CHAPTER 1

INTRODUCTION

1.1 GENERAL

Electric machine has been the workhorse of the industry for many years. Control of electric motors has been a significant requirement in all industrial applications. Historically, the performance parameters, such as, starting torque, torque-speed characteristics and so on were matters of concern. The considerations were centered mainly on steady state operation of the motor at any given operating point. For analysis and performance prediction at steady state conditions, equivalent circuit model of the machine was considered adequate as discussed in Bose (2002).

The modern era of electric motor control began with the advent of power semiconductors in the 1950’s as discussed in Bose (1988) and Bose (1992). Subsequent progress in power electronics and microelectronics has profoundly influenced the operations and performance of drive systems, ac variable speed drives - in particular. AC machines exhibit highly-coupled, nonlinear, multi-variable structures, as opposed to DC machines with their much simpler decoupled control structure as given in Sen (1988). Advancements in power electronics, microelectronics and microcomputers have made it possible to implement sophisticated control tasks at reasonable cost. This can be considered as the second phase in the development of motor control.
The introduction of quadrature and direct axes resolution of the dynamics of the motor gave rise to an independent set of differential equation model of the machine. This has transformed the electric machine model to a standard system theoretic model. This can be considered as the third phase in the development of motor control. In the third phase, electric motor control is considered as a special case of system theoretic control problem already well developed both in the linear and nonlinear system domains. With this approach, electric motor is looked upon as an object of system theoretic control which threw light on such fundamental system concepts as controllability, observability, linearizability, stability, robustness etc. which could not be adequately addressed in the previous approaches. The downside of the system theoretic approach is that the variables involved are not directly related to motor physical variables. This is not a main issue since transformations are available which can transform new system variables to the physical machine variables.

In this thesis, electrical machine model transformed to a standard system theoretic model is considered. Linearization which is a system theoretic method of control is applied to motor control. In particular, control of permanent magnet synchronous motors (PMSM) is considered due to its increasing use in various industrial fields, and numerous advantages over other kinds of motors, such as DC motors and induction motors, because of their high power density, large torque to inertia ratio, high torque to current ratio and high efficiency as given in Jahns and Kliman (1986).

1.2 OBJECTIVES OF THE RESEARCH

The main motivation to the work reported in this thesis came from the importance of the Permanent magnet synchronous motor as mentioned
above and the application of linearization theory to provide an effective control strategy to control PMSM.

PMSM has been dealt with in different ways in terms of control strategies employed. However, the provision of a state space model of electrical machines including PMSM by Bose (2002), Pillay and Krishnan (1989), based on d-q model has transformed it from an electrical object for control to a dynamic system under a control input. Powerful system theoretic methods can then be applied to the control of electrical machines and PMSM in particular.

Linearization of the system model is one such powerful technique. Bodson and Chiasson (1988) have applied what is called ‘exact linearization’ techniques to the control of electrical motors including PMSM. The main drawback with this method is the possible singularity of the state feedback used.

It is proposed in this thesis, perhaps for the first time, the use of approximate linearization technique to the control of PMSM. Approximate linearization technique was introduced by Krener as given in Krener (1973) and later modified to include control input as given in Krener (1984). Approximate linearization is applied to control affine systems containing a controllable linear part. Use of approximate linearization technique allows the singularity issue of state feedback to be avoided.

Since the model of PMSM as given in Bose (2002), Pillay and Krishnan (1989) was essentially quadratic, attention was turned to the method of quadratic linearization which is a specific case of approximate linearization as given in Kang and Krener (1990 and 1992). Since not every system can be
quadratic linearized, the question came up as to what are the conditions to be satisfied before a system can be quadratic linearized. Since this is a system theoretic problem, it had to be handled in a general setting of an n-dimensional system.

As there were no truly explicit conditions available in the literature for satisfying quadratic linearizability, the first objective was to derive an explicit algebraic condition for quadratic linearization of an n-dimensional system with a single input. Obviously, the result being general, can be applied to a larger class of engineering systems apart from electrical motors.

Though the application of approximate linearization to PMSM allows us to circumvent the problem of singularity as mentioned earlier, it introduces third and higher order terms while removing quadratic terms from the system, even though the model originally possessed only quadratic nonlinearity.

To address the problem of introduction of third and higher order terms when PMSM is quadratic linearized, the problem is approached in two ways. The first is a theoretical approach in which the concept of ‘generalized quadratic linearization’ is introduced and solved in a general setting. The approach seeks to find transformations to cancel not only quadratic terms in the system, but also to cancel third and higher order terms which would be introduced into the system in the process of quadratic linearization. Generalized quadratic linearization being a stronger condition, can only be satisfied for a class of systems. This brings the second objective of identifying the class of system which satisfy the ‘generalized quadratic linearization’ and check whether PMSM model belongs to that class.
The second approach to address the problem of introduction of third and higher order terms during quadratic linearization is an empirical one. In this approach, along with higher order nonlinearities, other problems in an actual PMSM machine are also addressed. Actual PMSM machine may have higher order nonlinearities other than quadratic, such as, core loss which may need to be considered for more accurate analysis. Also, one has to take into account the unmodelled dynamics of the machine. A practical solution to the problem is provided by tuning the linearizing transformation already derived for PMSM for quadratic linearization on the lines proposed by Narendra and his coworkers in Levin and Narendra (1993). This brings the third objective of this thesis, viz., to provide a tuning configuration of the linearizing transformation and to derive tuning rules for least square error minimization with respect to a linear canonical system.

An asymptotic stability analysis is to be provided for the essentially quadratic dynamic model of control affine system followed by a similar analysis of the system that remains after quadratic linearization viz. system with third and higher order terms. This constitutes the fourth objective. The results for PMSM will follow as a special case.

Finally, all the theoretical results derived in the thesis are to be verified using MATLAB simulation and experiments with actual PMSM machine which constitutes the fifth and final objective.

The objectives of the thesis can be summarized as follows:

(i) To find new, explicit, necessary and sufficient conditions of quadratic linearization (i.e. to cancel the quadratic terms in the system) using the approximate linearization approach as

(ii) To overcome the problem of introduction of higher order terms, a new concept called ‘generalized quadratic linearization’ is to be introduced, which seeks to transform a system with essentially quadratic nonlinearity to a linear system through coordinate and state feedback, without introducing higher order terms into the system in the process as is customary in quadratic linearization.

(iii) Since the PMSM model may have higher order nonlinearities as well as unmodelled dynamics, it is to be shown that the linearizing transformations can be tuned to cancel the effects of these.

(iv) To test the asymptotic stability of quadratic dynamic model of a control affine system followed by a similar analysis of the system that remains after quadratic linearization using Lyapunov theory. Stability of PMSM model is to follow from these results as a special case.

(v) Application of the linearization techniques to PMSM is to be verified through simulation and implementation of the control strategies on an actual machine.

1.3 LITERATURE REVIEW

1.3.1 Linearization

Control of nonlinear systems, in general, is gaining increasing attention in recent years due to its technical importance and its impact on various applications as well. There has been a major breakthrough in rigorous
linearization theory in early 1980’s which established the exact conditions under which a nonlinear plant can be linearized by static-state feedback and coordinate transformations.

Krener (1973) considers system of the form

\[ \dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \]  

(1.1)

where \( f(x) \) and \( g_i(x); i = 1, 2, ... m \) are analytic vector-valued functions. The nonlinearities arise due to terms not linear in \( x \) and also due to the product of functions of the state \( x \) and inputs \( u_i, i = 1, 2, ... m \). Such systems are termed control affine systems. Krener (1973) gave the necessary and sufficient conditions to be satisfied for the existence of the transformation \( \lambda: x \mapsto y \) which carries the nonlinear system (Equation (1.1)) into a linear system.

\[ \dot{y} = b_0 + \sum_{i=1}^{m} b_i(y_0)u_i \]  

(1.2)

where time is introduced as a coordinate i.e., \( y_0 = t \); \( b_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \); \( b_i = \begin{pmatrix} 0 \\ * \\ * \\ * \end{pmatrix} \); \( i = 1, 2, ... m \) and \( y_0(0) = y_0^0 = 0 \) and \( * \) denotes some real-valued function of \( y_0 \) alone. Krener (1973)’s conditions are called “exact linearizability” conditions since these completely linearize the system. Krener (1973) also showed the importance of the Lie algebra of vector fields associated with the system \( (f(x) \) and \( g_i(x) \) in Equation (1.1)) in studying the change of coordinates that carries the system into a linear one.
Jakubezyk and Repondek (1980) first established the exact conditions under which, in the single-input case \((m = 1)\), system (Equation (1.1)) can be carried into a linear controllable one by a suitable coordinate change in the state-space and a static–state-feedback control law of the form

\[ u = \alpha(x) + \beta(x)v \]  

(1.3)

where \(v\) is the transformed control input.

The same conditions were independently discovered by Su (1982) and Hunt et al (1983), who solved the general case \((m \geq 1)\) and, in addition, gave a method to actually construct the relevant transformations.

Given the system (Equation (1.1)), the controllability matrix defined in Hunt et al (1983) is \(n \times n\) matrix with columns \(g, [f, g], ..., (ad^{n-1} f, g)\), where \([f, g]\) denotes the Lie bracket of \(f\) and \(g\), which is also another vector field, given as \([f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g\)

Also, \((ad^2 f, g) = [f, [f, g]], ...,\)

\((ad^k f, g) = [f, [ad^{k-1} f, g]].\)

The assumption is that the controllability matrix is nonsingular at every point in \(\mathbb{R}^n\), and \(g, [f, g], ..., (ad^{n-1} f, g)\) are involutive on \(\mathbb{R}^n\). A set of \(C^\infty\) vector fields \((f_1, f_2, ..., f_r)\) on \(\mathbb{R}^n\) (i.e. function \(f_i; i = 1, 2, ..., r\) are infinitely differentiable) is called involutive if there exist \(C^\infty\) functions \(\gamma_{ijk}(x)\) such that \(\left[f_i, f_j\right](x) = \sum_{k=1}^r \gamma_{ijk}(x)f_k(x); \quad 1 \leq i, j \leq r; i \neq j\). That is, in simpler terms, the function resulting from Lie derivatives of any two functions can be expressed in terms of original functions themselves.
Considering the output function $y = h(x)$, a linearization technique known as ‘input-output linearization’ of Equation (1.1) was shown to be solvable using an algebraic state feedback control law by Isidori and Ruberti (1984).

Cheng et al (1988) discussed the problem of using feedback and coordinate transformation in order to transform a nonlinear system with outputs into a controllable and observable linear one. Also, extra conditions needed in addition to input-output linearization, to achieve full linearity of both state-space and output map was shown.

Just as linear algebra is an essential mathematical tool to study the structure of linear control system, differential geometry is an essential tool for the study of structural properties of nonlinear control systems. Isidori (2002) used differential geometric methods in the synthesis of feedback laws for nonlinear systems, more or less in the same way as linear geometric methods were used in the synthesis of feedback laws for linear systems. The result of this seminal work was the development of systematic methods addressing outstanding design problems like feedback linearization, noninteracting control, disturbance decoupling, and model matching.

1.3.2 Approximate linearization

In the last three decades, there has been a rapid increase of interest in the search for approximate solutions to the problem of linearizing nonlinear systems by state or output feedback. The main reason for that is the limited applicability of rigorous methods, and the complexity, sensitivity and design difficulties of the exact linearizing compensators. Also, noninvolutivity of the system prevents the application of exact linearization.
The approximate feedback linearization is required when a nonlinear system presents noninvolutivity property as given in Cardoso and Schnitman (2011). The process consists of finding approximate output functions that satisfy the involutivity condition up to a determined system order.

An overview of approximate linearization via feedback is given by Guardabassi and Savaresi (2001) who classified it into

(i) Partial-linearization technique
(ii) Linearization-oriented modeling
(iii) Nonlinearity measures and
(iv) Linear model-matching.

Partial-linearization is based upon the rigorous solution of a design problem, the objective of which is less demanding than exact linearization. One aspect of this method is dealt with in more detail in the next section. The basic idea behind linearization-oriented modeling is to carry the linearization objective back to the modeling phase. The rationale upon which nonlinearity measures is based is quite simple. Given a nonlinear plant, a bounded, real, positive function is defined, which gives a measure of its degree of nonlinearity. Approximate linearization of the plant is then achieved by selecting the feedback compensator which minimizes what is called the nonlinearity index of the overall control system. In linearization-oriented model-matching technique, a linear system tangent to the plant at a nominal operating condition is chosen as the reference model and a compensator design is done to make the dynamic behavior of the closed-loop system as close as possible to that of the linear reference model.
Research on partial linearization developed along three main streams. They are known as higher-order approximate linearization, pseudo-linearization and extended linearization. The work presented in this thesis pertains to a class of higher-order approximate linearization which will be considered in detail next.

1.3.3 Higher Order Approximate Linearization

Krener (1984) stated the problem of higher-order linearization as follows. Let $P$ be an affine plant described by Equation (1.1) or precisely, by

$$P: \dot{x} = f(x) + g(x)u$$

(1.4)

where $g(x) := (g_1(x) \ g_2(x) \ldots \ g_m(x))^T$, and superscript $T$ stands for transpose. Assuming that the origin $x = 0$ is an equilibrium state of $P$ relative to the constant input $u = 0$, i.e. $f(0) = 0$. Taylor expansion yields

$$\dot{x} = Ax + Bu + O(x, u)^2$$

(1.5)

where $A := \frac{\partial f}{\partial x}(0)$ and $B := g_x(0)$ and $O(x, u)^2$ contains terms of order 2 and higher. Dropping the $O(.)$ term, one gets the linear system $\delta P$ tangent to $P$ at the origin, namely the first-order approximation of $P$ around the origin. The question arises in Krener (1984) whether $\delta P$ can be made a higher-order approximation of $P$, around the origin, by means of state-feedback and coordinate transformation in the state space. One is interested in finding locally invertible transformations :

$$u = \alpha(x) + \beta(x)v; \quad y = T(x)$$

(1.6)

such that
\[
\dot{y} = Ay + Bv + O(x, v)^{p+1}
\]  
(1.7)

for some \(p \geq 1\). Necessary and sufficient conditions for the existence of \(\alpha, \beta\) and \(T\) are given in Krener (1984).

Poincare derived what are known as homological equations for approximate linearization of autonomous differential systems as given in Arnold (1983). The normal forms approach given in Arnold (1983) is a widely used technique in the analysis of bifurcations in nonlinear vector fields.

Krener et al (1987) extended Poincare’s approach with the introduction of control input. The procedure consists of finding a coordinate change by an appropriate feedback to achieve higher order linear approximations to nonlinear systems. The method of solving for linearizing transformations is based on the normal forms approach of Poincare as given in Arnold (1983). The results can be generalized as follows. Rewrite equations (1.4) and (1.5) and let the system be accurate to only order \(p - 1\), i.e.

\[
\dot{x} = Ax + Bu + O(x, u)^p; p \geq 2
\]

(1.8)

where \(O(x, u)^p\) consists of terms of order \(p\) and higher.

A coordinate change is sought as:

\[
x = y + \phi^{(p)}(y)
\]

(1.9)

along with feedback:
\[ u = \alpha^{(p)}(x) + \left( I + \beta^{(p-1)}(x) \right)v \quad (1.10) \]

which yields the homological equation to be solved as

\[ f^{(p)}(y) = -B\alpha^{(p)}(y) + [Ay, \phi^{(p)}(y)] \quad (1.11) \]

\[ g^{(p-1)}(y)v = -B\beta^{(p-1)}(y)v + [Bv, \phi^{(p)}(y)] \quad (1.12) \]

In Equations (1.11) and (1.12), \( \phi^{(p)}, f^{(p)}, \alpha^{(p)}, g^{(p-1)} \) and \( \beta^{(p-1)} \) are, respectively, homogeneous vector fields of orders corresponding to their superscripts. The resulting system is accurate up to order \( p \):

\[ \dot{y} = Ay + Bv + O(y, v)^{p+1} \quad (1.13) \]

One important issue is the following: when a solution exists, \( \alpha^{(p)}, \beta^{(p-1)} \) and \( \phi^{(p)} \) are not necessarily unique and the best choice of possible solutions needs more investigation. The equations, which need to be solved for finding the transformations, are a set of linear algebraic equations. However, the number of equations grows rapidly with increasing orders of linearization and with higher dimensional systems.

Krener et al (1988) presented a method to solve the approximate linearization problem of nonlinear systems and tried to solve the issues that were raised in Krener et al (1987) i.e. the non uniqueness of \( \alpha^{(p)}, \beta^{(p-1)} \) and \( \phi^{(p)} \), and the fact that the number of linear algebraic equations to be solved grew rapidly with order and dimension of the system to be linearized. Nonlinear system Equation (1.4) is revisited.

\[ \dot{x} = f(x) + g(x)u \quad (1.14) \]
where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \). The system is assumed to be at rest at the nominal operating point \((x^o; u^o = u(0) = 0)\). Considering linearization of system Equation (1.14) at \( x^o \):

\[
\dot{x} = Fx + Gu
\]

\[
F = \frac{\partial f}{\partial x}(0); \quad G = g(0)
\]

and the nonlinear system Equation (1.14) is rewritten as

\[
\dot{x} = Fx + f^{(2)}(x) + \left(G + g^{(1)}(x)\right) u + O(x,u)^3
\]

Transformation in state variable is

\[
y = x - \phi^{(2)}(x)
\]

A new input variable \( v \) is chosen as

\[
v = \alpha^{(2)}(x) + \left(I + \beta^{(1)}(x)\right) u \quad ; \quad \left(I + \beta^{(1)}(x)\right) \neq 0
\]

The desired system after transformation is

\[
\dot{y} = Fy + Gv + O(y,v)^3
\]

By taking time derivative of Equation (1.19), and introducing it into Equations (1.18) - (1.21), it follows that

\[
f^{(2)}(x) = [Fx, \phi^{(2)}(x)] + Ga^{(2)}(x)
\]
\[ g^{(1)}(x)u = [Gu, \phi^{(2)}(x)] + G\beta^{(1)}(x)u \ \forall \ \text{constant } u \]  

Equations (1.22) and (1.23) are called homological equations.

The above approximation is extended to an arbitrary degree as

\[ f^{(\rho)}(x) = [Fx, \phi^{(\rho)}(x)] + G\alpha^{(\rho)}(x) \]  

\[ g^{(\rho-1)}(x)u = [Gu, \phi^{(\rho)}(x)] + G\beta^{(\rho-1)}(x)u \ \forall \ \text{constant } u \]  

The resulting system is accurate up to degree \( \rho \):

\[ \dot{y} = Fy + Gv + O(y, v)^{\rho+1} \ ; \ \rho \geq 2 \]  

Hence by introducing an appropriate basis for expressing higher degree monomials in the vector field, a set of equations linear in the coefficients of the monomials are found. An exact solution to these set of equations is always not possible. Hence a least square solution is proposed that minimizes the error in the approximation.

Kang (1994) also addressed the problem of approximate linearization of a nonlinear control system by using dynamic state feedback. Some control problems involving dynamic state feedback amount to input-output decoupling problem and disturbance decoupling problem. In the dynamic input-output decoupling problem, one tries to design a compensator for the system so that the closed loop system is input-output decoupled. The dynamic disturbance decoupling problem requires one to construct, if possible, a dynamic feedback such that the output is independent of the disturbance, independent of the choice of new input.
Nam et al (1993) characterized approximate linearization to order \( m \) of a single input nonlinear system in terms of the existence of an integrating factor to order \((m-2)\), which is equivalent to the condition that a system of vector fields of order \((m-1)\) be involutive.

Barbot et al (1999) dealt with higher order approximation for discrete-time systems. It is shown that approximate feedback linearization at the second order can always be achieved under feedback compensation based on an approximated observer.

1.3.4 Quadratic Linearization and Normal Form

In linear control theory, different kinds of normal forms were found, of which the Brunovsky form was useful for the purpose of the design of control laws. From Brunovsky (1970) and Kailath (1980) it is evident that any controllable linear system can be transformed into a controller form by a linear change of coordinates. If, in addition, we also allow linear change of coordinates in the input space and linear state feedback, any controllable linear system can be transformed into a Brunovsky form. Hence Brunovsky form is the normal form of controllable linear systems.

Consider a single input, time-invariant linear system:

\[
\dot{x} = Fx + Gu
\] (1.27)

If it is controllable, then by a suitable change of coordinate and feedback as given in Equation (1.29), the linear system Equation (1.27) can be transformed into the following system of Brunovsky form:
The change of coordinate and feedback used are

\[
\begin{align*}
    \dot{x} &= T \dot{y} \\
    u &= \alpha y + \beta v
\end{align*}
\]  

(1.29)

where \( T \) is a constant \( n \times n \) nonsingular matrix, \( \alpha \) is a row vector and \( \beta \) is a nonzero real number.

As a starting point of finding the normal form for nonlinear dynamic system under change of coordinate and state feedback, in Kang and Krener (1990 and 1992), normal forms of the quadratic part was found. Consider the following nonlinear system:

\[
\dot{x} = f(x) + g(x)u
\]  

(1.30)

where \( f(x) \) and \( g(x) \) are nonlinear vector fields such that

\[
f(0) = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}; F = \frac{\partial f}{\partial x}(0); G = g(0)
\]

If \((F,G)\) is controllable, Equation (1.30) is called a linearly controllable system. Kang and Krener (1990) states that most linearly controllable nonlinear systems do not admit a controller normal form, therefore they cannot be transformed into the Brunovsky form. There exists a
linear change of coordinate and linear feedback Equation (1.29) which transforms a nonlinear system Equation (1.30) into a system:

$$\dot{y} = Ay + Bv + f^{(2)}(y) + g^{(1)}(y)v + O(y,v)^3 \quad (1.31)$$

where $$A = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{pmatrix}_{n \times n}$$, $$B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}_{n \times 1}$$

$$f^{(2)}(y) = \begin{pmatrix} f_1^{(2)}(y) \\ f_2^{(2)}(y) \\ \vdots \\ f_n^{(2)}(y) \end{pmatrix}$$, $$g^{(1)}(y) = \begin{pmatrix} g_1^{(1)}(y) \\ g_2^{(1)}(y) \\ \vdots \\ g_n^{(1)}(y) \end{pmatrix}$$

So, Brunovsky form is a normal form for the linear part of linearly controllable nonlinear system. To find the normal form of the quadratic terms in this system, consider the following nonlinear system, the linear part of which is in Brunovsky form:

$$\dot{x} = Ax + Bu + f^{(2)}(x) + g^{(1)}(x)u + O(x,u)^3 \quad (1.32)$$

Consider the change of coordinate and feedback of the following form:

$$\begin{cases} x = y + \phi^{(2)}(y) \\ u = v + \alpha^{(2)}(y) + \beta^{(1)}(y)v \end{cases} \quad (1.33)$$

Here $$\phi^{(2)}(y)$$ is an n-dimensional vector field whose entries are homogeneous polynomials of second degree in $$y$$, $$\alpha^{(2)}(y)$$ is a homogeneous polynomial of second degree and $$\beta^{(1)}(y)$$ is a polynomial of first degree.
Transformation Equation (1.33) leaves the linear part of Equation (1.32) invariant and the nonlinear coordinates $x$ and $y$ agree to the first degree.

According to Kang and Krener (1990), by a change of coordinate and feedback Equation (1.33), system Equation (1.32) can be transformed into one and only one of the following systems:

$$
\dot{y} = Ay + Bv + \tilde{f}^{(2)}(y) + O(y, v)^3
$$

(1.34)

where

$$
\tilde{f}^{(2)}(y) = \begin{pmatrix}
\tilde{f}_1^{(2)}(y) \\
\tilde{f}_2^{(2)}(y) \\
\vdots \\
\tilde{f}_n^{(2)}(y)
\end{pmatrix}
$$

$$
\tilde{f}_i^{(2)}(y) = \begin{cases}
\sum_{j=i+2}^{n} a_{ij} y_j^2 & 1 \leq i \leq n - 2 \\
0 & i = n - 1 \text{ or } n
\end{cases}
$$

A system such as Equation (1.34) is said to be in “extended quadratic controller form”. Since most nonlinear systems (Equation (1.30)) do not admit a controller form, they cannot be completely linearized by a change of coordinate and feedback.

The results in Kang (1994a), are the extension of the results in Kang and Krener (1990 and 1992) to higher degree. In Kang (1994a), the normal forms and invariants of a nonlinear control system with a single-input for each homogeneous part of degree greater than 2 is found. The conditions to be satisfied by the (homogeneous) transformation are given which are expressed in terms of Lie brackets so that the conditions can be
written as a set of linear algebraic conditions. But, the construction of transformation is not given explicitly.

Jin (2005) proposed a constructive method for studying the extended quadratic controller normal form given in Kang and Krener (1990 and 1992) for continuous linearly controllable, control systems with a single input. Jin transformed the homological equations to linear algebraic forms which need to be satisfied for some matrix terms corresponding to coordinate and state feedback.

Devanathan (2001) showed that if a nonlinear system is linearly controllable, then it can always lead to the linearization condition, provided the eigenvalues of the Jacobian of the system are made nonresonant.

Devanathan (2004) gave a constructive approach to the solution of homological equation for the coordinate and state feedback, after putting the linear part in diagonal form with non-resonant eigenvalues. The problem faced in the techniques given in Devanathan (2001 and 2004) is of the arbitrariness introduced in the selection of non-resonant eigenvalues of the linear part.

Tall (2010) expresses linearizability conditions in terms of involutive properties of the system quadratic polynomials without involving the coordinate and state feedback. However, these conditions are hard to verify. Tall (2010) provided iterative steps involving differentiation and integration to obtain coordinate and state feedback for arbitrary order linearization. These methods, being constructive in nature, implicitly assume the existence of coordinate and state feedback for the successful verification of quadratic linearization condition. The algorithms involve Lie derivative
and are complex. The linearizability conditions have to be verified at every step and the system may not be completely linearized at the end.

Tall (2005) constructed a normal form for multi-input nonlinear control systems with uncontrollable mode which generalizes the result obtained in the single input case given by Kang (1994a).

1.3.5 Control of PM motor drives

Permanent magnet (PM) motor drives have been a topic of interest since 1980’s. Due to its high torque to inertia ratio, superior power density, high efficiency and many other advantages, permanent magnet synchronous motor is widely used in many industrial applications. Various control techniques are used to regulate speed of PMSM drive. A review of different types of control techniques is given below.

Pillay and Krishnan (1988) presented the dynamic models and equivalent circuits of Permanent Magnet Synchronous Motor and Brushless DC Motor (BDCM). The d-q model of the wound rotor synchronous motor is easily adapted to PMSM while an abc phase variable model is necessary for the BDCM if a detailed study of its behavior is needed.

Jahns and Kliman (1986) describe interior permanent-magnet synchronous motor as a desirable candidate for adjustable-speed operation. Interior permanent-magnet synchronous motor is a robust high power density machine capable of operating at high motor and inverter efficiencies over wide speed ranges, including considerable ranges of constant-power operation. As a further advantage, it is noted that constant power characteristics could be achieved over an extended speed range by means of
flux weakening. The impact of the buried-magnet configuration on the motor's electromagnetic characteristics is also discussed.

Jahns (1987) discusses the flux-weakening operation of Interior permanent magnet synchronous motor which makes it compatible with extended-speed-range constant-power operation.

The classification of permanent magnet AC (PMAC) machines is given by Dwivedi et al (2010). Permanent magnet brushless DC (PMBLDC) motors and permanent magnet synchronous motors are described as the most popular permanent magnet brushless motors for various applications. The general classification of the different control techniques for variable frequency control of PMSM drive is given in Dwivedi et al (2010).

The control methods applied to PMSM can be divided into categories (a) scalar control (b) vector control and (c) new control techniques. These methods are given in detail by Bose (2002). The scalar control method is based on a simplified model which is only valid for steady state operation. In this method the magnitude and frequency of voltage, current and flux linkage space vectors are controlled. The scalar control method is not suitable for transient performance.

In the vector control method not only the magnitude and frequency of voltage, current and flux space vectors are controlled, but it also results in precise control of position of these space vectors. Vector control can be further subdivided into (i) Field Oriented Control (FOC) and (ii) Direct Torque Control (DTC).

The field oriented control methods utilize the coordinate transformation of motor equation in a synchronously rotating reference frame.
It allows decoupled control of flux and torque producing component of stator current and attempts to control motor flux and torque independently. FOC uses d-axis and q-axis current controllers to get decoupled control of drive. However FOC requires absolute position information of the magnet. Field oriented control of PMSM is given in detail by Bose (2002), Pillay and Krishnan (1989).

DTC allows direct control of motor flux and torque without the need of inner current loops. The DTC uses hysteresis controller for controlling the flux and torque. DTC suffers from various disadvantages viz., requirement of fast sampling time, need of two hysteresis controllers, variable switching frequency behaviour and higher ripples in torque and flux. Analysis of DTC in PMSM drives is reported by Zhong et al (1999) and Rahman et al (1997).

System theoretic methods of control of PMSM can be subdivided into (i) Feedback linearization control and (ii) Passivity based control. Recent advances in modern control techniques suggest that the controllers for electrical motors should be designed directly from nonlinear models as given in Taylor (1994). Feedback linearization techniques have been extensively applied to the control of electrical machines. The essence of this technique is to first transform a nonlinear system into a linear one by nonlinear feedback and then use the well-known linear design techniques to complete the controller design.

An overview of nonlinear control of electric machines is given by Taylor (1994). The design concept of exact linearization is reflected by the two-loop structure of the controller. Nonlinear compensation which explicitly cancels the nonlinearities present in the motor is implemented as an inner
control loop. Linear compensation which is derived on the basis of the resulting linear dynamics of the pre-compensated motor is implemented as an outer feedback loop. The advantage of linear closed-loop dynamics is clearly that selection of controller parameters is simplified and the achievable transient responses are very predictable.

Zhang et al (2011) discuss the decoupling control method of PMSM using exact linearization via state variable feedback. The conditions for exact linearization are verified for PMSM nonlinear system. Instead of applying differential geometry for the construction of output function, a new decoupling method is proposed which is verified using MATLAB simulation.

In Wu et al (2010), PMSM is abstracted into two-input, two-output PMSM mathematical model. It is verified that the model could be exactly linearized using differential geometric method. Using coordinate transformation and nonlinear state feedback, linear model of PMSM is obtained. Active rejection controllers are designed respectively for decoupled direct axis current and rotor speed linear subsystems. The on-line simulation results based on dSPACE illustrate the feasibility of the design method.

Differential geometric approach has led to effective ways of designing control systems for electric motors. Bodson and Chiasson (1988) have designed a controller using differential geometric method based on exact feedback linearization. The mathematical model for PM synchronous motor is given and the model is simplified as a result of $dq$ transformation into state variable differential equation model. The nonlinear feedback which results in a linear system is derived as

$$
\begin{pmatrix}
v_d \\
v_q
\end{pmatrix}
= \begin{pmatrix}
2L_g p/L_d & p(K_m + 2L_g i_d)/L_q \\
1/L_d & 0
\end{pmatrix}
\begin{pmatrix}
u_d \\
u_q
\end{pmatrix}
+ \begin{pmatrix}
f_1(\omega_r, i_d, i_q) \\
f_2(\omega_r, i_d, i_q)
\end{pmatrix}
$$

(1.35)
where \( u_q, u_d, i_q, i_d \) represent the quadrature and direct axis voltages and currents respectively and \( \omega_r \) represents rotor speed. \( L_d, L_q \) are the direct and quadrature inductances respectively. \( L_g \) denotes inductance due to non-uniform airgap. \( p \) denotes number of pole pairs and \( K_m \) is the flux in the stator phase due to the permanent magnet of the rotor. \( f_1(\omega_r, i_d, i_q) \) and \( f_2(\omega_r, i_d, i_q) \) are functions of \( \omega_r, i_d, i_q \) and motor parameters. \( v_q, v_d \) are the new inputs which are given in Equation (1.35).

For implementation of the controller, the inversion of the transformation matrix is required as shown below.

\[
\begin{pmatrix}
 u_d \\ u_q \\
\end{pmatrix} = \begin{pmatrix}
 2L_g p / L_d \\ 1 / L_d \\
\end{pmatrix} \begin{pmatrix}
 p(K_m + 2L_g i_d) / L_q \\ 0 \\
\end{pmatrix}^{-1} \begin{pmatrix}
 -f_1 + v_d \\ -f_2 + v_q \\
\end{pmatrix} (1.36)
\]

As can be seen, the transformation matrix is a function of state variables, and there may exist a condition when the matrix is singular and cannot be invertible. For example, when \( i_d = -K_m / 2L_g \), the transformation matrix is singular and even though the system is linearizable, the linearization method fails. Hence this method suffers from the issue of singularity and this puts a constraint on the practical implementation of the control strategy.

Zribi and Chiasson (1991) proposed a method for position control of PM stepper motor using exact linearization. Since the nonlinear controller takes into account the full dynamics of the stepper motor, the phase shift between voltage and current in each phase is accounted for automatically. The feedback linearization controller turns the stepper motor into a fast accurate positioning system.
Zhu et al (2001) have presented a scheme which combines exact-linearization based controller with a nonlinear state observer which is used to estimate the rotor position and speed. The stability of the closed loop system including the observer is demonstrated through Lyapunov stability theory. In Zhu et al (2001), it is assumed that the direct and quadrature axes inductances are equal.

A new approach to control design for electric machines, referred to as energy shaping, is based on Euler-Lagrange form of the dynamic equations as given in Taylor (1994). In contrast to the exact linearization design technique, which could be called a mathematically motivated approach, the energy shaping design approach has evolved from consideration of the physical properties of the system, such as energy conservation and passivity. Energy shaping design is claimed to have some advantages over exact linearization. As a consequence of being based on physical energy properties of the system, the energy shaping design can be robust with respect to modeling errors, provided the perturbations do not violate the skew-symmetry assumption of the model. The use of Euler-Lagrange equations also facilitates adaptive design since the key features of linear and minimal parameterization are present. In addition to these generic benefits, the energy shaping design results in a globally defined control law which is free from singularities, and initial conditions are also not restricted. The concept and utility of passivity-based control is explained in detail by Khalil (2002).

Belabbes et al (2009) presented the study of the behavior of passivity based control and difficulties due to synthesis of various operating conditions of permanent magnet synchronous motor. The paper describes the new sensorless control of the PMSM using a sliding mode observer. The proposed methodology incorporates Lyapunov’s algorithm to estimate the
rotor speed and the stator resistance so that it can overcome the problem of sensitivity in the face of motor parameter variation. Li Tao et al (2010) proposed a new nonlinear control strategy for three phase permanent magnet synchronous motor, in which passivity-based controller is used as current inner loop and the active disturbance rejection controller is used as speed outer loop of the control system.

Sliding mode variable structure control (SMVSC) has been studied by many researchers due to its favorable advantages, such as its insensitivity to parameter uncertainties and external disturbances as given in Hung et al (1993). But the drawback is the chattering, which limits the application of SMVSC. A robust speed control of PMSM using boundary layer integral sliding mode control technique is presented in Baik et al (2000) to reduce the chattering phenomenon.

The implementation of feedback linearization, as well as the other strategies, require an optical/mechanical sensor to obtain position and speed as part of the state to be fed back. However, mechanical sensors can be avoided when sensorless control strategies are designed. In such cases rotor position and speed must be estimated and these estimated values are used to compute the control law. Han et al (2000) have presented a new speed and position sensorless control method of the PMSM based on sliding-mode control observer. Lyapunov functions are chosen for determining the adaptive law for the speed and stator resistance estimator.

Yue et al (2003) proposed an observer-based robust adaptive position and speed-tracking control system for the PMSM based on Lyapunov stability theory. In the proposed methodology, the initial rotor position is treated as an uncertain constant parameter.
Kumar et al (2006) presented a critical analysis of intelligent tuned PID controllers for the PMSM drive. PID control strategies based on fuzzy logic, neural network and genetic algorithms are reviewed and implemented. The algorithm developed can be easily modified to study the dynamics of drive operation under different operating conditions.

Fayez (2007) has presented an intelligent sliding-mode controller for achieving favourable decoupling control and high precision position tracking performance of PMSM servo drive. A proposed on-line trained fuzzy-neural-network model-following controller is designed in addition to the sliding-mode controller to improve the dynamic performance.

Kumar et al (2007) have presented an artificial neural network based high performance speed control system for a PMSM. The rotor speed of PMSM is made to follow an arbitrarily selected trajectory so that accurate trajectory control of the speed could be achieved when the motor and load parameters are unknown. The unknown nonlinear dynamics of the motor and the load are captured by the ANN.

An and Sun (2008) have proposed an on-line identification of PMSM electromagnetic parameters based on model reference adaptive control. By means of Popov stability theory, the adaptive laws are derived with $d,q$-axis voltages, currents and their errors in the rotor reference frame. The model which runs in parallel with vector control can track the electromagnetic parameters rapidly.

Barkat et al (2011) have developed a decentralized adaptive fuzzy controller for PMSM based on fuzzy logic system. The adaptive law is designed to compensate for the effects of interconnection and the
reconstruction errors, using a sliding mode term. The fuzzy logic systems are used to model the unknown functions.

Sperb et al (2011) have presented a sensorless control strategy for PMSM control using a novel neural network algorithm. The proposed observer uses a neural network trained to learn the electrical and mechanical motor models using the current prediction error.

Rahideh et al (2007) have proposed a high performance direct torque controlled PMSM using fuzzy logic and genetic algorithm. In this approach, the electromagnetic torque error, stator flux linkage error and stator flux angle are fuzzified using three appropriate fuzzy sets. The best voltage voltage space vector is selected in order to minimize torque, flux and current ripples. The PI speed controller is tuned using genetic algorithm.

1.4 MAIN CONTRIBUTIONS

The main contributions of the thesis can be summarized as follows:

(i) Approximate linearization technique is successfully applied to remove the predominant quadratic nonlinearity in a PMSM machine through coordinate and state feedback. This is a new approach to the control of PMSM not found in the literature. Also, in contrast to the existing exact feedback linearization technique, the proposed technique does not suffer from singularity issue.

(ii) Truly explicit necessary and sufficient condition in algebraic form, to verify quadratic linearizability of an n-dimensional system with a single input is derived. The condition is easily
verifiable and is not available in literature. This is one of the main system theoretical contributions of the thesis.

(iii) A new concept called ‘generalized quadratic linearization’ is introduced for a class of n-dimensional system. The concept seeks to cancel the third and higher order terms which would be introduced into the system in the process of quadratic linearization. A closed form solution to the generalized quadratic linearization for a class of n-dimensional system is derived. PMSM is shown to belong to this class. The theoretical result obtained here can also be seen as a system theoretic contribution towards solution of arbitrary order linearization of a class of systems.

(iv) Adaptive tuning of the linearizing transformation is done by comparing against a linear canonical system. This adaptive tuning can provide an empirical solution to not only the problem of third and higher order terms that are introduced during the process of quadratic linearization, but also unmodelled dynamics and higher order nonlinearities in the model, such as core loss. This tuning technique which cancels the effect of higher order terms is novel and has not been applied before in the context of PMSM control.

(v) The proposed linearization techniques applied to PMSM are successfully verified through simulation and implementation of the control strategies with an actual machine.

(vi) An asymptotic stability analysis of control affine system containing only second order nonlinearity is carried out. The result is also extended to a system containing third and higher
order nonlinearities such as would arise subsequent to quadratic linearization. The stability analysis holds good in particular to PMSM model having predominantly quadratic nonlinearity.

1.5 SUMMARY

In this chapter, the objectives and main contributions of the thesis have been defined in detail followed by a review of linearization and control of PM motor drives. The rest of the thesis is organized as follows.

Since the model of PMSM is predominantly quadratic in normal operating conditions, it is sought to apply quadratic linearization, which is a special case of approximate linearization to PMSM model. In chapter 2, explicit algebraic and easily verifiable necessary and sufficient conditions for quadratic linearization of control affine systems for the case of single input is derived. Linearization of PMSM using these conditions is illustrated through an example. Also, the construction of linearizing transformations of input and state feedback is demonstrated.

In chapter 3, generalized quadratic linearization for a class of control affine systems is derived. The generalized quadratic linearization seeks to remove higher order terms introduced into the system due to quadratic approximate linearization. It is showed that induction motor and PM motor models belong to this class. Hence the result of generalized quadratic linearization is applied to PMSM. This result is in effect an approximate linearization analog of exact linearization for a class of systems.

In chapter 4, tuning of the transformation parameters is presented to cancel the effect of core loss, stray loss, unmodelled dynamics and third and
higher order terms introduced into the system due to quadratic linearization. Asymptotic stability analysis for a control affine system containing only second order nonlinearity is carried out and the result is generalized to a system containing third and higher order terms. This result can be applied to PMSM as a special case.

In chapter 5, PMSM machine model, together with the state and input transformations, are simulated using SIMULINK. The effectiveness of the linearization method and tuning technique proposed is verified using SIMULINK.

In chapter 6, the proposed method of linearization of PMSM is implemented using TMS320F2812 DSP on an actual PMSM machine. The effectiveness of the proposed linearization technique is verified. In chapter 7, the thesis is concluded and the possible future research work is discussed.