_{33} p^2 + 2\bar{C}_{34} p + \tilde{\mu}_e H_0^2 p^2, \]

\[ \bar{D}_5 = \frac{1}{\tau \star C_E} \left( \bar{K}_2 + \bar{K}_3 p^2 \right). \]

\[ P_2 = \sin \theta, P_3 = \cos \theta, \quad p = \frac{P_3}{P_2} = \cot \theta. \]

Here, \( p = -\cot \epsilon_1; \) \( p = -\cot \epsilon_2; \) \( p = -\cot \epsilon_3 \) are for incident qP waves, incident qT, and incident qSV waves respectively.

And \( p = -\cot \epsilon_4; \) \( p = \cot \epsilon_5; \) \( p = -\cot \epsilon_6 \) are for qP, qT, and qSV reflected waves respectively. The solution of equation (3.22), which is cubic in \((\bar{V}^2)\) can be obtained for \( \bar{V}^2 \) for chosen value of \( p \) and the three roots relates to qP, qT and qSV waves. Moreover the equation (3.22) is a six degree equation in \( p \) for a given value of \( \bar{V} \) and corresponds to incident qP, qT and qSV and reflected qP, qT and qSV waves. The positive roots and negative roots correspond to reflected waves and incident waves respectively.

Putting values of \( \bar{D}_1, \bar{D}_2, \bar{D}_3 \) and \( \bar{D}_5 \) in equation (3.22) and after simplification a six degree equation in \( p \) is obtained as

\[ g_0 p^6 + g_1 p^5 + g_2 p^4 + g_3 p^3 + g_4 p^2 + g_5 p + g_6 = 0, \]  

(3.23)

where, \( g_0 = \frac{1}{\tau \star C_E} \bar{K}_3 \left( \bar{C}_{33} - \bar{C}_{34}^2 + \tilde{\mu}_e H_0^2 \right), \)

\[ g_1 = \frac{2}{\tau \star C_E} \bar{K}_3 \left( \bar{C}_{24} \bar{C}_{33} - \bar{C}_{34} \bar{C}_{23} + \bar{C}_{24} \tilde{\mu}_e H_0^2 - \bar{C}_{34} \tilde{\mu}_e H_0^2 \right), \]

\[ g_2 = (\bar{C}_{34}^2 - \bar{C}_{33} - \bar{K}_0^2 - \tilde{\mu}_e H_0^2 - \frac{1}{\tau \star C_E} \bar{K}_3 (\Omega^* + \bar{C}_{33} \Omega^* + \Omega^* \tilde{\mu}_e H_0^2)) \bar{V}^2 \]

\[ + \frac{1}{\tau \star C_E} \bar{K}_3 (\bar{C}_{23} \bar{C}_{33} + 2\bar{C}_{24} \bar{C}_{34} + 1 - (1 + \tilde{\mu}_e H_0^2 + \tilde{\mu}_e H_0^2 \bar{C}_{22} + \bar{C}_{33} \tilde{\mu}_e H_0^2 - 2\tilde{\mu}_e H_0^2 - 2\bar{C}_{23} \tilde{\mu}_e H_0^2) \bar{V}^2 \]

\[ - 2\tilde{\mu}_e H_0^2 (\bar{C}_{23} \tilde{\mu}_e H_0^2) + \frac{1}{\tau \star C_E} \bar{K}_3 (\bar{C}_{33} - \bar{C}_{34}^2 + \tilde{\mu}_e H_0^2), \]
\[ g_3 = \{2\bar{C}_{34}\bar{C}_{23} - 2\bar{C}_{24}\bar{C}_{33} + 2\bar{C}_{34}\mu_e H_0^2 - 2\bar{C}_{24}\mu_e H_0^2 \]
\[ - \frac{2}{\tau * C_E} K_3(\Omega * \bar{C}_{34} + \Omega * C_{24}) + 2 \in (\bar{\beta}_0\bar{C}_{34} - \bar{\beta}_0^2\bar{C}_{24}))\bar{V}^2 \]
\[ + \frac{2}{\tau * C_E} K_3(\bar{C}_{22}\bar{C}_{34} - \bar{C}_{24}\bar{C}_{23} + \bar{C}_{34}\mu_e H_0^2 - \bar{C}_{24}\mu_e H_0^2) \]
\[ + \frac{2}{\tau * C_E} K_2(\bar{C}_{24}\bar{C}_{33} - \bar{C}_{34}\bar{C}_{23} + \bar{C}_{24}\mu_e H_0^2 - \bar{C}_{34}\mu_e H_0^2) \]
\[ g_4 = \{\Omega * +\bar{C}_{33}\Omega * +\Omega * \in \bar{\beta}_0^2 + \Omega * \bar{\mu}_e H_0^2 + \frac{1}{\tau * C_E} K_3(\Omega^{*2} - 4(\frac{\Omega}{\omega})^2)\bar{V}^4 \]
\[ + \{1 + \bar{C}_{23}\}^2 - 1 - \bar{C}_{22}\bar{C}_{33} - 2\bar{C}_{24}\bar{C}_{34} + 2\bar{\mu}_e H_0^2 + 2\bar{C}_{23}\bar{\mu}_e H_0^2 - \bar{C}_{22}\bar{\mu}_e H_0^2 \]
\[ - \bar{C}_{33}\bar{\mu}_e H_0^2 - \bar{C}_{22} \in \bar{\beta}_0^2 - \in \bar{C}_{33} + 2 \in \bar{\beta}_0 + 2\bar{C}_{23} \in \bar{\beta}_0 - \in \bar{\beta}_0^2 \bar{\mu}_e H_0^2 - \in \bar{\mu}_e H_0^2 \]
\[ + 2 \in \bar{\beta}_0\bar{\mu}_e H_0^2 - \frac{1}{\tau * C_E} K_3(\Omega * +\bar{C}_{23}\Omega * +\Omega * \bar{\mu}_e H_0^2) - \frac{1}{\tau * C_E} K_2(\Omega * +\bar{C}_{33}\Omega * +\Omega * \bar{\mu}_e H_0^2) \]
\[ \Omega * \bar{\mu}_e H_0^2)\bar{V}^2 + \frac{1}{\tau * C_E} K_2(\bar{C}_{22}\bar{C}_{33} + 2\bar{C}_{24}\bar{C}_{34} + 1 - (1 + \bar{C}_{23})^2 - 2\bar{\mu}_e H_0^2 \]
\[ - 2\bar{C}_{23}\bar{\mu}_e H_0^2 + \bar{C}_{22}\bar{\mu}_e H_0^2 + \bar{C}_{33}\bar{\mu}_e H_0^2 + \frac{1}{\tau * C_E} K_3(\bar{C}_{22} - \bar{C}_{24} + \bar{\mu}_e H_0^2), \]
\[ g_5 = 2[\Omega * (\bar{C}_{24} + \bar{C}_{34})\bar{V}^4 + \{\bar{C}_{24}\bar{C}_{23} - \bar{C}_{22}\bar{C}_{34} + \bar{C}_{24}\bar{\mu}_e H_0^2 - \bar{C}_{34}\bar{\mu}_e H_0^2 \]
\[ + \bar{C}_{24} \in \bar{\beta}_0 - \bar{C}_{34} \in \bar{\beta}_0^2 \} - \frac{1}{\tau * C_E} K_2(\Omega * \bar{\mu}_e H_0^2)\bar{V}^2 \]
\[ + \frac{1}{\tau * C_E} K_2(\bar{C}_{22}\bar{C}_{34} - \bar{C}_{24}\bar{C}_{23} - \bar{C}_{24}\bar{\mu}_e H_0^2 + \bar{C}_{34}\bar{\mu}_e H_0^2), \]
\[ g_6 = \{4(\frac{\Omega}{\omega})^2 - \Omega^{*2})\bar{V}^6 + \bar{C}_{22}\Omega * +\Omega * \in \bar{\mu}_e H_0^2 \]
\[ + \frac{1}{\tau C_E} K_2(\Omega^{*2} - 4(\frac{\Omega}{\omega})^2))\bar{V}^4 + \{\bar{C}_{24} - \bar{C}_{22} - \bar{\mu}_e H_0^2 - \in \]
\[ - \frac{1}{\tau * C_E} K_2(\Omega * +\bar{C}_{22}\Omega * +\Omega * \bar{\mu}_e H_0^2)\bar{V}^2 + \frac{1}{\tau * C_E} K_2(\bar{C}_{22} - \bar{C}_{24} + \bar{\mu}_e H_0^2). \]

Define \( q = \frac{1}{p} \) and then the equation (3.23) becomes
\[ g_6 q^6 + g_5 q^5 + g_4 q^4 + g_3 q^3 + g_2 q^2 + g_1 q + g_0 = 0, \quad (3.24) \]
For incident angle, all three qP, qSV and qT reflected wave exist, the equation (3.24) must have three positive roots. Out of three positive roots smaller one (say $q_5$) relates to reflected qT, the root ($q_6$) relates to reflected qSV and larger one ($q_4$) relates to reflected qP waves. Also,

$$e_4 = \tan^{-1}(q_4), e_5 = \tan^{-1}(q_5), e_6 = \tan^{-1}(q_6),$$

(3.25)

In case of isotropic thermo-elastic medium, putting

$$\bar{C}_{11} = \bar{C}_{22} = \bar{C}_{33} = \frac{\lambda + 2\mu}{\mu}, \quad \bar{C}_{12} = \bar{C}_{13} = \bar{C}_{23} = \frac{\lambda}{\mu},$$

$$\bar{p}_0 = 1, \quad \bar{K}_2 = \bar{K}_3 = \bar{K}, \quad \bar{C}_{14} = \bar{C}_{24} = \bar{C}_{34} = 0$$

$$\bar{C}_{44} = \bar{C}_{55} = \bar{C}_{66} = 1, \quad \gamma^* = \frac{\lambda + 2\mu}{\mu},$$

in equation (3.23), it is obtained

$$g_0'p^6 + g_1'p^5 + g_2'p^4 + g_3'p^3 + g_4'p^2 + g_5'p + g_6' = 0,$$

(3.26)

where, $g_0' = \frac{1}{\tau^* C_E} \bar{R} \left( \gamma^* + \bar{p}_e H_0^2 \right),$ $g_1' = 0,$

$$g_2' = \left[ \gamma^* + \bar{\varepsilon} + \bar{p}_e H_0^2 + \frac{1}{\tau^* C_E} \bar{R} (\Omega^* + \gamma^* \Omega^* + \Omega^* \bar{p}_e H_0^2) \right] \bar{V}^2$$

$$+ \frac{3}{\tau^* C_E} \bar{R} (\gamma^* + \bar{p}_e H_0^2),$$

$g_3' = 0,$

$$g_4' = \left[ (\Omega^* + \gamma^* \Omega^* + \bar{\varepsilon} \Omega^* + \bar{\gamma} \Omega^* \bar{p}_e H_0^2 + \frac{1}{\tau^* C_E} \bar{R} (\Omega^* + \gamma^* \Omega^* + \Omega^* \bar{p}_e H_0^2) \right] \bar{V}^4$$

$$+ \left[ -2\gamma^* - 2 \bar{\varepsilon} - 2 \bar{p}_e H_0^2 - \frac{2}{\tau^* C_E} \bar{R} (\gamma^* + \bar{p}_e H_0^2) \right] \bar{V}^2$$

$$+ \frac{3}{\tau^* C_E} \bar{R} (\gamma^* + \bar{p}_e H_0^2),$$

$g_5' = 0,$

$g_6' = 0,$
\[ g'_0 = \left\{ \frac{4(\frac{\Omega}{\omega})^2 - \Omega^*}{\Omega} \right\} \bar{V}_6 + \left\{ \frac{\Omega^* + \gamma^*}{\Omega} \right\} \bar{V}^* + \left\{ \frac{\Omega^* + \Omega^*}{\Omega} \right\} \bar{V}^* + \left\{ \frac{\Omega^* + \mu_e H_0^2}{\Omega} \right\} \bar{V}_6 + \frac{1}{\tau^* C_E} \bar{V}_6 \] 

Taking \( \frac{\bar{V}}{\tau^* C_E} \left( \gamma^* + \bar{\mu}_e H_0^2 \right) = S_{\gamma^*}, \quad \gamma^* + \bar{\mu}_e H_0^2 + \frac{1}{\tau^* C_E} \bar{\Omega}^* = r, \]

\[ \frac{r}{S_{\gamma^*}} + \Omega^* = d_1, \quad \frac{1}{S_{\gamma^*}} \left\{ \frac{\Omega^* + \Omega^*}{r - 4} \frac{1}{\tau^* C_E} \frac{\bar{K}(\frac{\Omega}{\omega})^2}{d_2}, \quad \frac{1}{S_{\gamma^*}} \left\{ \frac{(4(\frac{\Omega}{\omega})^2 - \Omega^*)}{d_3} \right\} \right. \]

the equation (3.26) becomes

\[ S_{\gamma^*} \left( p^2 - \delta_1^2 \right) \left( p^2 - \delta_2^2 \right) \left( p^2 - \delta_3^2 \right) = 0, \quad (3.27) \]

where, \( \delta_1^2 + \delta_2^2 + \delta_3^2 = d_1 \bar{V}_2 - 3, \)

\( \delta_1^2 \delta_2^2 + \delta_2^2 \delta_3^2 + \delta_3^2 \delta_1^2 = d_2 \bar{V}_4 - 2d_1 \bar{V}_4 - 2d_1 \bar{V}_2 + 3, \)

\( \delta_1^2 \delta_2^2 \delta_3^2 = -d_1 \bar{V}_6 + d_2 \bar{V}_4 - d_1 \bar{V}_2 + 1. \)

In this case the Snell’s law take the form of

\[ \frac{\sin e_1}{V_{qP}} = \frac{\sin e_2}{V_{qT}} = \frac{\sin e_3}{V_{qSV}}, \quad (3.28) \]

Therefore, the roots \( p^2 = \delta_1^2 = \cot^2 e_1, \quad p^2 = \delta_2^2 = \cot^2 e_2 \) and \( p^2 = \delta_3^2 = \cot^2 e_3 \) relate to qSV, qP and qT waves, respectively. For \( \frac{1}{p} = q, \)

\[ q_1 = -\tan e_1, \quad q_2 = -\tan e_2, \quad q_3 = -\tan e_3, \quad q_4 = \tan e_1, \quad q_5 = \tan e_2, \quad q_6 = \tan e_3 \]

are the six roots of the equation (3.24) for isotropic medium. This choice will act as a helping factor in calculating the reflected angles of qP, qT and qSV waves in a monoclinic magneto-thermoelastic rotating medium. In case of orthotropic, it can be proved that in (3.24) \( g_1, g_3, g_5 \) become zero i.e. \( g_1 = g_3 = g_5 = 0. \) Therefore equation (3.24) reduced to a cubic equation in \( q^2. \)

So, it can be taken as \( q_1 = -q_4, q_2 = -q_5, q_3 = -q_6. \) Therefore, reflected angle of qP, qT and qSV waves are equal to incident angle of qP, qT and qSV waves. For monoclinic case this is not
true. The angles of reflection for a particular incident wave for monoclinic magneto-thermoelastic rotating medium may be computed by following the above procedure.

3.5 Particular cases:

3.5.1 Monoclinic elastic case: If rotation, thermal field and magnetic field are neglected (i.e. \( H_0 = 0, \Omega = 0, \Omega^* = 1, D_4 = 0, \varepsilon = 0, K_2 = K_3 = 0, \beta_2 = \beta_3 = 0, \beta_0 = 0 \)) then the equation (3.14) reduces for monoclinic elastic case as

\[
E_0 \xi^2 + E_1 \xi + E_2 = 0,
\]

where, \( E_0 = 1 \),

\[
E_1 = -(D'_1 + D'_2),
\]

\[
E_2 = D'_1 D'_2 - D_2^2,
\]

\[
D'_1 = C_{22} \sin^2 \theta + (C_{44} \cos \theta \pm 2C_{24} \sin \theta) \cos \theta,
\]

\[
D'_2 = C_{34} \sin^2 \theta + (C_{33} \cos \theta \pm (C_{23} + C_{44}) \sin \theta) \cos \theta,
\]

\[
D'_3 = C_{44} \sin^2 \theta + (C_{33} \cos \theta \pm 2C_{34} \sin \theta) \cos \theta,
\]

Equation (3.23) reduces to

\[
L_0 p^4 + L_1 p^3 + L_2 p^2 + L_3 p + L_4 = 0,
\]

where,

\[
L_0 = (\bar{C}_{33} - \bar{C}_{34} q),
\]

\[
L_1 = 2(\bar{C}_{24} \bar{C}_{33} - \bar{C}_{34} \bar{C}_{23}),
\]

\[
L_2 = -(1 + \bar{C}_{33}) \bar{V}^2 + (\bar{C}_{22} \bar{C}_{33} + 2\bar{C}_{24} \bar{C}_{34} - \bar{C}_{23}^2 - 2\bar{C}_{23}),
\]

\[
L_3 = -2(C_{24} + \bar{C}_{34}) \bar{V}^2 + 2(\bar{C}_{22} \bar{C}_{34} - \bar{C}_{24} \bar{C}_{23}),
\]

\[
L_4 = \bar{V}^4 - (1 + \bar{C}_{22}) \bar{V}^2 + (\bar{C}_{22} - \bar{C}_{24}^2)
\]

Define \( q = \frac{1}{p} \) and then the equation (3.24) reduces to

\[
L_4 q^4 + L_3 q^3 + L_2 q^2 + L_1 q + L_0 = 0,
\]

For incident angle, for qP, and qSV reflected wave exist, the equation (3.31) has two roots which are positive smaller one say \( q_6 \), correspond to reflected qSV, and larger one say \( q_4 \) corresponds to reflected qP waves. Also, we have
\[ e_4 = \tan^{-1}(q_4), \quad e_6 = \tan^{-1}(q_6). \]

For isotropic case

the Snell’s law in this case becomes

\[
\frac{\sin e_1}{V_{qP}} = \frac{\sin e_3}{V_{qSV}} = \frac{1}{V_a},
\]

where, \( V = \frac{V_a}{\beta_0} = \frac{V}{P_2\beta_0}, \)

\[
\bar{V} = \frac{V_a}{\beta_0} = \frac{V}{P_2\beta_0} = \cos ec(e_3) = \sqrt{\gamma^*} \cos ec(e_2),
\]

\[
\alpha' = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad \beta' = \sqrt{\frac{\mu}{\rho}}, \quad \gamma^* = \frac{\lambda + 2\mu}{\mu} = (\alpha' / \beta')^2.
\]

and for this case the equation (3.26) reduces to

\[
\gamma^* p^4 + (2\gamma^* - (1 + \gamma^*)\bar{V}^2)p^2 + (\bar{V}^4 - (1 + \gamma^*)\bar{V}^2 + \gamma^*) = 0,
\]

which can be written as

\[
\gamma^* p^4 + (2\gamma^* - (1 + \gamma^*)\bar{V}^2)p^2 + [(\bar{V}^2 - 1)(\bar{V}^2 - \gamma^*)] = 0,
\]

This further can be simplified as

\[
\gamma^* (p^2 - \bar{V}^2 + 1)(p^2 - \bar{V}^2 / \gamma^* + 1) = 0,
\]

the roots given by equation (3.34) \( p^2 = \bar{V}^2 - 1 = \cot^2 e_1, \) relates to P wave and \( p^2 = \bar{V}^2 / \gamma^* - 1 = \cot^2 e_3, \) relates to SV wave and for \( q = \frac{1}{p} \)

\[ q_i = -\tan e_1, \quad q_3 = -\tan e_3, \quad q_4 = \tan e_1, \quad q_6 = \tan e_3. \]

This choice will act as a helping factor in calculating reflected angles of \( qP \) and \( qSV \) waves in a monoclinic medium. In case of orthotropic, it can be shown that \( L_1 = L_3 = 0 \). So equation (3.31) becomes quadratic in \( q^2 \). Thus, we can choose \( q_1 = -q_4, q_3 = -q_6 \). So, reflected angles of \( qP \), and \( qSV \) waves is equal to incident angle of \( qP \), and \( qSV \) waves. In case of monoclinic medium this is not true. The angles of reflection for a particular incident wave for monoclinic medium may be computed by following the above procedure. The result is same as obtained by Singh and Khurana [44].
3.5.2 Monoclinic thermoelastic case: If rotation and magnetic field are neglected (i.e. \( \Omega = 0, \Omega^* = 1, H_0 = 0 \), then the equation (3.14) reduces for monoclinic thermoelastic elastic case as

\[
M_0 \zeta^3 + M_1 \zeta^2 + M_2 \zeta + M_3 = 0,
\]

where, \( M_0 = 1 \),

\[
M_1 = -(D_1^\varepsilon + D_3^\varepsilon + D_5 + \varepsilon \sin^2 \theta + \beta_0^2 \varepsilon \cos^2 \theta),
\]

\[
M_2 = D_1^\varepsilon D_3^\varepsilon - D_2^\varepsilon^2 + (D_1^\varepsilon + D_3^\varepsilon)D_5
+ \varepsilon (\beta_0^2 D_1^\varepsilon \cos^2 \theta + D_3^\varepsilon \sin^2 \theta + 2\beta_0^2 D_5^\varepsilon \sin \theta \cos \theta),
\]

\[
M_3 = (D_2^\varepsilon^2 - D_1^\varepsilon D_3^\varepsilon)D_5,
\]

\[
D_1^\varepsilon = C_{22} \sin^2 \theta + (C_{44} \cos \theta \pm 2C_{24} \sin \theta) \cos \theta,
\]

\[
D_2^\varepsilon = C_{24} \sin^2 \theta + (C_{34} \cos \theta \pm (C_{23} + C_{44}) \sin \theta) \cos \theta,
\]

\[
D_3^\varepsilon = C_{44} \sin^2 \theta + (C_{33} \cos \theta \pm 2C_{34} \sin \theta) \cos \theta,
\]

For this case equation (3.23) reduces to

\[
N_0 p^6 + N_1 p^5 + N_2 p^4 + N_3 p^3 + N_4 p^2 + N_5 p + N_6 = 0,
\]

where, \( N_0 = \frac{1}{r^* C_E} K_3 (\bar{C}_{34}^2 - \bar{C}_{33}) \), \( N_1 = \frac{2}{r^* C_E} K_3 (\bar{C}_{34} \bar{C}_{23} - \bar{C}_{24} \bar{C}_{33}) \),

\[
N_2 = \{ \bar{C}_{33}^2 - \bar{C}_{34}^2 + \frac{1}{r^* C_E} K_3 (1 + \bar{C}_{23}) + \bar{C}_{23} \} \bar{V}^2 - \frac{1}{r^* C_E} K_3 (\bar{C}_{22} \bar{C}_{33}),
\]

\[
N_3 = \{ 2\bar{C}_{24} \bar{C}_{33} - 2\bar{C}_{34} \bar{C}_{23} + \frac{2}{r^* C_E} K_3 (\bar{C}_{34} + \bar{C}_{24} + \bar{C}_{23} \bar{C}_{34} - \bar{C}_{24} \bar{C}_{33}) \} \bar{V}^2
- \frac{2}{r^* C_E} K_3 (\bar{C}_{22} \bar{C}_{34} - \bar{C}_{24} \bar{C}_{33}) - \frac{2}{r^* C_E} K_3 (\bar{C}_{24} \bar{C}_{33} - \bar{C}_{34} \bar{C}_{23}),
\]
\[ N_4 = \{-1 - \bar{C}_{33} + \varepsilon \beta_0^2 + \frac{1}{\tau * C_E} \bar{K}_3\} \bar{V}^4 + \{\bar{C}_{22} \bar{C}_{33} + 2\bar{C}_{24} \bar{C}_{34} - \bar{C}_{23}^2 - 2\bar{C}_{23}\] \\
\[ + \bar{C}_{22} \varepsilon \beta_0^2 + \varepsilon \bar{C}_{33} - 2 \varepsilon \beta_0 - 2\bar{C}_{23} \varepsilon \beta_0 + \frac{1}{\tau * C_E} \bar{K}_3(1 + \bar{C}_{22}) \] \\
\[ + \frac{1}{\tau * C_E} \bar{K}_2(\bar{C}_{24}^2 + 2\bar{C}_{23} - \bar{C}_{22} \bar{C}_{33} - 2\bar{C}_{24} \bar{C}_{34}) \] \\
\[ + \frac{1}{\tau * C_E} \bar{K}_3(\bar{C}_{24}^2 - \bar{C}_{22}), \]

\[ N_5 = 2[(\bar{C}_{24} - \bar{C}_{34}) \bar{V}^4 + \{(\bar{C}_{22} \bar{C}_{34} - \bar{C}_{24} \bar{C}_{23} - \bar{C}_{24} \varepsilon \beta_0 + \bar{C}_{34} \varepsilon\}] + \frac{1}{\tau * C_E} \bar{K}_2(\bar{C}_{24} + \bar{C}_{23})\bar{V}^2 \] \\
\[ + \frac{1}{\tau * C_E} \bar{K}_2(\bar{C}_{24} \bar{C}_{23} - \bar{C}_{22} \bar{C}_{34})), \]

\[ N_6 = [\bar{V}^6 + \{-1 + \bar{C}_{22} + \varepsilon + \frac{1}{\tau * C_E} \bar{K}_2\}]\bar{V}^4 + \{\bar{C}_{22} - \bar{C}_{24}^2 + \varepsilon\] \\
\[ + \frac{1}{\tau * C_E} \bar{K}_2(1 + \bar{C}_{22})\bar{V}^2 + \frac{1}{\tau * C_E} \bar{K}_2(\bar{C}_{24}^2 - \bar{C}_{22})]. \]

Taking \( q = \frac{1}{p} \) and then for this case the equation (3.24) reduces to

\[ N_6 q^6 + N_5 q^5 + N_4 q^4 + N_3 q^3 + N_2 q^2 + N_1 q + N_0 = 0, \quad (3.37) \]

For isotropic case equation (3.26) reduces to

\[ \left(\frac{-\bar{K} \gamma^*}{\tau * C_E}\right) p^6 + \left(\{\gamma^* + \varepsilon + \frac{1}{\tau * C_E} \bar{K}(1 + \gamma^*)\} \bar{V}^2 + \frac{-3\bar{K} \gamma^*}{\tau * C_E}\right) p^4 \]
\[ + \left(\{-1 + \gamma^* + \varepsilon + \frac{\bar{K}}{\tau * C_E}\} \bar{V}^4 + \{2\gamma^* + 2 \varepsilon + \frac{2\bar{K}}{\tau * C_E} (1 + \gamma^*)\} \bar{V}^2 - \frac{3\bar{K} \gamma^*}{\tau * C_E}\right) p^2 \]
\[ + \left(\bar{V}^6 - \{1 + \gamma^* + \varepsilon + \frac{\bar{K}}{\tau * C_E}\} \bar{V}^4 + \{\gamma^* + \varepsilon + \frac{\bar{K}}{\tau * C_E} (1 + \gamma^*)\} \bar{V}^2 - \frac{\bar{K} \gamma^*}{\tau * C_E}\right) = 0, \quad (3.38) \]

Putting \( s' = \frac{-\bar{K}}{\tau * C_E}, \quad l^* = \gamma^* + \varepsilon + \frac{\bar{K}}{\tau * C_E}, \) in equation (3.38) we get

\[ (-s' \gamma^*) p^6 + \left(\{l^* + s' \gamma^*\} \bar{V}^2 - 3s' \gamma^*\right) p^4 + \left(\{-1 + l^*\} \bar{V}^4 + 2\bar{L} + s' \gamma^*\right) p^2 + \left(\bar{V}^6 - \{1 + l^*\} \bar{V}^4 + \bar{I} - s' \gamma^*\right) = 0, \quad (3.39) \]

Equation (3.39) can be written in this case as

\[ -s' \gamma^* (1 + p^2 - \frac{\alpha_{i}}{\gamma^*} \bar{V}^2)(1 + p^2 - \bar{V}^2)(1 + p^2 - \frac{\bar{K}}{\gamma^*} \bar{V}^2) = 0, \quad (3.40) \]
where, \( \alpha' = \frac{l' + \sqrt{l'^2 - 4s'\gamma'}}{2s'}, \quad \beta' = \frac{l' - \sqrt{l'^2 - 4s'\gamma'}}{2s'}, \)

using (3.28) Snell’s law the roots given by equation (3.40) \( p^2 = \frac{V^2}{\gamma} - 1 = \cot^2 e_3, \)
corresponding to SV wave and \( p^2 = \frac{\alpha' V^2}{\gamma} - 1 = \cot^2 e_1, \)
corresponding to P wave and \( p^2 = \frac{\beta' V^2}{\gamma} - 1 = \cot^2 e_2, \)
corresponding to T wave, and for \( \frac{1}{p} = q \)

\[ q_1 = -\tan e_1, \quad q_2 = -\tan e_2, \quad q_3 = -\tan e_3, \quad q_4 = \tan e_1, \quad q_5 = \tan e_2, \quad q_6 = \tan e_3, \]

are the six roots of equation (3.37) for isotropic case.

This choice will act as a helping factor in calculating reflected angles of \( qP, qT \) and \( qSV \) waves in a monoclinic thermoelastic medium. In case of orthotropic, it can be proved that \( N_1 = N_3 = N_5 = 0 \). Therefore equation (3.37) reduced to a cubic equation in \( q^2 \). Therefore, taking \( q_1 = -q_4, q_2 = -q_5, q_3 = -q_6 \). Therefore, reflected of \( qP, qT \) and \( qSV \) waves are equal to the incident angle of \( qP, qT \) and \( qSV \) waves. In case of monoclinic medium this is not true. The angles of reflection for a particular incident wave for monoclinic thermoelastic medium may be computed by following the above procedure. The result is same as obtained by Singh et al. [185].

### 3.6 Discussion of results

Numerical illustrations have been performed to calculate speeds of plane waves by taking the following relevant physical constants of the medium satisfying the inequalities between constants.

\[
C_{33} = 24.9 \times 10^9 \text{N.m}^{-2}, \quad C_{22} = 19.8 \times 10^9 \text{N.m}^{-2}, \quad C_{44} = 6.67 \times 10^9 \text{N.m}^{-2},
\]

\[
C_{23} = 7.8 \times 10^9 \text{N.m}^{-2}, \quad C_{34} = \frac{C_{44}}{5}, \quad C_{24} = \frac{C_{44}}{5},
\]

\[
C_{11} = 3.9 \times 10^2 \text{J.Kg}^{-1}\text{deg}^{-1}, \quad K_2 = 1.24 \times 10^2 \text{Wm}^{-1}\text{deg}^{-1}, \quad \rho = 2.714 \times 10^3 \text{Kg.m}^{-3},
\]

\[
K_3 = 1.34 \times 10^2 \text{Wm}^{-1}\text{deg}^{-1}, \quad \beta_5^* = 5.75 \times 10^6 \text{N.m}^{-2}\text{deg}^{-1}, \quad \tau_o = 0.05\text{s},
\]

\[
\beta_3^* = 5.17 \times 10^6 \text{N.m}^{-2}\text{deg}^{-1}, \quad \omega = 5\text{Hz}, \quad T_o = 296\text{K},
\]
Equation (3.14) is solved numerically to get the real propagation speeds \( V_j^* \) of qP, qSV and qT waves in rotating monoclinic magneto-thermoelastic medium. The \( V_j^* \) of qP, qSV and qT waves are plotted versus incident angle varying from \( 0^\circ \) to \( 90^\circ \) depicted in Figure 3.1(a) to 3.1(c) for \( H_0 = 10 \) and \( \Omega/\omega = 0, \) 2 and 10. The speed \( V_j^* \) of qP wave is 17.93 m.s\(^{-1}\) at \( \theta = 0^\circ \) for \( \Omega/\omega = 0. \) It increases gradually upto 19.6 m.s\(^{-1}\) at \( \theta = 90^\circ \). With the increase in rotation, it decreases at each incident angle. The speeds \( V_j^* \) of qSV wave is 1.567 m.s\(^{-1}\) at \( \theta = 0^\circ \) and \( \theta = 90^\circ \) for \( \Omega/\omega = 0. \) It increases to its highest value \( 1.682 \) m.s\(^{-1}\) \( \theta = 45^\circ \). With an increase in rotation, the speeds \( V_j^* \) of qSV waves decreases at each incident angle. The speeds \( V_j^* \) of qT wave is 0.2962 m.s\(^{-1}\) at \( \theta = 0^\circ \) for \( \Omega/\omega = 0. \) It first increases to 0.2967 ms\(^{-1}\) at \( \theta = 4^\circ \) and then decreases sharply to its least value 0.255 m.s\(^{-1}\) at \( \theta = 90^\circ \). From Figure 3.1(c), it can be observed that effect of rotation on qT wave increases with increase in rotation and is different from as observed in case of qP and qSV waves. The comparison of black curves with green and red curves in Figures 3.1(a) to 3.1(c) shows the effect of rotation on speeds \( V_j^* \) of qP, qSV and qT waves at each incident angle.

The speeds \( V_j^* \) of qP, qSV and qT waves are plotted versus incident angle varying from \( 0^\circ \) to \( 90^\circ \) shown in Figures 3.2(a) to 3.2(c) for \( H_0 = 0, 10, 20 \) and \( \Omega/\omega = 2. \) The speeds \( V_j^* \) of qP wave is 12.6 m.s\(^{-1}\) at \( \theta = 0^\circ \) for \( H_0 = 0. \) It increases gradually upto 13.9 m.s\(^{-1}\) at \( \theta = 90^\circ \). With the increase in magnetic field, it increases at each incident angle. The speeds \( V_j^* \) of qSV wave is 0.699 m.s\(^{-1}\) at \( \theta = 0^\circ \) and \( \theta = 90^\circ \) for \( H_0 = 0. \) It attains its highest value \( 0.740 \) m.s\(^{-1}\) \( \theta = 47^\circ \). With an increase in magnetic field, the speeds \( V_j^* \) of qSV waves also increases at each incident angle except at \( \theta = 0^\circ \) and \( \theta = 90^\circ \). The speeds \( V_j^* \) of qT wave is 0.140 m.s\(^{-1}\) at \( \theta = 0^\circ \) for \( H_0 = 0. \) It first increases to 0.141 m.s\(^{-1}\) at \( \theta = 3^\circ \) and then decreases gradually to its least value 0.108 m.s\(^{-1}\) at \( \theta = 90^\circ \). The speeds \( V_j^* \) of qT wave also increases at each incident angle with the increase in magnetic field. The comparison of black curves with green and red curves in Figures 3.2(a) to 3.2(c) shows the influence of magnetic field on qP, qSV and qT waves.

qP, qSV and qT wave speeds are plotted versus magnetic field varying from 0 to 50 in Figures 3.3(a) to 3.3(c) for angle of incidence \( \theta = 45^\circ \) and \( \Omega/\omega = 0, 4 \) and 8. From Figures 3.3(a) to 3.3(c), it is noticed that effect of rotation increases with the increase in magnetic field at a fixed incident angle.

The speeds \( V_j^* \) of qP, qSV and qT waves are also plotted versus the angle of reflection varying from \( 0^\circ \) to \( 90^\circ \) in Figures 3.4(a) to 3.4(c) for \( H_0 = 10 \) and \( \Omega/\omega = 0, 2 \) and 10. The variation of speeds \( V_j^* \) of qP, qSV and qT waves in Figures 3.4(a) to 3.4(c) is similar to those
in Figures 3.1(a) to 3.1(c). But the values of speeds of plane waves in Figures 3.4(a) to 3.4(c) are slightly different in decimal places from those obtained in case of angle of incidence in Figures 3.1(a) to 3.1(c). The comparison of black curves with green and red curves in Figure 3.4(a) to 3.4(c) shows the effect of rotation on qP, qSV and qT wave speeds at each reflected angle.

The speeds \( V_j^* \) of qP, qSV and qT waves are plotted versus the rotation parameter \( \Omega \) varying from 4 to 20 in Figures 3.5(a) to 3.5(c) for angle of reflection \( \theta = 45^\circ \), \( H_0 = 0, 10, 20 \) and \( \omega = 2 \). These speeds decrease rapidly in range \( 4 \leq \Omega \leq 20 \). For example at \( H_0 = 0 \), the speeds \( V_j^* \) of qP wave at \( \Omega = 4 \) is 13.27 m.s\(^{-1}\). It decreases to 1.81 m.s\(^{-1}\) at \( \Omega = 20 \). The speeds \( V_j^* \) of qSV wave at \( \Omega = 4 \) is 0.738 m.s\(^{-1}\). It decreases to 0.165 m.s\(^{-1}\) at \( \Omega = 20 \). The speeds \( V_j^* \) of qT at \( \Omega = 4 \) is 0.0725 m.s\(^{-1}\). It oscillates slightly for range \( 4 \leq \Omega \leq 20 \). From Figures 3.5(a) to 3.5(c), it is observed that qP, qSV and qT wave speeds are influenced by rotation and magnetic field at a fixed angle of reflection.
Figure 3.1(a) Variations of speeds ($V_j^*$) of qP plane waves versus incident angle ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$.

Figure 3.1(b) Variations of speeds ($V_j^*$) of qSV plane waves versus incident angle ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$. 
Figure 3.1(c) Variations of speeds ($V_j^*$) of qT plane waves versus incident angle ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$.

Figure 3.2(a) Variations of speeds ($V_j^*$) of qP plane waves versus incident angle ($\theta$) when rotation-frequency ratio $\Omega/\omega = 2$ and magnetic field $H_0 = 0, 10, 20$. 
Figure 3.2(b) Variations of speeds ($V_j^*$) of qSV plane waves versus incident angle ($\theta$) when rotation-frequency ratio $\Omega/\omega = 2$ and magnetic field $H_0 = 0, 10, 20$

Figure 3.2(c) Variations of speeds ($V_j^*$) of qT plane waves versus incident angle ($\theta$) when rotation-frequency ratio $\Omega/\omega = 2$ and magnetic field $H_0 = 0, 10, 20$
Figure 3.3(a) Variations of speeds ($V_j^*$) of qP plane waves versus the magnetic field $H_0$ when incident angle $\theta$ is $45^\circ$ and rotation-frequency ratio $\Omega/\omega = 0, 4, 8$.

Figure 3.3(b) Variations of speeds ($V_j^*$) of qSV plane waves against the magnetic field $H_0$ when angle of incidence $\theta$ is $45^\circ$ and rotation-frequency ratio $\Omega/\omega = 0, 4, 8$. 
Figure 3.3(c) Variations of speeds ($V_j^*$) of qT plane waves against the magnetic field $H_0$ when angle of incidence $\theta$ is 45° and rotation-frequency ratio $\Omega/\omega = 0, 4, 8$.

Figure 3.4(a) Variations of speeds ($V_j^*$) of qP plane waves against the angle of reflection ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$. 
Figure 3.4(b) Variations of speeds ($V_j^*$) of qSV plane waves against the angle of reflection ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$.

Figure 3.4(c) Variations of speeds ($V_j^*$) of qT plane waves against the angle of reflection ($\theta$) when magnetic field $H_0 = 10$ and rotation-frequency ratio $\Omega/\omega = 0, 2, 10$. 
Figure 3.5(a) Variations of speeds ($V_j^*$) of qP plane waves against the rotation $\Omega$ when angle of reflection ($\theta$) is 45°, angular frequency $\omega = 2$ Hz and magnetic field $H_0 = 0, 10, 20$.

Figure 3.5(b) Variations of speeds ($V_j^*$) of qSV plane waves against the rotation $\Omega$ when angle of reflection ($\theta$) is 45°, angular frequency $\omega = 2$ Hz and magnetic field $H_0 = 0, 10, 20$. 
Figure 3.5(c) Variations of speeds ($V_j^*$) of qT plane waves against the rotation $\Omega$ when angle of reflection ($\theta$) is $45^\circ$, angular frequency $\omega = 2$ Hz and magnetic field $H_0 = 0, 10, 20$. 