CHAPTER 4

FUZZY CLUSTERING FOR BRAIN TUMOR SEGMENTATION ON MRI

4.1 INTRODUCTION

This chapter starts with the preprocessing of MRI images for bias field correction or Image intensity inhomogeneity correction and Skull stripping or Extracranial tissue removal. The remaining part of the chapter is organized as follows: Section 4.3 describes the fuzzy clustering methods for brain tumor segmentation. Section 4.4 describes the proposed Enhanced Possibilistic Fuzzy C-Means (EPFCM) method. Section 4.5 presented the experimental results and discussion. Finally in Section 4.6 some conclusions are given.

4.2 PREPROCESSING

The images are processed to remove unwanted components present in it before applying to the any task. This process is called as image preprocessing. In this section, discussion on two preprocessing steps: Bias field correction or Image intensity inhomogeneity correction and Skull stripping or Extracranial tissue removal process are presented. Bias field correction is very important before performing any task on MRI images, especially more essential in the case of segmentation processes.
4.2.1 Bias field correction

A bias field is a low frequency smooth undesirable signal that corrupts MRI images because of the inhomogeneities in the magnetic fields of the MRI machine. Bias field blurs images and thus reduces the high frequency contents of the image such as edges and contours and changes the intensity values of image pixels so that the same tissue has different gray level distribution across the image. A low level variation will not have great impact on clinical diagnosis. However it degrades the performance of image processing algorithms such as segmentation and classification or any algorithm that is based on the assumption of spatial invariance of the processed image. A preprocessing step is needed to correct for the effect of bias field before doing segmentation or classification. Here is a list of the most common sources of MRI intensity inhomogeneity (Hou 2006).

- Imperfections in the gradient coils and abnormal currents through the gradient coils.
- Radio-Frequency (RF) coil related effects are caused by coil imperfections, by non-uniform sensitivity of the coil (e.g. in surface coils) or by ferro-magnetic objects in the imaged object (e.g. amalgam dental fillings, orthopaedic implants). The main effect is a variation of the intensity values across the MR image.
- Physical movement of the patient undergoing the scan
- Poor RF penetration due to absorption of the RF signal leads to darker inner regions of the imaged objects compared to the outer regions
Therefore, it is required to correct bias field in the image before processing the image for any task such as tumor segmentation. An automatic method based on entropy minimization introduced by Mangin (2000) have been used for bias field correction. Figure 4.1(b) shows the example of bias field corrected image of original image in Figure 4.1(a).

![Bias field correction](image)

**Figure 4.1** Bias field correction. (a) An axial slice of the original image. (b) Bias field corrected original image.

### 4.2.2 Skull stripping

The skull stripping problem is equivalent to the segmentation of the whole brain or the removal of non-cerebral tissue such as skull, scalp, veins or meninges. In this work, focus on the problem of extracting the brain from an MR Image i.e., removal of outer ring (skull) of the MR images. Many applications, such as presurgical planning, cortical surface reconstruction, delineation of anatomical structures and other regions of interests, known as image segmentation and brain morphometry, depend on the ability to accurately segment brain from non-brain tissue, e.g. remove extra-cerebral tissue such as skull, sclera, orbital fat, skin, etc. In addition to manual approaches, the primary bases for skull-stripping include intensity threshold, morphology, watershed, surface-modeling, and hybrid methods. In this research work the skull-stripping problem have been approached by employing the erosion, dilation and region filling morphological operations (Gonzalez and Woods 2011) iteratively. This gives us better removing of
skull for MRI images. One example of result of skull stripping using morphological operation is as shown in Figure 4.2

![Image](image.jpg)

**Figure 4.2** Skull stripping using morphological operation: (a) Original image (b) Skull removed image

### 4.3 FUZZY CLUSTERING METHODS FOR BRAIN TUMOR SEGMENTATION

Based on the discussion of different tumor segmentation methods in the chapter 3, region based clustering method called unsupervised fuzzy segmentation methods are of considerable benefits, because the uncertainty of MRI image is widely presented in data. In particular, the transitional regions between tissues like CSF, WM, GM, and tumor are not clearly defined and their memberships are intrinsically vague (Liu 2003). The fuzzy methods could retain much more information from the original image than other segmentation methods. Standard unsupervised fuzzy c-means (FCM) clustering algorithm is the best known and powerful method in fuzzy segmentations methods (Hou 2007). Unlike the crisp k-means clustering algorithm, which forces pixels to belong exclusively to one class, FCM allows pixels to belong to multiple clusters with varying degrees of membership. Because of medical images always include considerable uncertainty and unknown noise caused by operator performance, equipment, and the environment, it is very difficult for the standard FCM algorithm to segment
medical images with those condition. To segment the medical image with those condition mentioned above the spatial information on neighborhood also considered, this is not possible in standard FCM. So, standard FCM algorithm has proven to be problematic for medical images (Cai 2007).

Several researchers have proposed the approaches for increase the robustness of FCM to noise by modifying the objective function (Pham 2001, Ahmed 2002, Liew 2003, Shen 2005, Ma and Staunton 2007 and Hou 2007). Pham (2001) modified the objective function to discourage undesirable configurations according to the neighborhood of the pixels. In Liew (2003) the distance is weighted by a term based on the difference between the membership values of pixels in the neighborhood of the pixel. In the work of Ahmed (2002), Shen (2005) and Ma and Staunton (2007) a term is added to the objective function that allows the labeling of a pixel to be influenced by the labels in its immediate neighborhood. In the proposed methods the objective function is modified to make the algorithm to be indirectly similar to the Markov random field (MRF). In Hou (2007) a regularized FCM method is presented. Their method exploits a moving average filter as the regularizer, and the average of the fuzzy memberships of the neighboring pixels is introduced into the standard FCM objective function.

All these versions of the standard FCM clustering algorithm can reduce the noise effect to a certain extent, but they still suffer the following drawbacks. Firstly, these extension methods only incorporate local spatial information into the FCM. However, when the noise level in images is high, the adjacent pixels of a noise point may also contain abnormal features, only the neighborhood spatial constraint is thus not sufficient for perfect segmentation. In order to improve the segmentation result, non-local information must be taken into account. Secondly, in most of these FCM extension algorithms, there exists at least one parameter to control the tradeoff
between the original image feature and spatial constraint. The values of these
tune parameters have a crucial impact on the performance of the algorithm,
but their selection is generally difficult because the parameters should keep a
balance between insensitiveness to noise and effectiveness of prevent the
image details. In practice, the selection of these parameters is made by
experience or by trial-and-error experiments, which significantly increases the
computational burden (Hou 2007).

The robustness of FCM to noise is also increased in certain extent
by its direct updated versions such as PCM (Krishnapuram 1993), FPCM
(Pal 1997) and PFCM (Pal 2005). This methods also not including any local,
non-local spatial information of the image, in turn, not suitable for proper
segmentation of medical images.

In order to remove the above said issues, this research work
proposed the novel approach called enhanced possibilistic fuzzy c-means
(EPFCM). It is an unsupervised clustering method that uses the constraints
such as membership, possibility (typicality) and both local, non-local spatial
neighborhood information to classify each pixel by modifying the distance
metrics of possibilistic fuzzy c-means (PFCM) method. The medical images
consist of more complicated structure, both local and non local neighborhood
spatial information are necessary for proper segmentation of normal and brain
tumor images, otherwise misclassifications result. Therefore, the proposed
method is more reliable for medical images even with unknown noisy
condition. With the consideration of above mentioned constraints, which is
not present in other methods, the proposed method can greatly control the
unknown noise and segment the medical images properly.

Next, the discussion about the fuzzy clustering techniques such as
FCM, FPCM and PFCM for better understanding and comparison of
proposed method (EPFCM) is presented in the subsequent sections.
4.3.1 FCM clustering method

Fuzzy c-means clustering (FCM) also known as Fuzzy ISODATA is a clustering method that groups data elements to two or more clusters associating with each elements a set of membership levels. These membership levels indicate the strength of association between that data element and a particular cluster. Fuzzy clustering is a process of assigning these membership levels, and then using them to assign data elements to desired clusters (Bezdek 1980). This algorithm is proposed by Bezdek, which is an improvement of the hard k-means algorithm.

Let an image be represented as \( X = (x_1, x_2, x_3, \ldots, x_n) \) where \( x \) represents individual pixel within multidimensional (multiple feature ) data and ‘n’ denotes the numbers of pixels . The algorithm is iterative optimization that minimizes the objective function defined as follows,

\[
J_m(U, V) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m D_{ij}^2
\]  

(4.1)

Subject to

\[
\sum_{i=1}^{c} \mu_{ij} = 1, \forall j, \quad 0 < \sum_{j=1}^{n} \mu_{ij} < n, \quad \forall j \quad \text{and} \quad \sum_{j=1}^{n} \mu_{ij} > 0, \quad \forall i
\]  

(4.2)

where

\[
D^2(x_j, v_i) = D_{ij}^2 = \|x_j - v_i\|^2
\]  

(4.3)

is the distance metric used to measure the similarity between a element \( x_j \) and fuzzy cluster centroid of the \( i^{th} \) cluster \( (v_i) \), \( \| \cdot \| \) is a Euclidean distance norm , \( \mu_{ij} \) is the degree of membership of data elements \( x_j \) in the \( i^{th} \) cluster, \( c \) is the number of cluster to be partitioned and \( m \) is real constant greater than one \( (m>1) \) which controls the fuzziness of the resulting partition (Bezdek 1993).
The objective function is minimized when pixel is close to the centroid of their clusters are assigned high membership values and low membership values are assigned to pixels with data far from the centroid. The membership function represents the probability that a pixel belongs to a specific cluster. In the FCM algorithm, a probability depended on the distance between the pixel and each individual cluster centroid in the feature domain (Bezdek 1993).

The necessary conditions for $J(U,V)$ to reach a minimum can be found by taking first derivative of $J(U,V)$ with respect to $\mu, v$ and set it to zero. Then the necessary conditions are as follows,

Membership function:

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left[ \frac{||x_j - v_i||^{2/m}}{||x_j - v_k||^{2/m}} \right]}$$  \hspace{1cm} (4.4)

Cluster center:

$$v_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m x_j}{\sum_{j=1}^{n} \mu_{ij}^m}$$  \hspace{1cm} (4.5)

The FCM algorithm proceeds by iterating the two necessary conditions until a solution is reached. Each data point will be associated with a membership value for each class after FCM clustering. By assigning the data point to the class with the highest membership value, a segmentation of the data could be obtained.
4.3.1.1 FCM algorithm steps

1. Select the number of clusters ‘C’ and fuzziness factor ‘m’
2. Select initial class center prototypes \( v_i \); \( i=1,2,\ldots,C \), randomly and \( \epsilon \), a very small number called stopping criterion value. It takes the continues value (0,1)
3. Update membership function \( \mu_{ij} \) using Equation (4.4)
4. Update cluster centre \( v_i \) using Equation (4.5)
5. Repeat steps 3 to 4 until stopping criterion. The stopping criterion condition is defined as \( \| v_{t+1} - v_t \| \leq \epsilon \), where ‘t’ is the iteration steps; \( \| \cdot \| \) is the Euclidean distance norm.

The FCM method has limitations as follows,

1. The spatial information is not utilized
2. Very sensitive to noise and outliers in the image.
3. Noisy pixels always wrongly classified because of it is abnormal features

Therefore to obtain more accurate clustering result, the FCM method needs to be improved.

4.3.2 FPCM clustering method

To overcome the problem of FCM, a new clustering method named possibilistic c-mean (PCM) was proposed by Krishnapuram (1993). In this algorithm the objective function is modified and the normalization constraint, 
\[ \sum_{i=1}^{C} \mu_{ij} = 1, \forall j, \] is not considered and each element of j’th column can be any number between 0 and 1 (at least one of them is non zero). The authors named the value \( \mu_{ij} \) as typicality (typicality of \( x_j \) relative to cluster i). In fact each row
of $U$ is a possibility distribution over $X$. However this algorithm also has some problems. It is very sensitive to initialization and sometimes coincident clusters will occur. In addition it is very sensitive to additional parameters in this model.

To address the problems of FCM and PCM a new fuzzy possibilistic c-mean (FPCM) algorithm was proposed in Pal (1997) by combining these two algorithms. In data classification, both membership and typicality are mandatory for data structures interpretation and FPCM computes these two factors simultaneously. FPCM solves the noise sensitivity defect of FCM and overcomes the problem of coincident clusters of PCM. The objective function of FPCM is written as:

$$J_m(U,T,V;X) = \sum_{i=1}^{c} \sum_{j=1}^{n} (\mu_{ij}^m + t_{ij}^\eta)D_{ij}^2$$  \hspace{1cm} (4.6)

where $m>1$, $\eta>1$, $0 \leq \mu_{ij} \leq 1$, $0 \leq t_{ij} \leq 1$, $\sum_{i=1}^{c} \mu_{ij} = 1$, $\sum_{j=1}^{n} t_{ij} = 1$ \hspace{1cm} (4.7)

and $D^2(x_j,v_i) = D_{ij}^2 = \|x_j - v_i\|^2$ \hspace{1cm} (4.8)

Here $T = (t_{ij})$ is the typicality matrix.

The necessary conditions for $J(U,V,T;X)$ to reach a minimum can be found by taking first derivative of $J(U,V)$ with respect to $\mu,v$ and $t$. And set the resultant value to zero. Then the necessary conditions are as follows,

Membership function:

$$\mu_{ij} = \left[\frac{\sum_{k=1}^{c} \left(\frac{D_{ij}}{D_{kj}}\right)^{2/m-1}}{\sum_{k=1}^{c} \left(\frac{D_{ij}}{D_{kj}}\right)^{2/m-1}}\right]^{-1}$$ \hspace{1cm} (4.9)

Typicality:

$$t_{ij} = \left[\frac{\sum_{k=1}^{n} \left(\frac{D_{ij}}{D_{ik}}\right)^{2/\eta-1}}{\sum_{k=1}^{n} \left(\frac{D_{ij}}{D_{ik}}\right)^{2/\eta-1}}\right]^{-1}$$ \hspace{1cm} (4.10)
Cluster Center:

\[ V_i = \frac{\sum_{j=1}^{n} (\mu_{ij}^m + t_{ij}^j) x_j}{\sum_{j=1}^{n} (\mu_{ij}^m + t_{ij}^j)} \]  

(4.11)

4.3.2.1 FPCM algorithm steps

1. Select the number of clusters ‘C’ and fuzziness factor ‘m’
2. Select initial class center prototypes \( v_i \); \( i=1,2,...,C \), randomly and \( \varepsilon \), a very small number called stopping criterion value.
3. Update membership function \( \mu_{ij} \) using the Equation (4.9)
4. Update typicality using the Equation (4.10)
5. Update cluster centre \( v_i \) using the Equation (4.11)
6. Repeat steps 3 to 5 until stopping criterion. The stopping criterion condition is defined as \( \|v_{t+1} - v_t\| \leq \varepsilon \), where 't' is the iteration steps; \( \| . \| \) is the Euclidean distance norm.

The FPCM method has limitations as follows,

1. Although FPCM is less prone to the problems of FCM and PCM, in the case of a large data set such as medical MRI image, this algorithm does not work properly (it operates such as FCM), because FPCM normalizes the possibility values, so that the sum of typicality of all data points in each row of \( U \) is one. Hence the typicality values are very small in large data sets.
2. The spatial information is not utilized
4.3.3 PFCM clustering method

To address the problems of FPCM a new possibilistic fuzzy c-mean (PFCM) algorithm was proposed in Pal (2005). In this algorithm the constraint of the typicality values has been relaxed to overcome the problem of FPCM. The objective function of PFCM is written as:

\[ J_m(U, V; T; X) = \sum_{i=1}^{c} \sum_{j=1}^{n} (a \mu_{ij}^m + bt_{ij}^\eta) D_{ij}^2 + \sum_{i=1}^{c} \sum_{j=1}^{n} \gamma_i (1-t_{ij})^\eta \]  \hspace{1cm} (4.12)

where  \[ D_i^2(x_j, v_i) = D_{ij}^2 = \|x_j - v_i\|^2 \]  \hspace{1cm} (4.13)

\[ \sum_{i=1}^{c} \mu_{ij} = 1 \hspace{1cm} \forall j, \hspace{1cm} 0 \leq \mu_{ij}, \hspace{1cm} t_{ij} \leq 1 \hspace{1cm} \text{and} \hspace{1cm} a > 0, \hspace{1cm} b > 0, \hspace{1cm} m > 1, \hspace{1cm} \eta > 1 \hspace{1cm} \text{are user defined constants.} \]

The relative importance of fuzzy membership \( \mu_{ij} \) (as in FCM) and typicality \( t_{ij} \) (as in PCM) in the objective function are defined by the constants ‘a’ and ‘b’. If \( a = 1, b = 0 \) and \( \gamma_i = 0, \hspace{1cm} \forall j \), PFCM reduces to FCM and if \( a = 0 \) and \( b = 1 \), it reduces to PCM.

The necessary condition for \( J_m \) in Equation (4.12) to reach a minimum can be found by taking first derivative of it with respect to \( \mu, v, t \) and set it to zero. Then three necessary conditions are as follows,

Membership function:

\[ \mu_{ij} = \left[ \frac{D_i^2(x_j, v_i)}{\sum_{i=1}^{c} D_i^2(x_j, v_i)} \right]^{2/(m-1)} \]  \hspace{1cm} (4.14)

Typicality:

\[ t_{ij} = \frac{1}{1 + \left( \frac{b D_{ij}^2}{\gamma_i} \right)^{\frac{1}{\eta}}} \]  \hspace{1cm} (4.15)
Cluster center:

\[ v_i = \frac{\sum_{j=1}^{n} (a\mu_{ij}^m + bt_{ij}^r)x_j}{\sum_{j=1}^{n} (a\mu_{ij}^m + bt_{ij}^r)} \quad (4.16) \]

In Krishnapuram (1993) the following equation is suggested to compute typicality deciding parameter \( \gamma_i \):

\[ \gamma_i = \frac{K\sum_{j=1}^{n} \mu_{ij}^mD_{ij}^2}{\sum_{j=1}^{n} \mu_{ij}^m} \quad K > 1 \quad (4.17) \]

### 4.3.3.1 PFCM algorithm steps

1. Select the number of clusters ‘C’ and fuzziness factor ‘m’
2. Select initial class center prototypes \( v = \{v_i\}; i=1,2,...,C \), randomly and \( \epsilon \), a very small number called stopping criterion value.
3. Update membership function \( \mu_{ij} \) using the Equation (4.14)
4. Update typicality deciding parameter \( \gamma_i \); \( i=1,2,...,C \), using Equation (4.17)
5. Update typicality using the Equation (4.15)
6. Update cluster centre \( v_i \) using the Equation (4.16)
7. Repeat steps 3 to 6 until stopping criterion. The stopping criterion condition is defined as \( \|v_{t+1} - v_t\| \leq \epsilon \), where ‘t’ is the iteration steps; \( \| \cdot \| \) is the Euclidean distance norm.
It can easily be seen from Equation (4.12) that the objective function of PFCM does not take into account any spatial information. Hence, it is sensitive to noise and intensity in-homogeneity and its application for medical MRI image segmentation are very limited.

4.4 PROPOSED ENHANCED POSSIBILISTIC FUZZY C-MEANS (EPFCM) METHOD

As explained in section 4.3 all the fuzzy clustering versions can reduce the noise effect in the medical brain MRI images only to a certain extent, because these methods are not including both local, nonlocal spatial information of the image pixels or deal with images as the separate pixels. Moreover medical images always contain uncertainty and unknown noise caused by clinician performance, instrument and patient movement, these methods are not effective for truthful segmentation of medical images.

In this algorithm, distance metric $D_{ij}$ in PFCM is modified in such a way that it includes membership, typicality and both local, nonlocal spatial neighborhood information to overcome the noise effect in MRI brain medical images. This modified distance metric is incorporated into objective function of PFCM. Then resultant algorithm is called Enhanced Possibilistic Fuzzy C-means (EPFCM) is obtained for enhanced segmentation results. Therefore objective function of proposed EPFCM is defined as follows,

$$J_m(U, V, T; X) = \sum_{i=1}^{c} \sum_{j=1}^{n} (a\mu_{ij}^m + b\gamma_j)D_{ij}^2 + \sum_{j=1}^{c} \sum_{i=1}^{n} (1-t_{ij})^\eta$$

(4.18)

where $\sum_{i=1}^{c} u_{ij} = 1$, $0 \leq \mu_{ij}$, $t_{ij} \leq 1$ and $a > 0$, $b > 0$, $m > 1$, $\eta > 1$

(4.19)
And modified distance metric is given by

\[ D^2(m, v_i) = D^2_j = (1-\lambda_j)d^2_i(m, v_i) + \lambda_j d^2_n(m, v_i) \] (4.20)

where, \( d_i \) is the Local distance metric, \( d_n \) is the Non local distance metric and \( \lambda_j \) is the Tradeoff parameter.

Use the Equations (4.14), (4.15), (4.16) and (4.17) for the membership function, typicality, cluster center and typicality deciding parameter calculations.

4.4.1 Significance of Modified distance metric term

The modified distance metric or dissimilarity measure is rewritten from Equation (4.20) as follows

\[ D^2(m, v_i) = D^2_j = (1-\lambda_j)d^2_i(m, v_i) + \lambda_j d^2_n(m, v_i) \]

where, \( d_i \) is the distance metric influenced by local spatial information. This added local spatial neighborhood term is similar to the one which is used in modified FCM (MFCM) Ma and Staunton (2007) and Hassan Khotanlou (2009) to incorporate the neighborhood effects in the classic FCM. Similar terms are also used in Ahmed (2002), Shen (2005). The local spatial constraint is evaluated by the feature difference between neighboring pixels in the image.

\( d_n \) is the distance measurement influenced by non-local spatial information. This added non local term is obtained from the non local means (NL-means) algorithm (Buades 2005) for image denoising. It tries to take advantage of the high degree of redundancy in image. In other words, they assumed that for every pixel in an image, there is a set of samples with a similar neighborhood configuration of it. Then the pixel under consideration could be influenced by the weighted averaging over these samples. The
NL-means algorithm also can deal with the noise of image successfully and geometrical edges in the image can be retained perfectly. However, the most favorable case for the NL-means is the textured or periodic case, because these two kinds of images have a large redundancy. For MRI brain images, because of the complicated structures, noise, blur in acquisition and the partial volume effect originating from the low sensor resolution, the images may contain exception, non-repeated details. Such details can be smoothed out by the NL-means algorithm. The non-local constraint determined by all points whose neighborhood configurations look like the neighborhood of the pixel of interest.

\[ \lambda_j \] is the weighting factor controlling the tradeoff between local and nonlocal spatial information. It varies from zero to one.

For more understanding of local and nonlocal spatial neighborhood information, let us consider the Figure 4.3 shows an example of the local and non-local information influence approach. Let P, Q and R are the center pixel value (marked by red points) of chosen local neighborhood configurations (marked by red square box) of fixed size with respect to a center pixels and consider local neighborhood configurations with center pixel value of R for the explanation of local spatial information. If the value of any pixel \( x_j \) in local neighborhood configurations is close to the center pixel R, then \( x_j \) should be influenced greatly by it, otherwise, its influence to \( x_j \) should be small. This is applicable to local neighborhoods with P and Q center pixel also. Therefore, local spatial constraint is evaluated by the difference between neighboring pixels in the image with respect to center pixel. Consider all three local neighborhood configurations with center pixel values of P, Q and R for the explanation of non local spatial information. It can be noted from the Figure 4.3 that the pixel Q has the very same neighborhood configuration of pixel P, so its influence to P is relatively large. However, though the pixel R has the nearly same gray level as pixel P, the neighborhood configurations of them are much different and therefore its influence to P is relatively small.
Figure 4.3  An example of the local and non-local information influence approach

From this example, it is concluded that the neighborhood configurations with center pixels P and Q treated as single cluster by the nonlocal spatial measurement due to same neighborhood pixels. But in the case of local spatial measurement these two neighborhoods are treated as separate clusters, misclassification result. So, both local and non-local information measure is important for better clustering. Simply, local spatial information measure is the intra pixel measure within the neighborhood configuration and non local spatial information measure is the inter pixel measure between the neighborhood configurations.

4.4.2 Significance of Local Distance Metric

Let \( N_j \) denote a chosen local neighborhood configuration of fixed size with respect to a center pixel \( x_j \). If the value of a pixel \( x_k \) in \( N_j \) is close to the center pixel, then \( x_j \) should be influenced greatly by it, otherwise, its influence to \( x_j \) should be small. According to the above description, the distance measurement influenced by local information \( d_i \) is given by

\[
d_i^2(x_j, v) = \frac{\sum_{x_k \in N_j} e_i(x_k, x_j) d^2(x_k, v)}{\sum_{x_k \in N_j} e_i(x_k, x_j)}
\]

(4.21)
where \( d^2(x_k, v_i) = \| x_k - v_i \|^2 \) is the Euclidean distance metric measure the similarity between pixel pixel \( x_k \) and cluster centroid \( v_i \), \( \omega_i(x_k, x_j) \) is the weight of each pixel \( x_k \) in \( N_j \) and is given by

\[
\omega_i(x_k, x_j) = e^{-\frac{\| x_k - x_j \|^2}{\sigma^2}}
\]

where, \( \sigma^2 \) is the variance of \( N_j \).

### 4.4.3 Significance of Non Local Distance Metric

The distance measurement influenced by non-local information \( d_{nl} \) is computed as a weighted average of all the pixels in the image \( I \),

\[
d_{nl}^2(x_j, v_i) = \sum_{x_k \in I} \omega_{nl}(x_k, x_j)d^2(x_k, v_i)
\]

where the family of weight \( \omega_{nl}(x_k, x_j) \); \( x_k \in I \) depends on the similarity between the pixel \( x_k \) and \( x_j \), and satisfies the usual conditions \( 0 \leq \omega_{nl}(x_k, x_j) \leq 1 \) and \( \sum_{x_k \in I} \omega_{nl}(x_k, x_j) = 1 \).

The similarity between two pixels \( x_k \) and \( x_j \) depends on the similarity of the intensity gray level vector \( v(N_k) \) and \( v(N_j) \), where \( N_k \) denotes a square neighborhood of fixed size and centered at a pixel \( x_k \). This similarity is measured as a decreasing function of the weighted Euclidean distance \( \| v(N_k) - v(N_j) \|_{L_2,a}^2 \), where \( a>0 \) is the standard deviation of the Gaussian kernel. The pixels with a similar gray level neighborhood to \( v(N_j) \) have larger weights in the average. These weights are defined as,

\[
\omega_{nl}(x_k, x_j) = \frac{1}{Q(x_j)}S(x_k, x_j)
\]
where $S(x_k,x_j)$ is the exponential form of the similarity, and $Q(x_j)$ is the normalizing constant, these terms are defined as,

$$S(x_k, x_j) = e^{-\frac{|x(N_k) - x(N_j)|}{h^2}}$$  \hspace{1cm} (4.25)

$$Q(x_j) = \sum_{x_k \in \Omega} e^{-\frac{|x(N_k) - x(N_j)|}{h'}}$$  \hspace{1cm} (4.26)

where, the parameter ‘$h$’ acts as a degree of filtering. It controls the decay of the exponential function and therefore the decay of the weights as a function of the Euclidean distance.

### 4.4.4 Significance of Tradeoff Parameter

For computational purpose, the search of the similar neighborhood configuration always be restricted in a larger “search window” denoted by $\Omega_i$. Let $x_j$ be the pixel under consideration. For each pixel $x_k$ in the search window of size $S \times S$, calculate its exponential similarity to $x_j$ using Equation (4.25). The tradeoff parameter of $x_j$ is then defined as

$$\lambda_j = \frac{1}{m} \sum_{i=1}^{m} S_i(x_k, x_j)$$  \hspace{1cm} (4.27)

where, $S_i$ represents the $i^{th}$ exponential similarity term in the search window and choose $m=S-1$. The parameter $\lambda_j$ decides the tradeoff between local and non-local spatial information. For example fine structures in brain image always have no redundancy (or they have no redundancy in their search window); therefore in order to prevent them from being smoothed out, low degree should be given to non-local information constraint, and conversely, local information should be more important. From Equation (4.25), if $x_k$ has the same neighborhood configuration as $x_j$, the value of $S(x_k, x_j)$ would tend to
1. Furthermore, if \( x_j \) has some redundancy in its search window, which means there are at least \( S-1 \) pixels have similar neighborhood configuration with \( x_j \), even pixel \( x_j \) locates on an edge of the images. Hence, the \( \lambda_j \) value calculated by Equation (4.27) will also tend to 1. Otherwise, the value of \( \lambda_j \) will be very small.

### 4.4.5 Algorithm for Proposed EPFCM Method

The algorithm for proposed EPFCM method for segmentation of MRI brain images can be stated from the following steps,

1. Select the number of clusters ‘C’ and fuzziness factor ‘m’ 
2. Select initial class center prototypes \( \nu = \{v_i\}; i=1,2,\ldots,C \), randomly and \( \nu \), a very small number called stopping criterion value. 
3. Select the neighborhood size and search window size 
4. Calculate modified distance measurement \( D_{ij}^2 \) using the Equation (4.20) 
5. Update membership function \( \mu_{ij} \) (Equation 4.14) using \( D_{ij}^2 \) 
6. Update typicality deciding parameter \( \gamma_i ; i=1,2,\ldots,C \), using Equation (4.17) 
7. Update typicality using the Equation (4.15) 
8. Update cluster centre using equation (4.16) 
9. Repeat steps 4 to 8 until stopping criterion. The stopping criterion condition is as follows \( \|v_{t+1}-v_t\| \leq \varepsilon \), where ‘t’ is the iteration steps, \( \|\cdot\| \) is the Euclidean distance norm.
4.4.6 Flow chart for Proposed EPFCM Method

The flow chart for the proposed algorithm is as shown in Figure 4.4 below. Algorithm starts with get the MRI medical image then set the number of clusters, fuzziness factor, initial class center, stopping criterion value, neighborhood size and search window size. After setting these values calculate distance measurement then update membership function, typicality deciding parameter, typicality and cluster centre. In the next step calculate difference of two consecutive updated cluster centers, if it is less than the stopping criterion stop the segmentation process, else calculation of distance metric and updating steps are continue till stopping criterion condition is satisfied.

![Flowchart](image)

Figure 4.4 Flow chart for Proposed EPFCM Method
4.5 EXPERIMENTAL RESULTS AND DISCUSSIONS

The proposed algorithm is applied on to synthetic square image, simulated MRI Brain image for testing and finally, applied it to segment tumors on CE-T1w and FLAIR images. The comparison of the results with FCM, PCM, and PFCM demonstrates the superiority of the algorithm. It shows that proposed method is better than others. Also, the results are quantitatively evaluated. Based on the result of number test images, set the parameter $h = 500$, search window size is $7 \times 7$, neighborhood window size is $3 \times 3$, $m = 2$, $a = 5$, $b = 3$ and $\eta = 2$ for proper segmentation result.

4.5.1 Testing on Synthetic Square Image

A synthetic square image with intensity values of $0,105,145,200$ as shown in Figure 4.5a is simulated for segmentation of four square’s after adding gaussian white noise as shown in Figure 4.5b. The simulated result shows that, the proposed algorithm removes most of the noise and segmented properly as shown in Figure 4.5f. However in results of FCM, FPCM, PFCM as shown in Figure 4.5c, Figure 4.5d and Figure 4.5e shows that still there is some noise remaining at the edges of squares in compare with the proposed method. With reference to segmentation results obtained the following points are noted from the output square images: 1. FCM method not removing the noise fully, in turn segmented square regions contains noise. The edges between different square regions are blurred 2. FPCM also still having some noise remaining, in turn; perfect segmentation of square region is not present. The edges also not defined properly 3. PFCM method comparatively reduces the most of the noise, but the edges between different square regions are still blurred. 4. The proposed EPFCM method eliminates noise in the image and segments all square regions exactly without any blurring effect because of inclusion of both local, nonlocal spatial neighborhood information. Visually
segmentation results obtained by the proposed EPFCM clustering algorithm performs well in noisy environment and segment the image properly.

![Clustering results on synthetic square images](image)

**Figure 4.5** Clustering results on synthetic square images: (a) original synthetic four-level gray scale image; (b) original image corrupted by Gaussian noise; Visualized segmentation results using (c) FCM (d) FPCM (e) PFCM (f) Proposed EPFCM.

### 4.5.2 Testing on Simulated MRI Brain Image

The proposed algorithm is applied on to simulated T1-weighted normal brain MR image obtained from brain web. By the application of this algorithm, the image is segment into four tissue classes, which corresponds to background, CSF, WM and GM and compared with FCM, FPCM and PFCM. The segmentation results obtained by FCM, FPCM, PFCM and the proposed EPFCM method is as shown in Figure 4.6d, Figure 4.6e and Figure 4.6f and Figure 4.6g. The original image, original image corrupted with 9% gaussian noise and ground truth image shown in Figure 4.6a, Figure 4.6b and Figure 4.6c. With reference to segmentation results obtained the following points are noted from the output images: 1. FCM method not removing the
noise fully, in turn segmented tissue regions contains noise and not possible to
identity exact location of tissue regions . The edges between different tissue
regions are blurred 2. FPCM also still having some noise remaining, in turn,
perfect segmentation of tissue region is not present. The edges also not
defined properly 3. PFCM method comparatively reduces the most of the
noise, but the edges between different tissue regions are still blurred. 4. The
proposed EPFCM method eliminate noise in the image and segment all tissue
types exactly without any blurring effect because of inclusion of both local,
nonlocal spatial neighborhood information. Visually segmentation result by
this method is better than other three.

![Image](image.png)

**Figure 4.6** Comparison results of segmentation of image into 4 classes
on a simulated T1-weighted normal brain MR image: (a) Original image (b) One axial slice of the original image with
9% Gaussian noise (c) Ground truth image; Segmentation results using (d) FCM (e) FPCM (f) PFCM (g) Proposed EPFCM.

The evaluation of segmentation performance is also carried out quantitatively by employing three volume metrics namely, the similarity
index(S), false positive volume function (FPVF) and false negative volume
function (FNVF) in the experimental results. For a given image, suppose that $A_i$ and $B_i$ represent the sets of pixels belong to class $i$ in ground truth or manual segmentation and in segmentation result. $|A_i|$ denotes the number of pixels in $A_i$ . $|B_i|$ denotes the number of pixels in $B_i$.

The similarity index is an intuitive and clear index to consider the matching pixel between $A_i$ and $B_i$, and defined as

$$ S = \frac{2|A_i \cap B_i|}{|A_i| + |B_i|} \quad (4.28) $$

Similarity index $S > 70\%$ indicates an excellent one (Zijdenbos 1994).

The false positive volume function (FPVF) represents the error due to the misclassification in class $i$ and the false negative volume function (FNVF) represents the error due to the loss of desired pixels of class $i$, they are defined as in Equation (4.29) and Equation (4.30)

$$ \text{FPVF} = \frac{|B_i| - |A_i \cap B_i|}{|A_i|} \quad (4.29) $$

$$ \text{FNVF} = \frac{|A_i| - |A_i \cap B_i|}{|A_i|} \quad (4.30) $$

Higher value of $S$ and lower value of FPVF and FNVF gives better segmentation result (Zijdenbos 1994).

The comparison result of three volume metrics for four tissue classes, which corresponds to background, CSF, WM, GM as shown in Table 4.1 and Figure 4.7. This Table 4.1 compares the proposed EPFCM method with other three methods. From this table, it can be see that the similarity metrics of this method are greater than 95% for all 4 tissue types in compare with other three methods. Hence, the overlap degree between the
segmentation result and the ground truth is higher than other three methods. FPVF and FNVF values also very less in comparisons with other three methods. It shows misclassification and loss of desired pixels of tissue classes (background, CSF, WM, GM) is reduced. It clearly demonstrates that the proposed method has a better a segmentation performance than other three.

The evaluation of the proposed algorithm with different gaussian noise level added into the original image is also carried out. This evaluation metric is called segmentations accuracy $S_A$ which is defined as,

$$S_A = \frac{N_c}{N_t} \times 100\%$$  \hspace{1cm} (4.31)

is used for evaluation. Where $N_c$ is the number of correctly classified pixels and $N_t$ is the total number of pixels. This is performed by quantitative comparison with the ground truth segmented image.

Table 4.1  Quantitative comparison result of three volume metrics for four tissue classes

<table>
<thead>
<tr>
<th>Methods for Comparison</th>
<th>Tissue Type and Volume</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Background</td>
<td>WM</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>FPVF</td>
</tr>
<tr>
<td>FCM</td>
<td>88.16</td>
<td>10.21</td>
</tr>
<tr>
<td>FPCM</td>
<td>92.25</td>
<td>9.23</td>
</tr>
<tr>
<td>PFCM</td>
<td>95.98</td>
<td>5.38</td>
</tr>
<tr>
<td>EPFCM</td>
<td>96.17</td>
<td>3.16</td>
</tr>
</tbody>
</table>
Figure 4.7 Graph of the quantitative comparison results of three volume metrics for four tissue classes

The quantitative results obtained with different methods are shown in Table 4.2 and Figure 4.8 for original image in Figure 4.6a added with different levels of gaussian noise. As seen in Table 4.2 and Figure 4.8, when the noise is low, the differences between the segmentation accuracies of the different methods are low. When noise increases, the difference is high. But in comparison with other method segmentation accuracy of this method is varied with very small value when changing noise level from 3% to 9% (i.e., This algorithm is more effective in segmenting images even with noise). Moreover our algorithm produces 9% and 2% improvement with FCM and PFCM for 9 % noise level. This results show that the proposed method has more robust than other three algorithms.

The proposed algorithm also evaluated for different neighborhood (search window) size. By increasing search and neighborhood window size, keeping other values constant, the similarity index difference between neighborhood (search window) sizes is very small for all tissue types as shown in Figure 4.9 and Table 4.3 This means that the algorithm will not affect the segmentation result even though the neighborhood size and search window size are large.
Table 4.2 Quantitative comparison of segmentations accuracy obtained by different algorithms for different noise level

<table>
<thead>
<tr>
<th>Methods for Comparison</th>
<th>Gaussian Noise level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
</tr>
<tr>
<td>FCM</td>
<td>98.22</td>
</tr>
<tr>
<td>FPCM</td>
<td>98.43</td>
</tr>
<tr>
<td>PFCM</td>
<td>98.81</td>
</tr>
<tr>
<td>EPFCM</td>
<td>99.21</td>
</tr>
</tbody>
</table>

Figure 4.8 Graph of the quantitative comparison of segmentations accuracy obtained by different algorithms for different noise level

Table 4.3 Segmentation similarity index for different neighborhood and search window size

<table>
<thead>
<tr>
<th>Neighborhood (search window) size</th>
<th>Similarity index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Background</td>
</tr>
<tr>
<td>3x3 (9x9)</td>
<td>0.962</td>
</tr>
<tr>
<td>5x5 (15x15)</td>
<td>0.963</td>
</tr>
<tr>
<td>7x7 (21x21)</td>
<td>0.96</td>
</tr>
</tbody>
</table>
4.5.3 Results and discussion of proposed method for brain tumor segmentation

To segment tumors, consider the full-enhanced and ring-enhanced tumor on contrast enhanced T1-weighted images and non-enhanced tumors on FLAIR images. By the application of the method to 10 contrast enhanced T1-weighted images (5 full-enhanced and 5 ring-enhanced tumor) and 5 FLAIR images for non-enhanced tumors shows better tumor segmentation. The results of the 3 cases are discussed: one for full enhanced tumor, one for ring-enhanced tumor and one for non-enhanced tumors on FLAIR images.

- Segmentation of full-enhanced and ring-enhanced tumor

In the case of full-enhanced and ring-enhanced tumors, the extracted brain is classified into six classes: CSF, WM, GM, tumor, background and any other component, if it is present. The EFPCM method is applied to segment contrast enhanced T1-weighted images with enhanced tumor and ring enhanced tumor at different locations, with different sizes and shapes. In Figure 4.10, the segmentation result for an enhanced tumor obtained
by EPFCM for a relatively large tumor is compared with FCM, FPCM and PFCM results. In Figure 4.12 result for a ring enhanced tumor are shown. The comparisons of segmentation result with FCM, FPCM and PFCM shows that this EPFCM method is better for tumor segmentation on CE T1w images.

![Figure 4.10](image)

**Figure 4.10** Comparison of classification results obtained by FCM, FPCM, PFCM and EPFCM for a large full-enhanced tumor on CE T1-weighted image. (a) One axial slice of the contrast enhanced T1-weighted image (b) Result of FCM classification (c) Result of FPCM. (d) Result of PFCM. (e) Result of EPFCM.

To select the tumor class from Figure 4.10(e) and Figure 4.12(e), thresholding is applied to each, the resulting image is shown in Figure 4.11(a) and Figure 4.13(a). This resulting images contains some undesired additional pixels with tumor class. To eliminate these misclassified components, binary morphological operations are applied to the tumor class. An opening operation (erosion followed by dilation) (Gonzalez and Woods 2011) is first used to separate the components. Then, select the largest connected component, which proved to always correspond to the tumor, even if it has a small size. Here the elementary neighborhood of the morphological operations corresponds to 6-connectivity. The result of this operation gives exact tumor class without undesired pixels as shown in Figure 4.11 (b) for enhanced tumor and Figure 4.13 (b) for ring enhanced tumor.
Figure 4.11 Tumor detection result for a large full-enhanced tumor (Figure 4.10). (a) One axial slice of the selected tumor class after thresholding (b) Selected tumor class result after morphological operations.

Figure 4.12 Comparison of classification results obtained by FCM, FPCM, PFMC and EPFCM for a ring enhanced tumor on CE T1-weighted image. (a) One axial slice of the contrast enhanced T1-weighted image (b) Result of FCM classification (c) Result of FPCM. (d) Result of PFMC. (e) Result of EPFCM.

Figure 4.13 Tumor detection result for a ring enhanced tumor (Figure 4.12). (a) One axial slice of the selected tumor class after thresholding. (b) Selected tumor class result after morphological operations.
Segmentation of non-enhanced tumor

Since non-enhanced tumors have hyperintense appearance on FLAIR images and they have no other components, perform the segmentation step for this tumor on FLAIR images. In this case the FLAIR image is classified into 5 classes (background, CSF, GM, WM and tumor). In Figure 4.14 a comparison of segmentation results in a FLAIR image obtained by FCM, FPCM, PFCM and EPFCM is shown. These results show that the EPFCM algorithm is good candidate for segmenting non enhanced tumor on FLAIR image.

![Figure 4.14 Comparison of classification results obtained by FCM, FPCM, PFCM and EPFCM for a non-enhanced tumor on FLAIR image.](image)

Figure 4.14 Comparison of classification results obtained by FCM, FPCM, PFCM and EPFCM for a non-enhanced tumor on FLAIR image. (a) One axial slice of the original FLAIR image. (b) Result of FCM classification (c) Result of FPCM. (d) Result of PFCM. (e) Result of EPFCM.

![Figure 4.15 Tumor detection result for a non-enhanced tumor](image)

Figure 4.15 Tumor detection result for a non-enhanced tumor (Figure 4.5). (a) One axial slice of the selected tumor class after thresholding. (b) Selected tumor class result after morphological operations.
Such as in enhanced tumor detection, to select the tumor class from Figure 4.14(e), thresholding is applied, the resulting image is shown in Figure 4.15(a). This resulting image contains some undesired additional pixels with tumor class. To eliminate these misclassified components, binary morphological operations (Gonzalez and Woods 2011) are applied to the tumor class. The result of this operation gives correct tumor class as shown in Figure 4.15 (b).

This method is applied in the same way to all the images and segmentation results of 12 cases are shown in Figure 4.16. It consists of original images (Figure (a)): 4 full-enhanced tumors on CE-T1w images (first 4 images in the 1\textsuperscript{st} column) and 4 Ring-enhanced tumors on CE-T1w images (last 2 images in the 1\textsuperscript{st} column and first 2 images in the 4\textsuperscript{th} column) and 4 Non-enhanced tumors on FLAIR images (last 4 images in the 4\textsuperscript{th} column). The corresponding segmentation results are presented in the 3\textsuperscript{rd} and 6\textsuperscript{th} column (Figure(c)). The manual segmentation results by the medical expert are presented in the 2\textsuperscript{nd} and 5\textsuperscript{th} column (Figure(b)) for comparing and evaluating the segmentation result obtained by this EPFCM method.

The evaluation of segmentation results are performed quantitatively by employing four volume metrics namely, the similarity index(S), false positive volume function (FPVF), false negative volume function (FNVF) and Jaccard index. The definition for S, FPVF and FNVF are discussed in section 4.5.2. Jaccard index is defined as follows,

For a given image, suppose that $A_i$ and $B_i$ represent the sets of pixels belong to class $i$ in manual segmentation and in segmentation result. $|A_i|$ denotes the number of pixels in $A_i$, $|B_i|$ denotes the number of pixels in $B_i$. 
The Jaccard index is defined mathematically as follows,

$$J_J(A, M) = \frac{|A \cap B|}{|A \cup B|} \times 100\%$$  (4.32)

**Figure 4.16 Tumor Segmentation results using EPFCM Method.**

(a) Original image. (b) Manual segmentation result
(c) Segmentation result by EPFCM.

The result of four volume metrics for this method is as shown in Table 4.4 and Figure 4.17 and 4.18. From this Table 4.4 and Figure 4.17, it can see that an average similarity metrics(S) and Jaccard index (J) of this method is 88.74% and 77.5% that is, the overlap degree between segmentation result and the manual segmentation is acceptable one. The average FPVF and FNVF values are equal to 1.5% and 1.62%. It shows the misclassification and loss of desired tumor pixels is reduced at minimum level. Although this method produces compromising result, its detection at
tumor boundary is not appreciable one. Therefore this can be improved by combining result of this method with the boundary based method, which will be discussed in the next sections.

Table 4.4 Evaluation of the segmentation results of enhanced and non-enhanced tumor by EPFCM method on a few CE-T1w and FLAIR images. (FET, RET and NET denotes the Full enhanced tumor, Ring-enhanced tumor and non-enhanced tumor).

<table>
<thead>
<tr>
<th>MRI modality type</th>
<th>Type of tumor</th>
<th>Volume metric functions (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S</td>
<td>FPVF</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>FET1</td>
<td>90.2</td>
<td>0.7</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>FET2</td>
<td>89.5</td>
<td>1.4</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>FET3</td>
<td>88.6</td>
<td>1.8</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>FET4</td>
<td>87.6</td>
<td>0.9</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>FET5</td>
<td>89.1</td>
<td>0.8</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>RET6</td>
<td>86.6</td>
<td>0.4</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>RET7</td>
<td>89.8</td>
<td>1.7</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>RET8</td>
<td>92.4</td>
<td>0.6</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>RET9</td>
<td>85.2</td>
<td>2.3</td>
</tr>
<tr>
<td>CE-T1w</td>
<td>RET10</td>
<td>90.5</td>
<td>0.5</td>
</tr>
<tr>
<td>FLAIR</td>
<td>NET11</td>
<td>91.5</td>
<td>1.3</td>
</tr>
<tr>
<td>FLAIR</td>
<td>NET12</td>
<td>86.3</td>
<td>2.5</td>
</tr>
<tr>
<td>FLAIR</td>
<td>NET13</td>
<td>89.7</td>
<td>3.5</td>
</tr>
<tr>
<td>FLAIR</td>
<td>NET14</td>
<td>86.3</td>
<td>2.4</td>
</tr>
<tr>
<td>FLAIR</td>
<td>NET15</td>
<td>87.8</td>
<td>1.8</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>88.74</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure 4.17 Graph of the average values of the volume metric functions

Figure 4.18 Graph shows the values of the volume metric functions for 15 tumor cases mentioned in Table 4.4
4.6 CONCLUSION

This chapter proposed and presented the new fuzzy clustering method called as EPFCM for brain tumor segmentation on MRI images. In this method membership, typicality and both local, nonlocal spatial neighborhood information is incorporated into the objective function of PFCM by modifying distance metric of it to allow the labeling of a pixel to be influenced by other pixels. This algorithm is applied on to synthetic square image, simulated MRI Brain image and finally, applied it to segment tumors on CE-T1w and FLAIR images. The results are compared with FCM, PCM, and PFCM to demonstrate superiority of this algorithm. It shows that this method is better than others. But its performance at the border of the tumor region is less. It can be improved by refining the results at the border of the segmented objects by boundary based method. This will be discussed in the chapters 5-7.