CHAPTER 3

CHANNEL ESTIMATION FOR MIMO AMPLIFY AND FORWARD WIRELESS RELAY NETWORKS

3.1 PREAMBLE

In this section channel estimation for Multiple Input Multiple Output (MIMO) Amplify and Forward Wireless Relay Network (AFWRN) using BLUE algorithm is proposed. As MIMO technology combats multipath fading, (Xiaoyan Xu et al 2012) when incorporated with AFWRN it significantly improves communication system performance. The reason behind is that, the relay nodes with amplify and forward relaying strategy have the tendency to improve wireless coverage for users due to its linear processing capability even when they posses limited power resources and spectral resources (Ting Kong and Yingbo Hua 2011). But addition of MIMO technology to such relay nodes provides additional power requirements in comparison to the power and spectral savings provided by relay nodes itself since it exploits spatial diversity of multiple antennas. However, to obtain the benefits namely additional power and spectral savings (Ting Kong and Yingbo Hua 2011) in MIMO AFWRN, channel state information or MIMO channel matrices knowledge are required at the destination nodes. Hence, channel estimation attains importance in MIMO AFWRN systems which ultimately makes it beneficial for coherent data detection.
3.2 EXISTING METHODS FOR MIMO AFWRN

Existing methods available for estimating channel coefficients in multiple input multiple output amplify and forward wireless relay networks are mainly based on Pilot symbol based channel estimation techniques. In pilot symbol based channel estimation techniques a pilot signal or training sequence is employed to estimate channel coefficients in a relay network from overall channel coefficients (Panagiota Lioliou and MatsViberg 2008). In other existing works proposed, many pilot symbol based algorithms have been reported (Jun Ma et al 2009, Ting Kong and Yingbo Hua 2010, Jiyong Pang et al 2010, Yue Rong and Muhammed R.A.Khandekar 2011, Jun Ma et al 2011, Sun Sun and Yindi Jing 2011, Ting Kong and Yingbo Hua 2011 Panagiota Lioliou et al 2011, Nam Tran Nguyen et al 2011, Yue Rong et al 2012, Panagiota Lioliou et al 2012, Raphel and Sameer 2012, Choo W.R. Chiong et al 2012, Xinwei Yu and Yindi Jing 2012, and Xiaoyan Xu et al 2012).

Panagiota Lioliou and MatsViberg (2008) proposed a MIMO relay channel estimation algorithm using Least Squares (LS). In this work, equal number of training blocks and relay nodes are employed to estimate individual channels using Singular Value Decomposition (SVD) of a cascaded channel. However, this sort of estimation is not efficient as it is well known that least squares only reduces signal error rather than channel estimation error as it does not require knowledge of channel parameters.

As the work of Panagiota Lioliou and MatsViberg 2008 proposed least squares approach which has its own drawback, Jun Ma et al (2009) developed a novel interim channel estimation approach where the relay node is not aware of the structure of the received signal and not capable of performing complicated signal processing tasks. Also, in this work necessary and sufficient conditions for pilot amplifying matrix sequence at the relay
node are derived to ensure successful interim channel estimation at the destination node using linear minimum mean square error algorithm. This work, employed low complexity pilot amplifying matrices to satisfy these conditions. However, the drawback of this estimation scheme is a scalar ambiguity on the estimates of individual MIMO channel matrices between the source node to relay node and relay node and destination node.

Ting Kong and Yingbo Hua (2010) proposed a channel estimation technique for MIMO amplify and forward wireless network by using LMMSE algorithm. This work suggested usage of fast algorithms for computing optimal training matrices at the source node and the relay node. These algorithms were developed using convex optimization and majorization theory. Moreover, channel matrices are correlated and are modeled as Kronecker product of transmit and receive correlations.

Jiyong Pang et al (2010) studied channel estimation for time division Amplify and Forward (AF) MIMO relay channels in spatial fading correlation environment. Initially, this work derived an LMMSE estimator with no correlations among relay antennas and then extended it to multiple relay antennas which are spatially correlated. The impact of spatial correlations, relay power gain and number of relay antennas on mean square error performance is analyzed. This work infers that channel estimation and optimal training sequences are not concerned with spatial correlation at the destination node and the derived estimator and designed sequences are directly extended to multihop relay channels.

Yue Rong and Muhammed R.A. Khandekar (2011) developed a novel channel estimation algorithm for two-hop MIMO relay systems using the parallel factor analysis. The destination node is assumed to know the full knowledge of all channel matrices involved in communication. This algorithm requires only two training blocks and compared with the existing approaches
it requires only lesser number of training data blocks for one way relay networks.

As an extension to the work of Jun Ma et al 2009, Jun Ma et al (2011) investigated the estimation of two cascaded relay channels at the destination node based on a predefined amplifying matrix at the relay node. The corresponding overall channel is obtained through the conventional channel estimation algorithms with the help of pilots transmitted by the source node. This work also suggests necessary and sufficient conditions on the pilot amplifying matrix sequence at the relay node to ensure feasible relay channel estimation at the destination node. The pilot amplifying matrices are diagonal or quasi diagonal and facilitate channel estimation with minimum complexity at the relay node. Simulation results are obtained using linear least squares estimator for the relay channels with pilot matrix.

Sun Sun and Yindi Jing (2011) suggested a work on channel training design for distributed space time coding in multi antenna relay networks. LMMSE estimator is employed at receiver and optimal pilot design which minimizes estimation error is also derived. In addition to this, the minimum training time requirement which leads to full diversity in data transmission is proposed. Moreover lower bound, upper bound and an adaptive training design where the training time is adaptive to the quality of relay- receiver channels is also investigated.

Ting Kong and Yingbo Hua (2011) focused on two-hop nonregenerative MIMO system channel estimation schemes with no direct line of sight component between the source node and the destination node. The channel estimation scheme required at the relay node to generate a source pilot in the first phase, is slightly more complex than the other channel estimation schemes. The main advantage of this work is that the new scheme is not subject to any scalar ambiguity unlike that of Panagiota Lioliou and
MatsViberg 2008 and Jun Ma et al 2009. LMMSE scheme is considered for estimation of channel matrices which also allows the use of prior knowledge of channel correlations. This approach of channel estimation is useful in practical scenarios when a source node and a destination node are blocked from each other by a large building and a relay is added in between them around the building.

Panagiota Lioliou et al (2011) proposed a novel LMMSE and Expectation-Maximization (EM) based Maximum APriori (MAP) channel estimation algorithm for MIMO AFWRN. Simulation results demonstrate that incorporation of prior knowledge into the channel estimation algorithm offers significant improvement especially in lower signal to noise ratio regimes.

Nam Tran Nguyen et al (2011) proposed an optimal training design for an AFWRN system comprising of multiple antennas with spatial correlation. The training design problem is formulated as a convex optimization problem which can be efficiently solved with the iterative bisection procedure. Simulation results demonstrate the performance advantage of the proposed training design for the LMMSE channel vector estimation over the training design for the LMMSE channel matrix estimation.

Yue Rong et al (2012) proposed an extension to the previous work of Yue Rong and Muhammed R.A.Khandekar (2011) by performing channel estimation for MIMO AFWRN in correlated MIMO channels.

Panagiota Lioliou et al (2012) proposed an extension to their own work presented in 2011. This work also addresses the open problem of deriving analytical expressions for the Bayesian Cramer-Rao bound (BCRB). A major difficulty in calculating the BCRB is the computation of the expectation of the Fisher Information Matrix (FIM) with respect to unknown
random parameters. Expectation involves multi-dimension integration over the parameters. With these difficulties BCRB asymptotes at low and high SNRs are presented.

Raphel and Sameer (2012) proposed channel estimation technique for estimating interim relay channels at Destination Node (DN) without introducing complexity at Relay Node (RN) for generic two hop AF cooperative relay system into a MIMO MiMicking (MM) AF system. This work employs predefined Pilot Enhancement Matrix (PEM) at the RN and the corresponding overall channel estimation is performed at the DN with the help of pilots broadcast from the Source Node (SN). Linear least square estimation technique is used for channel estimation. Simulation results for the proposed technique results in better BER performance without compromise in the transmission rate.

Choo W.R. Chiong et al (2012), investigated the impact of CSI mismatch on the performance of channel estimation algorithm. In practical relay systems there exists a mismatch between the estimated and true relay destination channel. CSI mismatch affects the accuracy of the source-relay channel estimation. By explicitly taking into account the CSI mismatch, a robust algorithm to estimate the source relay channel is proposed.

Xinwei Yu and Yindi Jing (2012) proposed a channel estimation approach by taking into account the special structure of the channel matrix. By using singular value decomposition, the channel matrix is parameterized by its largest singular value left and right singular vectors. Estimations of the largest singular value and left and right singular vectors are first derived based on the maximum likelihood criterion. Then it is used to construct an estimation of the channel matrix. In comparison to the entry based estimation, it ignores the special structure of the end-to-end channel matrix and estimates
each channel entry significantly. The proposed estimation shown through simulation is superior in mean square error performance in comparison to other performances.

Xiaoyan Xu et al (2012) introduced superimposed training strategy for MIMO AFWRN systems to perform channel estimation at the destination node to overcome the critical drawback of Ting Kong and Yingbo Hua 2011 as it is based on three phase training strategy. This three phase training strategy is incompatible with two phase data transmission. Hence, in this work Xiaoyan Xu et al 2012 proposed a two phase training strategy along with superimposition of its own training signal over the received signal before forwarding it, to provide separate channel estimation. From the extracted individual channel information, the actions of the relay for data transmission, such as power allocation and carrier permutation are performed.

From the existing methods suggested from Panagiota Lioliou and MatsViberg (2008) to Xiaoyan Xu et al (2012), all of them employed traditional channel estimation algorithms namely least squares and linear minimum mean square error estimation algorithms which provided significant estimates of channels in terms of Normalized Mean Square Error (NMSE). However, those traditional channel estimation algorithms does not provide minimal variance attributes. As a result, it explores the possibility of developing estimators which can provide improvement in NMSE performance with minimal variance attributes. Among the family of unbiased estimators Best Linear Unbiased Estimator (BLUE) algorithm, posses the minimal variance characteristics. Hence, BLUE algorithm for MIMO AFWRN is developed.
3.3 PROBLEM FORMULATION

Although many existing methods have been reported for channel estimation for MIMO amplify and forward wireless relay network, all those methods are based on existing approaches like least squares and linear minimum mean square error. However these estimators do not result in minimal variance. Further, LS reduces signal error rather than estimation error (Mehrzad Biguesh and Alex B. Gershman 2004), while MMSE is based on the conditional Probability Density Function (PDF) (Steven M. Kay 1993) and second order channel statistics. As mentioned in chapter 2, the complexity of MMSE increases as the number of samples grows exponentially. In order to overcome the computational complexities experienced by LS and MMSE estimators, the concept of BLUE is extended for channel estimation in MIMO amplify and forward wireless relay networks.

3.4 CHANNEL ESTIMATION FOR MIMO AMPLIFY AND FORWARD WIRELESS RELAY NETWORK

3.4.1 System Model for MIMO AFWRN

Consider a AFWRN system with a single source node $S$, equipped with $L$ antennas, a single destination node $D$ equipped with $L$ antennas and $M$ randomly placed relays $R_i; i = 1, 2, ..., M$ equipped with $N_{rel}$ antennas as shown in Figure 3.1. The MIMO channel matrix between the source node $S$ to the $i^{th}$ relay node $R_i$ is represented by $N_{rel} \times L$ matrix $G_i$ and the channel from the $i^{th}$ relay node to the destination node $D$ by $H_i L \times N_{rel}$ matrix. It is assumed that the elements of channel matrices are Independent Identically Distributed (IID) circularly symmetric complex Gaussian random variables with zero mean and unit variance.
If the source node $S$, intends to transmit a training data sequence $z$ through the amplify and forward relays, to the destination node $D$, the transmission is accomplished in two phases, namely phase I and phase II. In phase I, the source node $S$ transmits the training data sequence $z$ of size $L \times 1$ to all the relays $R_i; i = 1, 2, \ldots, M$ with $N_{rel}$ antennas. It is assumed that the training data sequence $z$ satisfies the power constraint $\mathbb{E}[z^H z] = \frac{P_S}{L}$, where $P_S$ is the source node power.

In this thesis, training data sequence is chosen to be $z = \sqrt{P_S} \mathbf{1}_L$ since it releases peak to average power ratio problem at the source node $S$ and it satisfies the power constraint. $\mathbf{1}_L$ is an $L \times 1$ vector with all its elements are unity. In phase I, the $N_{rel} \times 1$ received signal vector at the relays $R_i; i = 1, 2, \ldots, M$ is expressed as

$$r_i = G_i z + n_i, \quad 1 \leq i \leq M$$  \hspace{1cm} (3.1)
where $\mathbf{G}_i$ is the $N_{rel} \times L$ channel matrix, $\mathbf{z}$ is the $L \times 1$ training data sequence and $\mathbf{n}_d$ is the $N_{rel} \times 1$ complex Gaussian noise vector at the relay with zero mean and covariance matrix of $\sigma_n^2 \mathbf{I}_{N_{rel}}$. In phase II, each MIMO relay processes its $N_{rel} \times 1$ received signal vector $\mathbf{r}_i$ to result in the $N_{rel} \times 1$ vector $\mathbf{t}_i$ which is then forwarded to the destination node. The $N_{rel} \times 1$ signal vector transmitted from the $i^{th}$ relay is represented as

$$\mathbf{t}_i = \xi \mathbf{A}_i \mathbf{r}_i \quad 1 \leq i \leq M \tag{3.2}$$

where $\xi$ is the real power scaling factor (or) power limit factor which is independent of $i$ (Youhua fu et al 2008). It ensures that the transmit power of relay terminals is not more than the relay node power $P_r$. The scaling factor is defined as $\xi = \sqrt{\frac{P_r}{\max_i \text{trace} \left( (P_s / L) \mathbf{G}_i^H \mathbf{G}_i + \sigma_n^2 \mathbf{I}_{N} \right) \mathbf{A}_i}}$. $\mathbf{A}_i$ represents the $N_{rel} \times N_{rel}$ unitary precoding matrix. The $N_{rel} \times 1$ forwarded signal vector from the relays satisfies the average power constraint $E\{\mathbf{t}_i^H \mathbf{t}_i\} \leq P_r$. The transmitted signal vector from each relay is received by the destination node $D$ and the $L \times 1$ received signal vector is given as

$$\mathbf{y} = \sum_{i=1}^{M} \mathbf{H}_i \mathbf{t}_i + \mathbf{n}_d \tag{3.3}$$

where $\mathbf{n}_d$ is an $L \times 1$ additive circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_{N_d}$. Substituting Equation (3.2) the $L \times 1$ received signal vector is expanded as

$$\mathbf{y} = \left( \sum_{i=1}^{M} \mathbf{H}_i \xi \mathbf{A}_i \mathbf{G}_i \right) + \sum_{i=1}^{M} \mathbf{H}_i \xi \mathbf{A}_i \mathbf{n}_d + \mathbf{n}_d \tag{3.4}$$
Let $\hat{H}$ be the $L \times N_{rel}M$ matrix defined as $\hat{H} = [H_1^T \ H_2^T \ \ldots \ H_{N_{rel}}^T]^T$ is an matrix, $\hat{A}$ be the $N_{rel}M \times N_{rel}M$ matrix defined as $\hat{A} = \text{diag}(A_1 \ A_2 \ \ldots \ A_{N_{rel}})$, $\hat{G}$ be a $N_{rel}M \times L$ matrix defined as $\hat{G} = [G_1 \ G_2 \ \ldots \ G_{N_{rel}}]^T$, $\hat{n}_r$ be the $N_{rel}M \times 1$ noise vector at the relays defined as $\hat{n}_r = [n_{r1}^T \ n_{r2}^T \ \ldots \ n_{rN_{rel}}^T]^T$ and $\hat{n}_d$ is the $N_{rel}M \times 1$ noise vector at the destination node. Now the received signal vector is written as

$$y = \xi \hat{H} \hat{A} \hat{G}z + \xi \hat{H} \hat{A} \hat{n}_r + \hat{n}_d$$

(3.5)

The overall $L \times L$ channel matrix between the source node $S$ and the destination node $D$ is defined as $W = \hat{H} \hat{A} \hat{G}$ and $n_d = \xi \hat{H} \hat{A} \hat{n}_r + \hat{n}_d$ is the $L \times 1$ overall noise vector at the destination node $D$. Using these definitions, the $L \times 1$ received signal vector at the destination node $D$ is given as

$$y = \xi Wz + n_d$$

(3.6)

### 3.4.2 Proposed Channel Estimation Algorithm

In this section, the estimation of overall MIMO channel matrix $W$ between the source node $S$ and the destination node $D$ in MIMO AFWRN systems is carried out. The proposed channel estimation algorithm for determining the $L \times L$ overall channel matrix $W$ is considered as the linear combination of multiple Least Squares (LS) estimate (Mehrzad Biguesh and Alex.B.Gershman 2006) of the channel. The number of linear estimates $J$ should be equal to the length of the training sequence $L$ as the objective function to obtain the unknown weight coefficients $\nu_j$ is formulated based on minimizing the mean square error. It is given by
\[ \hat{W}_{\text{BLUE}} = \sum_{j=1}^{J} v_j W_{LS} \]  (3.7)

where \( v_j \) represents the weight coefficients (Mehrzad Biguesh and Alex.B.Gershman 2004) and \( W_{LS} \) represents the \( j^{th} \) least squares estimate of \( W \) and it is represented as \( W_{LS} = y_j \zeta_j^{-1} \), \( 1 \leq j \leq J \) where \( \zeta_j \) represents the pseudo inverse of the training data sequence and \( \zeta_j^{-1} \) represents the inverse of the real scaling factor. The weight coefficients \( v_j;1 \leq j \leq J \) are determined by solving the constraint optimization problem stated as

\[
\min_{v_1, \ldots, v_J} E \left\{ \left\| W - \sum_{j=1}^{J} v_j \hat{W}_{LS} \right\|_F^2 \right\}
\]

subject to \( \sum_{j=1}^{J} v_j = 1 \)  (3.8)

Now, substituting the LS estimates \( W_{LS} = y_j \zeta_j^{-1} \), \( 1 \leq j \leq J \) in Equation (3.8) the objective function is written as

\[
\min_{v_1, \ldots, v_J} E \left\{ \left\| y \zeta^{-1} - n_d \zeta^{-1} - \sum_{j=1}^{J} v_j y_j \zeta_j \right\|_F^2 \right\}
\]  (3.9)

Using the identity \( \|D\|_F^2 = tr(D^H D)^{1/2} \) (Mehrzad Biguesh and Alex B. Gershman 2006) the objective function is rewritten as

\[
\min_{v_1, \ldots, v_J} E \left\{ tr \left( \sum_{m=1}^{J} v_m \zeta_m n_m z_m^H \right)^H \left( \sum_{p=1}^{J} v_p \zeta_p n_p z_p^H \right) \right\}
\]  (3.10)
Since the autocorrelation matrix of overall noise vector is 
\[ E[n, n^H] = \sigma_n^2 L \], the objective function is simplified as

\[
\min_{v_1, \ldots, v_J} \sigma_n^2 L \text{Tr} \left\{ \sum_{j=1}^{J} v_j^2 \left( \frac{z_j^2}{s_j} \right)^{-1} \left( z_j z_j^H \right)^{-1} \right\}
\]

(3.11)

Using Equation (3.11), the optimization problem in Equation (3.8) is rewritten as

\[
\min_{v_1, \ldots, v_J} \sigma_n^2 L \text{Tr} \left\{ \sum_{j=1}^{J} v_j^2 \left( \frac{z_j^2}{s_j} \right)^{-1} \left( z_j z_j^H \right)^{-1} \right\}
\]

subject to \( \sum_{j=1}^{J} v_j = 1 \)

(3.12)

The derived optimization problem is solved using Lagrangian method by minimizing the cost function

\[
Q = \min_{v_1, \ldots, v_J} \sigma_n^2 L \text{Tr} \left\{ \sum_{j=1}^{J} v_j^2 \left( \frac{z_j^2}{s_j} \right)^{-1} \left( z_j z_j^H \right)^{-1} \right\} + \lambda \left( \sum_{j=1}^{J} v_j - 1 \right)
\]

(3.13)

where \( \lambda \) is Lagrangian multiplier.

The gradient of Equation (3.13) with respect to \( v_j \) is determined as

\[
2\sigma_n^2 L \text{Tr} \left\{ \sum_{j=1}^{J} v_j \left( \frac{z_j^2}{s_j} \right)^{-1} \left( z_j z_j^H \right)^{-1} \right\} + \lambda = 0, \ 1 \leq j \leq J.
\]

By equating the gradient to zero, \( v_j ; 1 \leq j \leq J \) is determined as

\[
\sum_{j=1}^{J} v_j \left( \frac{z_j^2}{s_j} \right)^{-1} \text{tr} \left( z_j z_j^H \right)^{-1} = \frac{-\lambda}{2\sigma_n^2 L}, \ 1 \leq j \leq J
\]

(3.14)
Let \( \mathbf{v} = [v_1, v_2, \ldots, v_J]^T \) is the \( J \times 1 \) vector of weight coefficients and 
\( \mathbf{p} = [p_1, p_2, \ldots, p_J]^T \) is a \( J \times 1 \) vector and it is given as 
\[ p_j = (\xi_j^2)^{1/2} \text{tr}(\mathbf{z}_j \mathbf{z}_j^H) \].

Now, Equation (3.14) can be written in vector form as

\[
-\frac{\lambda}{2\sigma_n^2 L} = \mathbf{v}^T \mathbf{p} \tag{3.15}
\]

Solving for \( \mathbf{v} \), it is written as,

\[
\mathbf{v} = \frac{-\lambda}{2\sigma_n^2 L} (\mathbf{p}^T)^T \tag{3.16}
\]

Similarly the constraint in Equation (3.12) is written as

\[
\mathbf{v}^T \mathbf{1}_J = 1 \tag{3.17}
\]

where \( \mathbf{1}_J = [1, 1, \ldots, 1]^T \) is a \( J \times 1 \) vector of all ones. The Lagrangian parameter \( \lambda \) is determined by substituting Equation (3.16) in Equation (3.17). It is given as

\[
\lambda = -2\sigma_n^2 L (\mathbf{p} \mathbf{1}_J)^{-1} \tag{3.18}
\]

Further, substituting the Lagrangian parameter of Equation (3.18) in Equation (3.16), the weight coefficients in vector form is given by

\[
\mathbf{v} = (\mathbf{p}^T) (\mathbf{p} \mathbf{1}_J)^{-1} \tag{3.19}
\]

Now in Equation (3.19) using suitable substitutions, the weight coefficients \( v_j \) are represented as

\[
v_j = \left[ (\xi_j^2)^{-1} \text{tr}(\mathbf{z}_j \mathbf{z}_j^H)^{-1} \sum_{p=1}^{J} 1/\text{tr} \left( (\xi_j^2)^{-1} (\mathbf{z}_p \mathbf{z}_p^H)^{-1} \right) \right]^{-1} \tag{3.20}
\]
By substituting the expression of \( v_j \) and \( W_{LS} = yz^{-1} \) in Equation (3.7) the proposed channel estimation algorithm for MIMO AFWRN using BLUE is given by

\[
\hat{W}_{BLUE} = \sum_{j=1}^{J} \left( \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{J} 1/\xi_j^2 tr(z_pz_p^H) \right) y_j z_j^{-1} \tag{3.21}
\]

The performance of the proposed channel estimation algorithm for MIMO AFWRN system is analyzed in terms of MSE as in Chapter 2. The MSE of the proposed algorithm is defined as

\[
MSE_{BLUE}^{MIMO} = tr \left\{ E \left[ (\hat{W}_{BLUE} - W)(\hat{W}_{BLUE} - W)^H \right] \right\} \tag{3.22}
\]

Substituting Equation (3.6) in Equation (3.21) and applying simple mathematical steps, MSE of the proposed estimator is determined as

\[
\hat{W}_{BLUE} = \sum_{j=1}^{J} \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{J} 1/\xi_j^2 tr(z_pz_p^H) \right) \xi_j Wzz_j^\dagger + \\
\sum_{j=1}^{J} \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{J} 1/\xi_j^2 tr(z_pz_p^H) \right) n_d z_j^\dagger \xi_j^\dagger \tag{3.23}
\]

In Equation (3.23), the first term simplifies to the \( L \times L \) matrix \( W \) and the second term which comprises of destination noise vector is the \( L \times L \) error matrix \( E \) which is given as

\[
E = \sum_{j=1}^{J} \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{J} 1/\xi_j^2 tr(z_pz_p^H) \right) \xi_j z_j^\dagger \xi_j^\dagger \tag{3.24}
\]

Using Equation (3.24), Equation (3.23) is simply defined as

\[
E = \hat{W}_{BLUE} - W \tag{3.25}
\]
Substituting Equation (3.25) in Equation (3.22), $MSE^{\text{MIMO}}_{\text{BLUE}}$ is defined as

$$MSE^{\text{MIMO}}_{\text{BLUE}} = tr \left\{ E \left\{ EE^H \right\} \right\}$$

(3.26)

By substituting Equation (3.24), MSE of the proposed estimator is written as

$$MSE^{\text{MIMO}}_{\text{BLUE}} = tr \left\{ E \left\{ \left( \sum_{j=1}^{J} \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{P} 1/ \xi_j^2 tr\left( \left(z_pz_p^H \right)^{-1} \right) \right)^{-1} n_jz_j \xi_j^H \right\} \right\}$$

(3.27)

As the correlation matrix of the overall noise vector $E(\mathbf{n}_j\mathbf{n}_j^H) = \sigma_n^2 \mathbf{L}_I$, and $E[\mathbf{z}^H \mathbf{z}] = \frac{P}{L}$, Equation (3.27) simplifies to

$$MSE^{\text{MIMO}}_{\text{BLUE}} = tr \left\{ E \left\{ \left( \sum_{j=1}^{J} \xi_j^2 \left( tr(z_jz_j^H) \right)^{-1} \sum_{p=1}^{P} 1/ \xi_j^2 tr\left( \left(z_pz_p^H \right)^{-1} \right) \right)^{-1} \sigma_n^2 \mathbf{L}_I \frac{P}{L} \xi_j^{-1} (\xi_j^{-1})^H \right\}$$

(3.28)

Using Equation (3.20) in Equation (3.28) the expression for $MSE^{\text{MIMO}}_{\text{BLUE}}$ of the proposed algorithm is derived as

$$MSE^{\text{MIMO}}_{\text{BLUE}} = tr \left\{ \sigma_n^2 \mathbf{I} \sum_{j=1}^{J} v_j P_{\xi_j} \frac{1}{\xi_j} (\xi_j^{-1})^H \sum_{j=1}^{J} v_j \right\}$$

(3.29)
3.5 RESULTS AND DISCUSSIONS

In this section, Normalized Mean Square Error (NMSE) performance of the proposed channel estimation algorithm is analyzed. Theoretical MSE are analyzed using training sequence and precoding matrix (Tao Cui et al. 2007 and Feifei Gao et al. 2008) with channels $G_i$, $H_i$ and noise assumed as zero mean complex Gaussian random variables with unit variances. The Signal to Noise Ratio is defined as $SNR = (P_s \times 1) / N_0 = P_s$, where $P_s$ is the source node power. In all the simulations, 1000 independent Monte-Carlo runs are used.

![Normalized MSE performance of Channel Estimation Algorithms for MIMO AFWRN with $N_{rel} = 2$](image)

**Figure 3.2** NMSE performance of Channel Estimation Algorithms for MIMO AFWRN with $N_{rel} = 2$

Figure 3.2, shows the normalized MSE performance of proposed algorithm with two relay nodes $M=2$, length of the optimal training sequence $L=2$, the number of source and destination antennas $L=2$, and number of antennas in the relay nodes $N_{rel}=2$ and $J=2$ represents the number of linear estimates to obtain the proposed BLUE algorithm. Further NMSE
performance of LS and MMSE channel estimators are also shown for comparison. It is observed that the proposed algorithm achieves the NMSE of $10^{-2}$ at 27 dB, whereas conventional LS and MMSE estimators take 29 dB and 28 dB respectively. The NMSE is calculated as $\frac{\|W - \hat{W}_{\text{BLUE}}\|^2}{\|W\|^2}$. The proposed BLUE estimator results in lower MSE performance in comparison to LS and MMSE estimators as it has minimal variance characteristics. Also, the number of linear estimates $J$ should always be equal to the length of the training sequence $L$ as the objective function to determine the unknown weight coefficients $v_j$ is formulated based on minimizing the error. Also, the number of linear estimates $J$ are different due to the selection of different training data sequences which are taken from a properly normalized submatrix of normalized Discrete Fourier Transform (DFT) matrix (Mehrzad Biguesh and Alex.B.Gershman 2006). The simulation results are validated with the derived theoretical MSE of the BLUE algorithm.

![Figure 3.3](image)

Figure 3.3  NMSE performance of Channel Estimation Algorithms for MIMO AFWRN with $N_{rel} = 4$
Figure 3.3, shows the normalized MSE performance of proposed algorithm with two relay nodes M=4, length of the optimal training sequence L=4, the number of source and destination antennas L=4, and number of antennas in the relay nodes N_{rel}=4. BLUE achieves NMSE of $10^{-2}$ at 25 dB, whereas conventional LS and MMSE estimators take high SNR values to achieve similar NMSE performance. Besides BLUE algorithm exhibiting MVU characteristics, the addition of increased number of relays leads to substantial improvement in MSE performance for the BLUE estimator.

![Figure 3.4 NMSE performance of Channel Estimation Algorithms for MIMO AFWRN with N_{rel} = 8](image)

Figure 3.4 NMSE performance of Channel Estimation Algorithms for MIMO AFWRN with N_{rel} = 8

Figure 3.4, shows the normalized MSE performance of proposed algorithm with eight relay nodes M=8, length of the optimal training sequence L=8, the number of source and destination antennas L=8, and number of antennas in the relay nodes N_{rel}=8. It is observed that the proposed algorithm achieves the NMSE of $10^{-2}$ at 22 dB, whereas conventional LS and MMSE estimators take 24 dB and 23 dB respectively. By comparison in all scenarios
BLUE algorithm exhibits better performance than its traditional estimation counterparts as shown by Table 3.1.

**Table 3.1** SNR requirement of Channel Estimation Algorithms of MIMO AFWRN at MSE of $10^{-2}$

<table>
<thead>
<tr>
<th></th>
<th>M=2</th>
<th>M=4</th>
<th>M=8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L=2</td>
<td>L=4</td>
<td>L=8</td>
</tr>
<tr>
<td>$N_{rel}$=2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>29 dB</td>
<td>27 dB</td>
<td>15 dB</td>
</tr>
<tr>
<td>MMSE</td>
<td>28 dB</td>
<td>26 dB</td>
<td>10 dB</td>
</tr>
<tr>
<td>BLUE</td>
<td>27 dB</td>
<td>25 dB</td>
<td>9 dB</td>
</tr>
</tbody>
</table>

**3.6 SUMMARY**

In this chapter, a channel estimation algorithm based on Best Linear Unbiased Estimator is developed for MIMO AFWRN. It is proven by theoretical MSE derivation and simulation that the proposed algorithm shows better MSE performance than that of the LS and MMSE estimators for the same precoding matrix employed at relays and the same training data sequence broadcast from the source node as employed for SISO AFWRN. Naturally the enhanced performance of BLUE estimator for MIMO AFWRN system is due to its inherent minimum variance characteristic. Although, BLUE is an existing algorithm it is still largely unexplored and not yet applied for channel estimation for MIMO AFWRN. Hence, it is a noteworthy performance in terms of NMSE from the BLUE algorithm in this particular system model of MIMO AFWRN system. The considered MIMO AFWRN system model is equivalent to the SISO AFWRN system model for $N_{rel}=1; L=1$ under the condition that if the number of time slots in the SISO AFWRN system model is one as MIMO AFWRN system employs a
single time slot for transmission of training data signal in phase I. This is the important contribution pertaining to this chapter of this thesis. In future works, this chapter can be extended to select the best number of AF relays in MIMO environments for activation using the BLUE based channel estimation algorithm.