CHAPTER 2

PRELIMINARIES

CHAPTER OUTLINE

In this chapter, relevant technical background for the accomplished research is introduced. In section 2.1, uncertain nonlinear delayed systems with single point delay and multiple point delay are surveyed. Types of nonlinearities are discussed in section 2.2. Section 2.3 deals with the stability analysis in brief. The controllability and observability issues of a nonlinear system are discussed in section 2.4. Section 2.5 addresses the existing nonlinear control strategies. Adaptive control for the nonlinear systems containing uncertainties is discussed in section 2.6 while section 2.7 reveals the adaptive nonlinear observer strategy. Due to the richness of nonlinear system theory, only some definitions and theorems are introduced in this chapter, which are essential to the dissertation.
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2.1 Nonlinear Systems

This research is confined to the class of systems represented by continuous dynamics which can be written as Ordinary Differential Equations (ODEs). Since the objective is to control the plant dynamics, it is natural to inquire as to the existence and uniqueness of the solutions for the ODEs. The general form of ODE considered is

\[ \dot{x} = f(x, t), \quad x_0 = x(t_0) \]  

(2.1)

where \( x \in \mathbb{R}^n \) represents the state of the system, \( t \) is time, \( t_0 \) is the initial time, and \( x_0 \) is the initial state of the system. For this class of systems, the state derivatives are some nonlinear functions of states, which are represented by \( f(x) \). Presence of these nonlinear functions makes the system modeling and analysis complicated up to certain extent. As cited by few researchers, \( f(x, t) \) is required to be continuous in time and locally Lipschitz in the state variable for the existence and uniqueness of solutions for (2.1) \([1,2]\). However these conditions impose certain constraints over the system nonlinearities, which are not normally satisfied by most of the system dynamics. Also this restriction offers a locally stable and conservative solution for the system under consideration.

Since the goal is to design a control strategy that makes the plant trajectories follow desirable paths, the ODEs are considered to be forced by exogenous inputs. Thus, the differential equation with exogenous input can be written in the most general form as

\[ \dot{x} = f(x, u, t), \quad x_0 = x(t_0) \]  

(2.2)

where \( u \in \mathbb{R}^m \) is the command input. However, in this work the system is assumed to be autonomous thus the explicit dependence on time is removed, and for a wide class of systems, \( f(x, u) \) can be separated into two parts. The resulting equation is referred to as linear in the controls or affine and can be written as

\[ \dot{x} = f(x) + G(x)u, \quad y = h(x), \quad x_0 = x(t_0) \]  

(2.3)

where \( f(x): \mathbb{R}^n \rightarrow \mathbb{R}^n \) is referred as the homogenous dynamics of the model and \( G(x): \mathbb{R}^n \rightarrow \mathbb{R}^n \) as the control multiplier dynamics and are smooth vector fields defined on \( D \subset \mathbb{R}^n \) and \( h() \) is a bounded smooth function on \( D \). The dynamics of nonlinear systems are
richer than that of linear systems, due to which the analysis of nonlinear systems is more complex. One common way of analyzing nonlinear system is to linearize it, if possible, about some nominal operating point and then to analyze the resulting linear model. However there are two basic limitations of linearization. First, since linearization is an approximation in the neighborhood of an operating point, it can only predict the local behavior of the nonlinear system in the vicinity of that point, and hence it cannot predict the global behavior throughout the state space. Secondly, there are essentially nonlinear phenomena like finite escape time, multiple isolated equilibria, harmonic oscillations, chaos and multiple modes of behavior etc, which can take place only in the presence of nonlinearity and cannot be described or predicted by linear models. Another aspect of nonlinear systems which makes their analysis very complex is the uncertainty in the plant model. Due to the mathematical inaccuracies and the limitations of the physical laws governing the dynamics of the system, the developed model of the system is not accurate and contains some uncertainties, i.e. $f(x)$ is a function of some uncertain parameters. These uncertainties may be parametric uncertainties or functional uncertainties depending upon the system under consideration. The presence of nonlinear uncertainties in the dynamics of the system offers a challenging area of research in the field of control system.

### 2.1.1 Delayed Nonlinear systems

In many physical, industrial and engineering systems, delay in the states or the input of the dynamical system, occurs due to the finite capabilities of information process and data transmission among various parts of the system. Delay could arise as well from inherent physical phenomena like mass transport flow and recycling. Such delay could be constant or time varying, known or unknown, deterministic or stochastic, depending on the system under consideration. Some complex behaviors like limit cycles, chaotic behavior and even instability can be induced in the system due to presence of the delayed states or input in the dynamics [2]. Delay present in the nonlinear systems complicates the controller designing aspects. Therefore the researchers are inclined towards the subject of Time delay systems (TDS) from the last few years [3-7]. By incorporating the modeling uncertainties due to component aging, parameter variations or modeling errors, in the modeling of time delay systems, a special class of systems is obtained;
uncertain time delay systems (UTDS), whose dynamics can be represented by the following delayed differential equation
\[
\dot{x} = f(x(t), x(t-\tau)) + G(x(t), x(t-\tau))u, \quad x_0 = x(t_c)
\] (2.4)
where \( \tau \) is the time delay present in the states. The delay in (2.4) is single point delay. In many dynamic systems the delay is multiple point or distributed as well, which can be represented as
\[
\dot{x} = f(x(t), x(t-\tau_1), \ldots, x(t-\tau_n)) + G(x(t), x(t-\tau_1), \ldots, x(t-\tau_n))u, \quad x_0 = x(t_0)
\] (2.5)
where \( \tau_1, \tau_2, \ldots, \tau_n \) are the delays present in the system at different points.

The presence of delays can have an adverse effect on system stability and performance, so ignoring them may lead to design flaws and incorrect analysis conclusions. Stability is classified as delay-independent if it is retained irrespective of the size of the delays and delay-dependent if it is lost at a certain delay value. In this work, delay independent strategy is proposed for single point as well as multiple point delay systems, which has no constraint over the amount of delay present in the system.

### 2.2 Types of nonlinearities

The dynamics of most of the real time systems always contain some nonsmooth nonlinearity due to its inherent characteristic. Commonly existing nonlinear phenomenon like backlash, dead-zone, hysteresis and saturation in industrial processes severely limit the system performance as they are ill known and time varying in nature. These nonsmooth nonlinearities should always be taken into account while designing an effective control strategy. In particular, the major cause of the static (e.g., dead-zone) and dynamic (e.g., backlash, hysteresis) nonlinearities in control systems is the actuators used in control system due to their inherent characteristics. Due to the non-analytic nature of the actuator nonlinearities and the fact that their exact nonlinear functions are unknown, such systems present a challenge for the control design engineer. The static and dynamic performance of feedback control systems is highly prone to the nonsmooth nonlinearities as they usually lead to a relevant deterioration of system response. The complexity associated with these nonsmooth nonlinear characteristics, has inclined many researchers towards the control design for the class of systems subject to these nonlinearities during the last few years [5-7]. Some common nonlinear phenomena exist in practical systems are discussed below.
2.2.1 Backlash

Backlash is one of the most important nonlinearities that limit the performance of speed and position control in industrial, robotics, automotive, automation and other applications. It is a dynamic nonlinear relationship between input and output, which is present in every mechanical system where a driving member (motor) is not directly connected with the driven member (load). This is the case in many mechanical systems, notably those with gears, e.g., the drive train in cars, rolling mills, printing presses, and industrial robots. The analytical expression of the backlash characteristic is

\[
\dot{u}(t) = \begin{cases} 
  m \dot{v}(t) & \text{if } \dot{v}(t) \geq 0 \text{ and } u(t) = m(v(t) + c_r), \text{ or} \\
  0 & \text{if } \dot{v}(t) \leq 0 \text{ and } u(t) = m(v(t) - c_l) \\
  \text{otherwise} & 
\end{cases}
\]  
(2.6)

where \( m > 0, c_r < c_l \) are constant parameters. The motion on any inner segment is characterized by \( \dot{u}(t) = 0 \). A widely accepted characteristic of backlash is shown in Figure 2.1 where \( v \) is the input, \( u \) is the output, and \( c_r \) and \( c_l \) are the right and left “crossing”.

The systems subject to backlash often exhibit steady-state errors or limit cycles which makes the system oscillates, often in an irregular fashion which may lead the system to instability.

![Figure 2.1 Backlash Nonlinearity](image-url)
2.2.2 Dead-Zone

A dead-zone is a kind of non linearity in which the system does not respond to the given input until the input reaches a particular level. It is a static input-output relationship. Once the output appears, the slope between the input and the output remains constant. It is one of the most important nonsmooth nonlinearities arisen in actuators, such as servo valves and DC servo motors. In most practical motion systems, the dead-zone parameters are poorly known. The analytical expression of the dead-zone characteristic is

\[
    u(t) = \begin{cases} 
    m_r(v(t) - b_r) & v(t) \geq b_r \\
    0 & b_r < v(t) < b_l \\
    m_l(v(t) - b_l) & v(t) \leq b_l \end{cases} \tag{2.7}
\]

A graphical representation of the dead-zone is shown in Figure 6.6, where \( v \) is the input and \( u \) is the output. In general, neither the break-points \( b_r \geq 0, b_l < 0 \) nor the slopes \( m_r, m_l > 0 \) are equal. There is no loss of generality in assuming that the zero input point is inside the dead-zone because this can always be achieved with a redefinition of the input \( v \). Dead-zone can severely limit the system performances. Due to the non analytic nature of dead-zone in actuators and the fact that the exact parameters (e.g. width of dead-zone) are unknown, systems with dead-zones present a challenge for the control design engineers and attracted many researchers since last decade [3].

![Figure 2.2 Dead Zone Nonlinearity](image_url)
2.2.3 Hysteresis

Several models are developed to represent the hysteresis behaviors without necessarily providing the physical insight the system like Preissach model, Prandtl-Ishlinskii model etc. As an example, Prandtl-Ishlinskii model of hysteresis effect present in the actuator is discussed here. The models set up by composition of play and stop operators are referred as Prandtl-Ishlinskii model. Suppose $E_c[u]$ are basic elastic plastic elements or stop operators for all $r \in [0, \mathcal{R}]$, then the model can be expressed as

$$\nu(t) = \int_{0}^{r} p(r) E_c(u)(t) dr$$

(2.8)

where $p(r)$ is a given density function satisfying $p(r) \geq 0$ with $\int_{0}^{r} p(r) dr < \infty$, which is supposed to be identified from experimental data. With thus defined density function, this operator maps $C(t_0, \infty)$ into $C(t_0, \infty)$, i.e. Lipschitz continuous inputs will yield Lipschitz continuous outputs. The PI model can also be expressed in terms of the play operator $F_c(u)$ as

$$\nu(t) = p_u u(t) - \int_{0}^{r} p(r) F_c(u)(t) dr$$

(2.9)

where $p_u = \int_{0}^{r} p(r) dr$ is a constant, depending on the density function. Equation (2.9) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part and the second term gives the hysteresis. This decomposition is crucial for the design of controller, because the currently available robust control technique can be utilized for the controller design.

![Figure 2.3 Hysteresis Nonlinearity](image)
2.2.4 Saturation.

The physical constraints of the dynamical systems due to its inherent characteristics or due to the actuator in the closed loop, always imposes some hard limits on the amplitude of control system input. This hard limit constraint is modeled by saturation nonlinearity which is always a potential problem for actuators of control systems and may lead to severe performance deterioration and even instability in some cases. Thus, the impact of these constraints upon the closed-loop feedback control system has inclined many researchers to address this problem since last decade [19-22]. Saturation nonlinearity is defined as follows and shown in Figure 2.4.

\[
    u(t) = \text{sat}(v(t)) = \begin{cases} 
    \text{sign}(v(t))u_M & |v(t)| \geq u_M \\
    v(t) & |v(t)| \leq u_M 
\end{cases}
\]

(2.10)

where \( u_M \) is a known bound of \( u(t) \). The relationship between the applied control \( u(t) \) and the control input \( v(t) \) has a sharp corner when \( |v(t)| = u_M \).

![Saturation Nonlinearity Diagram](image)

Figure 2.4 Saturation Nonlinearity

Nonlinear behavior of the actuator causes the detuning of plant as well as controller parameters which may lead to the poor performance or even may cause the destabilization of the system. Saturation is addressed by several researchers, but most of the research in this topic is based on augmentation of baseline controller with additional saturation compensation dynamics [19,20].
2.3 Stability

In this section some definitions are given to make some qualitative statements about the response of the system with the control strategy.

**Definition 2.1 (Equilibrium State)**

A point, \( x_e \in \mathbb{R}^n \) is an equilibrium state of (2.3) if \( f(x_e) = 0 \). Thus, at equilibrium, if \( x(t_0) = x_e \) and \( u = 0 \) then \( x \) remains at \( x_e \) for all time \( t \geq t_0 \). In general terms, equilibrium can be referred as stable if, for given small perturbations, the state tends to return to equilibrium, or at least not go too far from the equilibrium. This notion of stability has been quantified in various ways and resulted in a variety of theorems[1-3].

**Definition 2.2 (Lyapunov Stability):**

An equilibrium point is said to be stable (in the sense of Lyapunov) if for any \( \epsilon > 0 \) there exists \( a\delta(\epsilon) > 0 \) such that \( x(t) - x_e < \epsilon \Rightarrow x(t) - x_e < \epsilon \forall t < t_0 \). Thus, Lyapunov stability implies that if we start close enough to the equilibrium, we will remain close. A more desirable type of stability is that of asymptotic stability [1-3].

**Definition 2.3 (Asymptotic Stability):**

An equilibrium point is said to be asymptotically stable if the equilibrium is stable, and if there exists \( a\delta > 0 \) such that \( x(t) - x_e \rightarrow 0 \).

Therefore, with asymptotic stability the solution is assured to approach the exact desired result as time passes [2]. In some situations, stability as defined above may not be achievable. In those cases the solution is insured to be bounded.

**Definition 2.4 (Boundedness):**

A solution \( x(t, t_0, x_0) \) is bounded if there exists \( \beta > 0 \) such that \( |x(t, t_0, x_0)| < \beta \forall t > t_0 \) where \( \beta \) may depend on the initial condition, \( x_0 \). (A system is said to be Lagrange stable if all solutions are bounded [2]). To insure that a system eventually settles to a bounded condition, there is the concept of ultimate boundedness.
Definition 2.5  (Ultimate Boundedness):

A solution $x(t,x_0)$ is ultimately bounded if there exists $\beta > 0$ and a time, $T = T(\delta)$ where $\delta > 0$ such that $|x_0| < \delta \rightarrow |x(t,x_0)| < \beta \forall t \geq T$.

Again, for autonomous plant models, ultimate boundedness is equivalent to uniform ultimate boundedness [2]. In order to make stability conclusions of the forms given by definitions 2.2, 2.3, 2.4 and 2.5, a variety of stability theorems are often used. These are often predicated on the definition of what is referred to as a Lyapunov function.

Definition 2.6  (Lyapunov Function):

If for $x \in B_r$, where $B_r$ is some ball of radius $r$, the function $V(x)$ has continuous partial derivatives, is positive definite and $\dot{V}(x)$ is at least negative semi-definite, then $V(x)$ is referred to as a Lyapunov Function [2]. With definition 2.6, a basic Lyapunov stability theorem can be stated.

Definition 2.7  (Lyapunov-Krasovskii Functional):

Consider the functional differential equation

$$
\dot{x}(t) = f(t,x_t), t \geq t_0
$$

$$
x_{\theta} = \phi(t + \theta), \forall \theta \in [-\tau, 0]
$$

where $x(\theta), t \geq t_0$ denotes the restriction of $x(.)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$, that is

$$
x(\theta) = \phi(t + \theta), \forall \theta \in [-\tau, 0]
$$

with $\phi \in C_{\tau,r}$. Let the function $f : \mathbb{R} \times C_{\tau,r} \rightarrow \mathbb{R}^n$ take bounded sets of $C_{\tau,r}$ in bounded sets of $\mathbb{R}^n$ and $\alpha, \beta, \gamma : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be continuous and non-decreasing functions with

$\alpha(r), \beta(r) > 0; r \neq 0$

$\alpha(0) = 0, \beta(0) = 0$

If there exists a continuous function $V : \mathbb{R} \times C_{\tau,r} \rightarrow \mathbb{R}$ such that

a. $\alpha(\|\phi(0)\|) \leq V(t,x) \leq \beta(\|\phi(0)\|), t \in \mathbb{R}, x \in \mathbb{R}^n$
b. \( \dot{V}(t, \phi) \leq -\gamma(\|\phi(0)\|) \)

Then the trivial solution of \((C,1)\) is uniformly stable.

If \( \alpha(r) \to \infty \) as \( r \to \infty \), then the solutions are uniformly bounded.

If \( \gamma(r) > 0 \) for \( r > 0 \), then the solution \( x = 0 \) is uniformly asymptotically stable and \( V(t, x) \) is referred to as a Lyapunov-Krasovskii Functional [2].

2.4 Nonlinear Controllability and Observerability

Two of the fundamental concepts appearing in the earlier studies on control systems are controllability and observability. With the stability of motion defined and the criteria for the stability introduced, naturally, the controllability and observability problems of nonlinear dynamic systems come up to our attention. Controllability denotes the ability to maneuver the states of a dynamic system around its operational domain by admissible inputs; whereas observability denotes the ability to uniquely determine the states of a dynamic system from its outputs. The appearance of nonlinearities in the state equations renders the controllability & observability problem for nonlinear systems more complicated than that of linear systems. Extensive research for nonlinear systems was accomplished by several researchers on this topic with profound result. Due to the importance of controllability and observability in output feedback tracking control, the concept and the criterion of controllability and observability are to be reviewed in this section. Next the differential geometric tools like Lie derivatives and Lie Brackets are reviewed, which forms a natural starting point for extending the standard results on linear controllability and observability to nonlinear systems.

**Definition 2.8 (Lie derivatives):**

For the system (2.3), let \( h : D \to \mathbb{R} \) be a smooth function and \( f : D \to \mathbb{R}^n \) be a smooth vector field. The Lie Derivative of \( h \) with respect to \( f \), written as \( L_f h \), is defined by[1]

\[
L_f h(x) = \frac{\partial h}{\partial x} f(x)
\]  \hspace{1cm} (2.12)
**Definition 2.9 (Lie Bracket):**

For the system (2.3), let \( f \) and \( g \) be two smooth vector fields on \( D \to \mathbb{R}^n \). The Lie Bracket of \( f \) and \( g \), written as \( \left[ f, g \right] \), is the third vector field defined by

\[
\left[ f, g \right](x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x) \quad (2.13)
\]

The bracketing of \( g \) with \( f \) may be repeated. The following notation is used to facilitate this process:

\[
\begin{align*}
ad^0 f g(x) &= g(x) \\
ad^1 f g(x) &= \left[ f, g \right](x) \\
ad^2 f g(x) &= \left[ f, ad^1 f g \right](x)
\end{align*}
\]

**2.4.1 Nonlinear Controllability**

The controllability of nonlinear system is defined as

**Definition 2.10 (Controllability):**

A nonlinear system is said to be controllable if for any two points \( x_0 \) and \( x_1 \), there exist a finite time \( T \) and an admissible control defined on \( [0, T] \) such that for \( x(0) = x_0 \), we have \( x(T) = x_1 \).

The controllability of nonlinear system is assessed using control Lie derivatives. System (2.3) is controllable if the rank of the matrix \( C \) is \( n \),

\[
C = \begin{bmatrix}
g_1 & g_2 & \ldots & g_n & [ad^1 f g(x)] & \ldots & ad^n f g(x)
\end{bmatrix}
\]

**2.4.2 Nonlinear Observability**

The observability of the system (2.3) is defined by the concept of indistinguishability. Let \( \psi_n(t, x_0) \) denote the solution of system (2.3) at time \( t \), with initial condition \( x_0 \) at time \( t_0 \) and control \( u(t) \). Besançon provided observability definitions for nonlinear systems in [1]. They are stated as follows.
**Definition 2.11 (Indistinguishability):**

A pair \((x_0, \dot{x}_0)\) will be said to be indistinguishable by \(u\), if \(\forall t \geq 0, h(y(t, x_0)) \equiv h(\dot{y}(t, x_0))\). The pair is just said to be indistinguishable, if it is so for any \(u\).

**Definition 2.12 (Observability):**

A nonlinear system (2.3) is observable if it does not have any pairs of indistinguishable states.

### 2.5 Nonlinear Control Schemes

Control theory for nonlinear dynamic systems has experienced a rapid development with many successful applications. In this section, the conventional nonlinear control techniques are briefly reviewed. The focus is on the output tracking of a nonlinear dynamic system described by a time-varying state equation of the form described in (2.3).

#### 2.5.1 Feedback Linearization

In a feedback linearization control (FLC) design, a suitable nonlinear coordinate transformation is applied on a nonlinear dynamic system to transform it to its linear counterpart and a nonlinear state feedback is used to cancel the nonlinearity in the transformed coordinates. The nonlinear coordinate transformation is achieved by solving a system of nonlinear partial differential equations. Then an LTI controller is designed for the transformed linear system to satisfy the desired performance parameters for the overall system. The whole state dynamics are linearized in an input-state linearization with an assumption that all the states are available for measurement. On the other hand, if full state measurement is not available, input-output linearization is developed. In input-output linearization the input-output map is linearized while the state equation is only partially linearized as illustrated in fig. 2.5. A command filter is required in the implementation of FLC to generate the command's derivatives from the given command. [3].
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Figure 2.5 Output Feedback Linearization Control Structure

One disadvantage of output linearization control is that the linearizing feedback may make some of the states unobservable. These unobservable internal dynamics of the input-output "linearized" system are referred to as zero dynamics of the nonlinear plant. Zero dynamics are required to be stable or bounded for FLC to be applicable. A system with stable zero dynamics is called a minimum-phase system. The requirement of minimum-phase prevents FLC being applicable to the non minimum-phase systems.

2.5.2 Dynamic Inversion

The dynamic inversion control (DIC) is similar to FLC except that in DIC, feedforward signal is generated from the command via the nonlinear dynamic inversion. Then an LTI controller is designed to handle the transient performance and robustness requirement of the system. The structure of DIC is illustrated in Figure 2.2. In the DIC, the dynamic inversion does not affect the closed-loop feedback controller stability, as it is an open-loop control. Unlike the FLC, the dynamic inversion control is able to control the non minimum-phase system also. The major challenge in DIC is to generate a stable inversion for a non minimum-phase system given the fact that the unstable system zeros are not affected by the feedback control law. One possible solution for this problem is to restrict the command trajectories in the range of the system generated command [3]. Another method is to redefine the system output using the concept of differential flatness, such that the redefined system is full-state linearizable by state feedback.
The feedback control in DIC is based on LTI techniques. However, the error dynamics between the actual system state trajectory and the desired state trajectory are still a nonlinear system due to modeling error, initial conditions and system uncertainty. The LTI feedback controller may not effectively handle the nonlinear time-varying error dynamics. It is known that the linearization of the error dynamics along the predefined trajectory yields a linear time varying (LTV) system.

### 2.5.3 Backstepping Control

Backstepping is a recursive control design procedure for the nonlinear system in a pure feedback form or an upper triangular form. In backstepping control the design problem of a high order system is broken into a sequence of design problems for lower-order systems, and hence making it capable of solving control problems under less restrictive conditions than those encountered in other methods [3]. The matching condition or extended matching condition assumption can also be relaxed using backstepping. Many backstepping methods are proved to be asymptotically stable, which is not as practical as exponential stability in engineering applications, as mere asymptotical stability does not guarantee (finite gain) BIBO stability. Moreover, the requirement of pure feedback form of the plant is in itself a rather severe restriction.
2.5.4 Sliding Mode Control and Variable Structure Control

Sliding mode control (SMC) is a robust nonlinear control algorithm which employs a discontinuous control input to enforce the system state trajectory over a prescribed sliding surface designed to achieve the desired control objectives. SMC has been widely used for its robustness to model parameter uncertainties and external disturbances. It also presents a major drawback caused by the finite bandwidth of real control actuators. The non-ideal behavior of the system and the actuator can produce the chattering phenomenon, which is a high frequency motion that makes the state trajectories rapidly oscillate about the sliding surface. Chattering and high control energy consumption, are the main limitations of SMC. Most SMC applications use a straightforward approach to avoid chattering: approximating the signum function of the discontinuous control by the continuous saturation function. As a result, the system motion is confined within a boundary layer of the sliding manifold.

The above mentioned control schemes relies on the nonlinearity cancellation in the transformed coordinate via nonlinear state feedback. In an actual control system, the cancellation of nonlinear terms will not be exact due to modeling errors, uncertainties, measurement noise and lag, and the existence of immeasurable states. The LTI feedback controller designed under the nominal conditions may not effectively handle the residual nonlinear time-varying error dynamics. In order to have an effective control solution, these classical control schemes are to be augmented with some adaptive components which tunes themselves online as per the system characteristics and compensate the system uncertainties and modeling inaccuracies, thereby leading to adaptive control strategies for nonlinear control systems. Incorporation of the adaptive components offers a very efficient solution for the nonlinear systems in global or semi global regime [1-3]. Adaptive tools like Neural Network (NN) are having universal approximation capabilities; they can effectively approximate the nonlinear functions over a specified set of interest. Theoretical development and the consequent applications of these tools in control applications have reflected the potential of NN as function approximators. Designing of NN based adaptive control schemes to offer an effective solution for a wide class of uncertain nonlinear systems is an active area of research.
2.6 Adaptive Control

Most of the complex industrial processes are highly prone to parasitic variations as it is neither possible nor economical to make a thorough investigation of the dynamics of the process. Adaptive controllers can be a good alternative in such cases. An adaptive controller is a controller with adjustable parameters and a mechanism for adjusting those parameters. Adaptive control system, in general, has two loops; one loop is a normal feedback loop with the process and the controller and the other is the parameter adjustment loop as shown in fig 2.1. It is a control technique dealing with partially known systems [8].

![Figure 2.7 Block Diagram of Adaptive Control System](image)

Adaptive control can be classified in two categories, linear and nonlinear, depending upon the type of system, as discussed below.

2.6.1 Linear Adaptive Control

Adaptive control for linear systems with unknown parameters was the most active area of research in the field of control engineering from last two decades [8]. Adaptive linear control techniques can further be classified as “indirect” or “direct” adaptive control. In the indirect adaptive control, the plant parameters are estimated on-line and the controller is adjusted according to the parameters of the identified plant model. In the direct adaptive control, there is no explicit plant parameter identification. Controller parameters are adjusted directly to improve the performance index.
Adaptive control can also be classified as model-reference adaptive control (MRAC) and self-tuning regulation (STR). In an MRAC controller, a reference model based on a priori information of the plant is chosen to satisfy the performance requirement, then an adaptive control law, direct or indirect, is designed such that the plant output can follow the reference model response. On the other hand, STR is developed from stochastic minimum variance control. Similar to MRAC, an explicit STR consists of an explicit plant dynamic estimation and a controller parameter regulator; whereas, an implicit STR consists of a direct update law of controller parameters with an implicit system estimation. The stability analysis of a linear adaptive control system is accomplished using the second method of Lyapunov [8]. The stability analysis usually treats the estimated parameters as new state variables, and is able to guarantee the stability of both the plant dynamics and parameter adaptation dynamics. The early results are based on the assumption that there is no modeling error besides parametric uncertainty. However, in practical applications, it is found that even very small modeling errors could destabilize the adaptive control system. The stability of linear adaptive control also requires the persistent excitation assumption to guarantee the convergence of the adaptive parameters, which is rather restrictive in application.

### 2.6.2 Nonlinear Adaptive Control

Nonlinear adaptive control, as an extension of the linear adaptive control and nonlinear control with a known dynamic model, gathered much interest in the last decade [9-12]. Nonlinear adaptive control is more challenging than its linear counterpart. One reason is that nonlinear systems, different from linear systems, do not have a unified representation. Nonlinear adaptive control based on feedback linearization [8] is proposed for a family of linearizable systems. Similar to FLC based on a known dynamic model, this method can only be applied to minimum-phase linearizable systems. The computational complexity increased rapidly with the system relative degree. Adaptive FLC has achieved great progress since late 1990’s [8-12]. In adaptive FLC, the uncertainty of the system is assumed with linear parameters, and an adaptive method is used to approximate the unknown parameters. In the early studies, the system is required to satisfy a matching condition, and there is an over-parameterization problem. These restrictions normally offer a solution which is confined over a small local set. The region of stability can be...
enhanced by using universal approximation tools like NN and WNN, which also relaxes the linear in parameters constraints of the uncertain nonlinearities, and thus offer a control solution for a wide class of nonlinear systems over an extended set.

### 2.7 Nonlinear Observers

In most practical cases, the physical state of the system cannot be determined by direct observation. Instead, indirect effects of the internal state are observed by way of the system outputs. In control theory, a state observer is a system that models a real system in order to provide an estimate of its internal state, given measurements of the input and output of the real system. It is typically a computer-implemented mathematical model.

**Definition 2.13 (Observer):**

For the nonlinear system given in (2.3), if we can find \( \phi(.) \) and \( \rho(.) \) such that

\[
\begin{align*}
\dot{\eta} &= \phi(\eta, y) \\
\dot{x} &= \rho(\eta, y)
\end{align*}
\]

(2.14)

where \( \eta \in \mathbb{R}^p \) and \( x \in \mathbb{R}^n \), with the properties

\[
\lim_{t \to x} \left[ x(t, x_0) - x(t, \eta_0, x_0) \right] = 0 \quad \forall \ x_0 \in \mathbb{R}^n \ \text{and} \ \eta_0 \in \mathbb{R}^p
\]

(2.15)

Then we say the dynamic (2.14) is a global asymptotic observer of the original system (2.3). If (2.15) holds only for some neighborhood, \( x_0 \in B_r(0) \subseteq \mathbb{R}^n \) and \( \eta_0 \in B_r(0) \subseteq \mathbb{R}^p \) with \( B_r(\xi) \) denoting a ball with radius \( r \) centered at \( \xi \), then we say that the dynamic (2.14) is a local asymptotic observer of system (2.15)[1,2].

### 2.7.1 Background

In many practical applications, all the state variables are either not available or uneconomical for direct measurement due the feasibility of sensors. This makes the high performance estimation of unmeasured state variables very necessary, especially in the model-based output feedback controllers. In order to reconstruct the unmeasurable state variables, state observers are usually employed. Kalman filter and Luenberger observer theory offer a satisfactory solution to the estimation problem in case of linear systems. But for nonlinear
systems, the nonlinear observers are employed which is more complex than their linear counterpart. The output-feedback control design for nonlinear systems has further increased the importance of developing effective observers for nonlinear systems with full or partial states unmeasurable. There have been many results cited in the literature on observer design for nonlinear systems [24-27]. Most of the nonlinear controller design techniques have been applied to observer design. Using the concept of FLC, Global linearization observers and pseudo-linearization observers are proposed in which the diffeomorphism and the correctly chosen observer gain are used to cancel the nonlinearity by converting the nonlinear system into nonlinear observer canonical form. But the exact knowledge of the system is needed for the satisfactory performance of the system by applying these two methods. Extended linearization observers [25] are also proposed observers using gain scheduling. In these observers, a family of linear systems is derived by linearizing the nonlinear systems about the equilibrium point and then the linear time invariant (LTI) methods are employed. Thus, the same slow-time-varying constraint for gain scheduling controller design applies to this extended linearization method. However the observer design problem for nonlinear system becomes more complex where the dynamics of the systems are prone to the modeling uncertainties. In such cases adaptive observers has to be implemented.

2.7.2 **Adaptive Observers**

When the system further depends on some unknown parameters, the observer design has to be modified so that both state variables and parameters can be estimated, leading to so-called adaptive observers. Design of state observers is based on mathematical model of the plant and it is usually presumed that the available model information is precise. However, in reality, the model cannot be accurate due to nonlinearities, model and parameter uncertainties and disturbances. To overcome this problem adaptive observers were proposed by several researchers [26,27]. The appearance of adaptive observers is natural due to the vigorous development of observer design and adaptive control for nonlinear systems. An observer-based adaptive control system performs tri-tasks of parameter identification, state estimation and state regulation. All existing adaptive control techniques can be applied to observer-based adaptive control design. However, the stability and convergence of the overall system need to be reevaluated due to the
additional dynamics and the extra feedback loop caused by the adaptive observer. There are also several other methods that were initially and particularly developed for nonlinear observer design, such as nonlinear Kalman filters [3]. The Kalman filter is an optimal filter for stochastic processes and state and measurement noise models that allows the estimation of the states based on a linear state space model. It has been available as a powerful engineering tool for more than four decades. The nonlinear Kalman filters use local linearization along the nominal values to extend the scope of the Kalman filter to systems described by nonlinear differential equations. The nominal trajectory can be defined online as the current best estimate of the actual trajectory. This approach is called extended Kalman filter (EKF). The Kalman gain is obtained by solving the Riccati equation based on the knowledge of state noises and measurement noises. Theoretically, Kalman filter provides the optimal filtering strategy under the assumed noise covariance matrix. Practically, however, it is difficult to obtain precise noises models, especially the state noise models. Even if the full information of the noise is available, the computational load and storage requirements associated with the propagation of the error covariance matrix would become a burden to the real time system. For some applications, the nominal values of the state variables are predetermined, such as guidance and control problems for which staying close to the pre-calculated trajectory is the key to good performance. For these applications, the Kalman gains can be computed beforehand to relieve the real-time computational burden. This approach is referred to as linearized Kalman filter [3]. The major disadvantage of linearized Kalman filter is that the perturbations from any predefined nominal trajectory are generally larger than the perturbations that include only the state estimation errors, which is the case for EKF, and therefore more difficult conditioned for linearization. Stability is another essential problem that hampers the application of nonlinear Kalman filter. From the classic works [29,30] and the references therein, the stability is not guaranteed, the performance bounds cannot be easily computed and the convergence rates are not known in general. Moreover, when large discrepancy occurs in the initial values, in the presence of relevant unmodelled dynamics or in the presence of disturbance, stability is often difficult to be fortified.
The publications regarding the controller designing aspects for the class of systems dealt in this thesis are:


