CHAPTER 2

BLUE BASED CHANNEL ESTIMATION TECHNIQUE FOR AMPLIFY AND FORWARD WIRELESS RELAY NETWORKS

2.1 PREAMBLE

Wireless relay network is a promising technology, which has gained profound interest, due to their ability to achieve spatial diversity through cooperation (Andrew Sendonaris et al 2003) among relays. This spatial diversity overcomes the requirement of employing multiple antennas in a transceiver, thereby providing an effective means of improving spectral efficiency and power efficiency. However, to realize the aforementioned advantages of wireless relay networks, accurate Channel State Information (CSI) is required at the destination node for coherent detection (Feifei Gao et al 2008). CSI is obtained by transmitting a training sequence from the source node to the destination node and the CSI is estimated using statistical estimator at the destination node.

2.2 EXISTING METHODS FOR CHANNEL ESTIMATION IN AMPLIFY AND FORWARD WIRELESS RELAY NETWORKS

Existing methods for estimating channel coefficients in Single Input Single Output (SISO) amplify and forward wireless relay networks can be classified into two types namely pilot symbol based channel estimation

In semiblind channel estimation techniques, a short training sequence and statistical parameter of the training sequence are used to estimate channel coefficients (Aris S. Lalos et al 2008, Yang Lu et al 2009). An Expectation-Maximization (EM) based channel estimation was also proposed by Yi Zhang et al 2011. In 2012, iterative channel estimation schemes along with interference cancellation was proposed for wireless relay networks by Yi Zhang et al 2012. The semiblind channel estimation algorithms require less computational overhead and are also bandwidth efficient.

2.2.1 Pilot Symbol Based Channel Estimation Techniques

Hiroyuki Yomo and Elisabeth de Carvalho (2007) proposed a method for estimating CSI in an uplink wireless relay network. The proposed method exploits the fact that the link condition between fixed Base Station (BS) and fixed Relay Station (RS) tends to be stable and frequent CSI update is not necessary for this link. In order for the BS to obtain the CSI of the distant link, RS amplifies a pilot signal received from the MS with a predefined amplification factor and forwards it to the BS. This enables the BS
to obtain the CSI of the RS-MS link based on the received pilot signal and
pre-knowledge on the CSI of the BS-RS link which reduces the required
overhead to explicitly exchange CSI. Also, this is an efficient method for a
node to obtain CSI of all the links in a relay network with minimal
information exchange among nodes.

A suboptimal Linear Minimum Mean Square Error (LMMSE)
estimator was considered by Chirag S. Patel and Gordon L. Stuber (2007) to
estimate the channel coefficients in a downlink wireless relay network. In this
method selection of appropriate channel models, relay mobility, relay gain,
and impact of the underlying channel on the pilot insertion strategy are
considered for estimator design. Bit error rate performance was also analyzed
by taking into account channel estimation errors.

Tao Cui et al (2007) studied training based channel estimation for
relay networks using amplify and forward transmission scheme. Popular Least
Squares (LS) estimator and Minimum Mean Square Error (MMSE) estimator
are studied with the derivation of optimal training sequences and optimal
precoding matrices.

Alireza Shahan Behbhani and Ahmed Eltawil (2008) proposed a
training based LMMSE channel estimator for time division multiplex amplify
and forward relay networks with multiple relays. In this work the following
three scenarios were considered; estimation of the backward and forward
channels by relay, estimation of channel statistics at destination node
assuming that relay knows its backward and forward channels perfectly and in
the third scenario relay do not have knowledge of the backward and forward
channels.

Feifei Gao et al (2008) provided a complete study on the training
based channel estimation issues for relay networks which employ AF
transmission scheme. Estimating the channel individually from source node to relay node and relay node to destination node suffers from many drawbacks. For example, the relay must inform the CSI of phase I to the destination node. It not only reduces bandwidth efficiency but also consumes additional transmitting power. Besides, transmitting the estimated channel may suffer from further distortions. Hence, to overcome these drawbacks, a training based channel estimation schemes to directly estimate the overall channels from source node to the destination node was proposed. A training sequence is broadcasted to all the relays during first phase and the relays forward the received signals after performing a linear transformation to the destination node, during phase II.

Chao Zhang et al (2009) proposed a novel channel estimation scheme referred to as Two Pilot Estimation (TPE) for distributed space time coding amplify and forward wireless relay networks. Maximum Likelihood (ML) decoding is used at the destination node, to estimate the product of source relay channel and relay-destination channel directly. However this is complex and the receiver performance may get degraded. To overcome the drawback, TPE scheme employs two pilot signals from the source node and the relay node. A MMSE estimator is designed at the destination node to estimate the overall channel between source node to relay node and relay node to destination node.

Duy H.N. Nguyen and Ha H. Nguyen (2009) derived optimum power allocation scheme to minimize mean square error of the channel estimate obtained with either ML or MMSE criterion. It also proves that the mismatched decoder of Distributed Space Time Coding (DSTC) which uses the imperfect channel estimation is able to achieve the same diversity order as the coherent decoder (which has perfect channel estimation).

Berna Gedik and Urat Uysal (2009), investigated the effect of channel estimation error on the performance of orthogonal amplify and forward and non-orthogonal amplify and forward relaying systems. Mismatched coherent and partially coherent receivers are also proposed. In mismatched coherent receiver, the channel coefficients are first estimated through pilot symbols based on a LMMSE approach and then fed to a coherent maximum likelihood decoder as if the channel is perfectly known. In a partially coherent receiver the estimates of channel phase information are obtained through a Phase Locked Loop (PLL) while no effort was made for estimation of channel amplitudes.

Aris S. Lalos et al (2010) suggested efficient channel estimation techniques for wideband cooperative systems operating in the amplify and forward strategy.

In 2010, a method for the design of optimal training sequence and selection of appropriate pilot subcarrier for channel estimation in a frequency selective amplify and forward relay network was proposed by (Christos Mavrokefalidis et al 2010). An LMMSE estimator and variants of Least Squares estimator are analyzed for the proposed method.

A novel algorithm was proposed to design pilot codes which minimizes the trace of error covariance matrix (Sun et al 2010). Further, this type of codes are used in channel estimation in multi antenna relay networks that employs distributed space time coding.
Space-Alternating Generalized Expectation-Maximization (SAGE) algorithm was proposed to perform code aided iterative channel estimation from the broadcast signals in amplify and forward relay networks (Nico Aerts et al 2011). In this method only a few pilot symbols are used to derive initial estimates. Estimates of channel coefficients are obtained using the SAGE algorithm exploiting statistics of the data symbols. This iterative approach yields a substantial gain over the pilot based estimates as it requires only a few pilot symbols and iterations to achieve a BER performance closer to the systems with perfect CSI.

### 2.2.2 Semiblind Channel Estimation Techniques

Aris S. Lalos et al (2008) proposed a semi blind channel estimation algorithm for amplify and forward based wireless relay networks. In this algorithm one of the channels from the source node through the relays to the destination node is estimated semiblindly employing a very short training sequence. The other links are estimated blindly with phase ambiguities vector which contains the phase of the direct source node to the destination node channel frequency response. The phase ambiguities are resolved effectively by a small number of pilot symbols. Through simulations, the proposed methods are proved to exhibit high estimation accuracy for a shorter training sequence and outperform direct training based estimation.

Yang Lu et al (2009) proposed a semi-blind channel estimation for space-time coded amplify and forward relay networks. This work requires less training for successful channel estimation and provides better estimates, if the same amount of training is used for comparison to Maximum Likelihood estimators or LMMSE estimators.

Yi Zhang et al (2011) proposed a semiblind method of channel estimation based on Expectation-Maximization (EM) algorithm with coherent
distributed space time coding performed at the relays. By employing an iterative channel estimation scheme, a considerable improvement in signal detections is achieved with only a few iterations.

Yi Zhang et al (2012) proposed an iterative channel estimation algorithm for amplify and forward wireless relay networks when orthogonal coherent distributed space time coding is done at the relays. The proposed schemes are based on the hard and soft estimates of the transmitted data symbols. Also this work proposes semiblind channel estimation together with partial interference cancellation at the receiver. With the employment of hard estimates of symbols in the iterative receiver results in less computational complexity.

2.3 PROBLEM FORMULATION

Although many existing methods have been reported for channel estimation for SISO amplify and forward wireless relay networks, all those methods are based on traditional estimation approaches like LS and MMSE which reduce the mean square error to a significant extent but does not result in minimal variance. Further, LS reduces signal error rather than estimation error, (Mehrzd Biguesh and Alex B. Gershman 2006) while MMSE is based on the conditional Probability Density Function (PDF), in particular the second order channel statistics. Further, the complexity of MMSE estimator increases as the number of samples grows exponentially. In order to overcome the computational complexities experienced by LS and MMSE estimators, an estimator which posses minimal variance characteristic is to be proposed. In this chapter Best Linear Unbiased Estimator (BLUE) is chosen for estimating the channel coefficients for amplify and forward wireless relay networks. BLUE is most suitable for practical application and can be determined with the knowledge of only the first and second moments of the PDF (Steven M. Kay 1993). Although, BLUE is an existing algorithm it is still largely
unexplored and has not yet been applied to amplify and forward based wireless relay networks.

2.4 CHANNEL ESTIMATION FOR AFWRN

2.4.1 System Model

Consider a WRN with a single source node $S$, a single destination node $D$ and $M$ randomly placed relays, $R_i; i = 1, 2, \ldots M$ as shown in Figure 2.1. The channel between each node pair is assumed to be quasi-stationary Rayleigh flat fading. It implies that the channel is constant within one frame but may vary from frame to frame. Denote the Line Of Sight (LOS) channel from source node $S$ to destination node $D$ as $f$, from source node $S$ to $i^{th}$ relay as $g_i$ and from the $i^{th}$ relay to destination node as $h_i$ respectively. Mathematically, $CN(\mu, \sigma^2)$ be a circularly symmetric complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$. The channels $f$, $g_i$ and $h_i$ are represented as $f \in CN(0, \sigma_f^2)$, $g_i \in CN(0, \sigma_{g_i}^2)$ and $h_i \in CN(0, \sigma_{h_i}^2)$ respectively.

![Figure 2.1 Amplify and Forward Wireless Relay Network](image-url)
If the source node $S$ intends to transmit a training data sequence $z$ of size $N \times 1$ through the $i^{th}$ relay $R_i; i = 1, 2 \ldots M$ to the destination node $D$ the transmission is accomplished in two phases, namely phase I and phase II. Each phase contains $T$ consecutive time slots for individual relays and then for relaying information towards the destination node. In phase I, the source node $S$ broadcasts the training data sequence $z$ of size $N \times 1$ to all the relays $R_i; i = 1, 2 \ldots M$. It is assumed that the training data sequence $z$ satisfies the power constraint which is represented as $z^H z \leq N P_s = E_s$, where $P_s$ is the source node power and $E_s$ is the source node energy. In this thesis, training data sequence $z$ is chosen to be $z = \sqrt{P_s} \mathbf{1}_N$ since it releases Peak to Average Power Ratio (PAPR) problem at the source node (Feifei Gao et al 2008) and satisfies the power constraint. $\mathbf{1}_N$ is an $N \times 1$ vector with all its elements are unity. In phase I, received signals at the relays $R_i, i = 1, 2 \ldots M$ are represented by

$$r_i = g_i z + n_i, \quad \text{for } 1 \leq i \leq M$$

(2.1)

where $n_i$ is an $N \times 1$ circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $\mathbf{N}_o \mathbf{I}$, where $\mathbf{N}_o$ is the noise variance.

In phase II, the relays send the transformed training signal to the destination node $D$. At each relay $R_i, i = 1, 2 \ldots M$, a linear transformation $t_i$ is applied to exploit diversity. It is expressed as

$$t_i = \alpha_i A_i r_i, \quad 1 \leq i \leq M$$

(2.2)

where $\alpha_i$ is the real scaling factor and is chosen as $\alpha_i = \sqrt{\frac{P_n}{\sigma^2_{g_i} + \mathbf{N}_o}}$. The scaling factor in linear transformation satisfies the individual power constraint
at each relay \( R_i \) as \( P_n \). This constraint becomes essential since the relay networks conduct distributed transmissions. \( \mathbf{A}_i \) is an \( N \times N \) unitary precoding matrix at \( i^{th} \) relay. The precoding matrix \( \mathbf{A}_i \) is designed such that it satisfies the condition \( z \mathbf{A}_i \mathbf{A}_i^H z = \begin{cases} 0 & \text{if } i \neq j \\ NP_s & \text{if } i = j \end{cases} \). In phase II, the \( N \times 1 \) received signal vector at the destination node \( D \) is represented as

\[
\mathbf{d}_2 = \sum_{i=1}^{M} h_i \mathbf{t}_i + \mathbf{n}_{d2} \tag{2.3}
\]

where \( \mathbf{n}_{d2} \in \mathbb{C}N(0, N_0 \mathbf{I}) \) at destination node \( D \). By substituting Equation (2.2) in Equation (2.3), \( \mathbf{d}_2 \) is written as

\[
\mathbf{d}_2 = \sum_{i=1}^{M} h_i g_i \alpha_i \mathbf{A}_i \mathbf{z} + \sum_{i=1}^{M} h_i \alpha_i \mathbf{A}_i \mathbf{n}_n + \mathbf{n}_{d2} \tag{2.4}
\]

Let the overall channel coefficients \( w_i = h_i g_i \alpha_i \) for \( 1 \leq i \leq M \). Hence Equation (2.4) becomes

\[
\mathbf{d}_2 = \sum_{i=1}^{M} w_i \alpha_i \mathbf{A}_i \mathbf{z} + \sum_{i=1}^{M} h_i \alpha_i \mathbf{A}_i \mathbf{n}_n + \mathbf{n}_{d2} \tag{2.5}
\]

Let the \( M \times 1 \) overall channel coefficient vector be \( \mathbf{w} = [w_1 w_2 \ldots w_M]^{\top}, \mathbf{C} = [\mathbf{A}_1 \mathbf{z}, \mathbf{A}_2 \mathbf{z}, \ldots \mathbf{A}_M \mathbf{z}] \) is an \( N \times M \) matrix, and \( \mathbf{\Lambda} = \text{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_M\} \) is an \( M \times M \) matrix of scaling factors and

\[
\mathbf{n}_d = \sum_{i=1}^{M} h_i \alpha_i \mathbf{A}_i \mathbf{n}_n + \mathbf{n}_{d2}. \] In matrix form, \( \mathbf{d}_2 \) is given by

\[
\mathbf{d}_2 = \mathbf{C} \mathbf{\Lambda} \mathbf{w} + \mathbf{n}_d \tag{2.6}
\]
Further, using the property $A_iA_i^H = I$ it could be easily checked that the covariance matrix $R_n$ of $n_d$, conditioned on a specific realization of $h_i$ is given as

$$R_n = \text{cov}(n_d / h_i, i = 1,2,...M) = \left[ \sum_{i=1}^{M} |h_i|^2 \alpha_i^2 + 1 \right] N_0 I \quad (2.7)$$

Therefore the overall noise, under a specific realization of $h_i$, is still white Gaussian but with a scaled covariance of $N_0$. (Feifei Gao et al 2008).

### 2.4.2 Proposed Channel Estimation Algorithm

The linear estimator for determining the $M \times 1$ overall channel coefficient vector $w$, is given by

$$\hat{w}_i = \sum_{n=0}^{N-1} b_{in}d_2(n) \quad \text{for} \quad i = 1,2,...M \quad (2.8)$$

where $b_{in}$ is weight coefficient of $n^{th}$ sample in $i^{th}$ relay. The weight coefficients are represented in vector form as $b_i = [b_{i0} \ b_{i1} \ldots \ b_{i(N-1)}]^T$. In matrix form, Equation (2.8) is written as

$$\hat{w}_{\text{BLUE}} = Bd_2 \quad (2.9)$$

where the $M \times N$ matrix given by

$$B = [b_1^T \ b_2^T \ldots \ b_M^T]^T \quad (2.10)$$

By substituting Equation (2.6) in Equation (2.9), it is written as

$$\hat{w}_{\text{BLUE}} = BC \Delta w + Bn_d \quad (2.11)$$
It is known that the condition for unbiased estimation of $\mathbf{w}$ is $E(\hat{\mathbf{w}}_{\text{BLUE}}) = \mathbf{w}$. This is satisfied only if $\mathbf{BU} = \mathbf{I}$, where $\mathbf{U} = \mathbf{C} \mathbf{A}$ and $\mathbf{I}$ is an identity matrix. Let $\mathbf{b}_i$ and $\mathbf{u}_j$ denote the $i^{th}$ and $j^{th}$ column of $\mathbf{B}$ and $\mathbf{U}$ respectively. Then, the constraint for unbiased estimation is reduced to

$$\mathbf{b}_i^T \mathbf{u}_j = \delta_{ij}; \text{ for } i = 1,2,\ldots,M ; j = 1,2,\ldots,M$$

(2.12)

The variance of the estimator is determined as

$$\sigma^2_{\hat{\mathbf{w}}_i} = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i$$

(2.13)

where $\sigma^2_{\hat{\mathbf{w}}_i} = \text{var}(\hat{\mathbf{w}}_i)$ and $\mathbf{R}_n = E[\mathbf{n}_n \mathbf{n}_d^H] = \sigma^2 \mathbf{I}$ is an $N \times N$ noise covariance matrix with variance $\sigma^2$. In addition to the unbiased constraint, the expression $\mathbf{BU} = \mathbf{I}$ also satisfies the individual relay power constraint of the relays. The estimate of $\mathbf{w}$ is obtained by minimizing the variance of the estimator in Equation (2.13) subject to the unbiased constraint in Equation (2.12). Mathematically, it is represented as

$$\text{Minimize } \sigma^2_{\hat{\mathbf{w}}_i} = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i$$

subject to $\mathbf{b}_i^T \mathbf{u}_j = \delta_{ij}; i = 1,2,\ldots,M ; j = 1,2,\ldots,M$

(2.14)

This constraint optimization problem is solved using Lagrangian method by minimizing the cost function $J_i$ which is given as

$$J_i = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i + \sum_{j=1}^{M} \lambda_j^{(i)} (\mathbf{b}_i^T \mathbf{u}_j - \delta_{ij})$$

(2.15)
The gradient of Equation (2.15) with respect to $\mathbf{b}_i$ is derived to be as

$$\frac{\partial J_i}{\partial \mathbf{b}_i} = 2\mathbf{R}^{-1}_n \mathbf{b}_i + \sum_{j=1}^{M} \lambda_j^{(i)} \mathbf{u}_j$$

(2.16)

where $\lambda_j^{(i)}, i = 1,2,...M$, and $j = 1,2,...M$ are the Lagrangian multiplier.

Let $\lambda_i = [\lambda_1^{(i)} \lambda_2^{(i)} \ldots \lambda_M^{(i)}]^T$ and $\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \ldots \quad \mathbf{u}_M]$ the gradient Equation (2.16) is simplified as

$$\frac{\partial J_i}{\partial \mathbf{b}_i} = 2\mathbf{R}^{-1}_n \mathbf{b}_i + \mathbf{U} \lambda_i, \quad i = 1,2,...M$$

(2.17)

By equating the gradient to zero, and substituting $\mathbf{U} = \mathbf{C} \Delta$, $\mathbf{b}_i$ is determined as

$$\mathbf{b}_i = -\frac{1}{2} \mathbf{R}^{-1}_n \mathbf{C} \Delta \lambda_i$$

(2.18)

Let $\mathbf{k}_i$ be the vector of all zeros except in $i^{th}$ place,. Now using Equation (2.18), the constraint Equation (2.14) is written as

$$(\mathbf{C} \Delta)^T (-\frac{1}{2} \mathbf{R}^{-1}_n \mathbf{C} \lambda_i) = \mathbf{k}_i$$

(2.19)

By solving Equation (2.19), $\lambda_i$ is given by

$$\lambda_i = -2((\mathbf{C} \Delta)^T \mathbf{R}^{-1}_n (\mathbf{C} \Delta))^{-1} \mathbf{k}_i$$

(2.20)

By substituting Equation (2.20) in Equation (2.18), $\mathbf{b}_i$ is written as

$$\mathbf{b}_i = \mathbf{R}^{-1}_n \mathbf{C} \Delta ((\mathbf{C} \Delta)^T \mathbf{R}^{-1}_n (\mathbf{C} \Delta))^{-1} \mathbf{k}_i$$

(2.21)
By substituting Equation (2.21) in Equation (2.7), the overall channel coefficients using BLUE is given by

$$\hat{w}_{BLUE} = \begin{bmatrix} k_1^T \left[ R_n^{-1} C A (C A)^T R_n^{-1} C A \right]^{-1} d_2 \\ k_2^T \left[ R_n^{-1} C A (C A)^T R_n^{-1} C A \right]^{-1} d_2 \\ \vdots \\ k_M^T \left[ R_n^{-1} C A (C A)^T R_n^{-1} C A \right]^{-1} d_2 \end{bmatrix}$$

(2.22)

Further Equation (2.22) can be rewritten as

$$\hat{w}_{BLUE} = \begin{bmatrix} k_1^T \\ k_2^T \\ \vdots \\ k_M^T \end{bmatrix} ((C A)^T R_n^{-1} C A)^{-1} (C A)^T R_n^{-1} d_2$$

(2.23)

Since \( k_i \) is vector of all zeros except in \( i^{th} \) place, the BLUE estimator for amplify forward wireless relay network is given by

$$\hat{w}_{BLUE} = ((C A)^T R_n^{-1} C A)^{-1} (C A)^T R_n^{-1} d_2$$

(2.24)

By substituting Equation (2.21) in Equation (2.13), variance of the estimator is

$$\sigma^2_{\hat{w}_i} = k_i^T [(C A)^T R_n^{-1} (C A)]^{-1} k_i$$

(2.25)

The performance of the proposed BLUE algorithm for SISO AFWRN system is analyzed in terms of MSE. The MSE of the proposed estimator is defined as

$$MSE_{BLUE} = tr \left( E \left( \hat{w}_{BLUE} - w \right) (\hat{w}_{BLUE} - w)^T \right)$$

(2.26)
By substituting Equation (2.6) in Equation (2.24), the estimator is written as

$$\hat{\mathbf{w}}_{\text{BLUE}} = (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A})^{-1} (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} (\mathbf{C} \mathbf{A} \mathbf{w} + \mathbf{n}_d))$$  \hspace{1cm} (2.27)

Expanding Equation (2.27) further it can be written as

$$\hat{\mathbf{w}}_{\text{BLUE}} = (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A})^{-1} (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A} \mathbf{w} + ((\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A})^{-1} (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{n}_d))$$  \hspace{1cm} (2.28)

In Equation (2.28), the first term simplifies to $\mathbf{w}$. The second term is a $M \times 1$ error vector $\mathbf{e}$ which is defined as

$$\mathbf{e} = (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A})^{-1} (\mathbf{C} \mathbf{A}^T \mathbf{R}_n^{-1} \mathbf{n}_d)$$  \hspace{1cm} (2.29)

Using Equation (2.29), Equation (2.28) is expressed alternatively as

$$\mathbf{e} = \mathbf{w} - \hat{\mathbf{w}}_{\text{BLUE}}$$  \hspace{1cm} (2.30)

Substituting Equation (2.30) in Equation (2.26) $\text{MSE}_{\text{BLUE}}$ is given as

$$\text{MSE}_{\text{BLUE}} = tr \left\{ E \left\{ \mathbf{e} \mathbf{e}^H \right\} \right\}$$  \hspace{1cm} (2.31)

Further, by substituting Equation (2.29) in Equation (2.31) and applying simple mathematical steps $\text{MSE}_{\text{BLUE}}$ is determined as

$$\text{MSE}_{\text{BLUE}} = tr \left\{ (\mathbf{C} \mathbf{A})^{-1} \left( (\mathbf{C} \mathbf{A})^T \right)^H \left( ((\mathbf{C} \mathbf{A})^T \mathbf{R}_n^{-1} \mathbf{C} \mathbf{A})^{-1} \right)^H \right\}$$  \hspace{1cm} (2.32)
2.4.3 CRLB for Channel Estimation in Amplify and Forward Wireless Relay Network

For coherent detection in amplify forward mode, maximum likelihood detection is performed in destination node $D$ based on a specific channel realization vector $w$ while treating $n_d$ as the overall white Gaussian noise. Hence, likelihood function for the received signal model in Equation (2.6) is determined as

\[ p(d_z;w) = \frac{1}{(2\pi \det(\sigma^2 I))^{N/2}} \exp\left\{-\frac{1}{2\sigma^2} (x^H x)\right\} \tag{2.33} \]

where $x = (d_z - CAw)$, $\det(\sigma^2 I)$ is the determinant of the $N \times N$ noise covariance matrix of $n_d$, which is $R_n = \sigma^2 I$. By taking natural logarithm on both sides of the likelihood function of Equation (2.33), it results in

\[ \ln(p(d_z;w)) = \ln\left(\frac{1}{(2\pi)^{N/2} \sigma^N}\right) - \frac{1}{2\sigma^2} \{x^H x\} \tag{2.34} \]

Omitting first term in Equation (2.34) and using the definition $x = (d_z - CAw)$, the log likelihood function is written as

\[ \ln(p(d_z;w)) = -\frac{1}{2\sigma^2} \left\{d_z^H d_z - d_z^H CAw - (CAw)^H d_z\right\} + (CAw)^H CAw \tag{2.35} \]

The first term in Equation (2.35) is independent of $w$, and the terms $d_z^H CAw$, $(CAw)^H d_z$ and $(CAw)^H CAw$ in Equation (2.35) are expanded as follows, to find derivatives of individual weight coefficients
Substituting Equation (2.36), Equation (2.37) and Equation (2.38), in Equation (2.35) the log likelihood function can be expanded as

\[
\ln(p(\mathbf{d}_2; \mathbf{w})) = -\frac{1}{2\sigma^2} \left\{ \mathbf{d}_2^T \mathbf{d}_2 - \sum_{k=1}^{N} \sum_{l=1}^{N} \Delta_i \Delta_l c_{kl}^* w_i w_l^* \right\} \\
+ \frac{1}{2\sigma^2} \left\{ \sum_{k=1}^{N} \sum_{l=1}^{N} \Delta_i^* c_{kl}^* w_i^* d_2(k) \right\} \\
- \frac{1}{2\sigma^2} \left\{ \sum_{k=1}^{N} \sum_{l=1}^{N} \Delta_i \Delta_l^* c_{kl}^* w_l^* w_i^* \right\} \\
(2.39)
\]

The Cramer-Rao Lower bound (CRLB) for a vector parameter (Steven M. Kay 1993)

\[
\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \ldots & w_M \end{bmatrix}^T \text{ is defined as}
\]

\[
\sigma^2_{w_i} \geq [\mathbf{F}^{-1}(\mathbf{w})]_{ii} \text{ for } i = 1, 2 \ldots M \\
(2.40)
\]

where \( \mathbf{F}(\mathbf{w}) \) is the Fisher Information Matrix (FIM). It is defined as

\[
[\mathbf{F}(\mathbf{w})]_{ij} = -E \left[ \frac{\partial^2 \ln(p(\mathbf{d}_2; \mathbf{w}))}{\partial w_i \partial w_j^*} \right] \text{ for } i, j = 1, 2 \ldots M \\
(2.41)
\]
and the elements of \([F(w)]\) for \(i, j = 1, 2 \cdots M\) are expressed as

\[
[F(w)] = \begin{bmatrix}
-\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_1; w))}{\partial w_1 \partial w_1} \right] & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_1; w))}{\partial w_1 \partial w_2} \right] & \cdots & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_1; w))}{\partial w_1 \partial w_M} \right] \\
-\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_2; w))}{\partial w_1 \partial w_1} \right] & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_2; w))}{\partial w_1 \partial w_2} \right] & \cdots & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_2; w))}{\partial w_1 \partial w_M} \right] \\
\vdots & \vdots & \ddots & \vdots \\
-\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_M; w))}{\partial w_1 \partial w_1} \right] & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_M; w))}{\partial w_1 \partial w_2} \right] & \cdots & -\mathbb{E} \left[ \frac{\partial^2 \ln(p(d_M; w))}{\partial w_1 \partial w_M} \right]
\end{bmatrix}
\]

(2.42)

Substituting Equation (2.39) in Equation (2.42) and applying simple mathematical steps, \(F(w)\) is determined as

\[
F(w) = \begin{bmatrix}
\frac{\Delta^2_1 \left( \sum_{m=1}^{M} c_{m1}^2 \right)}{\sigma^2} & 0 & \cdots & 0 \\
0 & \frac{\Delta^2_2 \left( \sum_{m=1}^{M} c_{m2}^2 \right)}{\sigma^2} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{\Delta^2_M \left( \sum_{m=1}^{M} c_{mM}^2 \right)}{\sigma^2}
\end{bmatrix}
\]

(2.43)

Since \(F(w)\) is a diagonal matrix, it can be easily inverted to obtain CRLB for the overall channel coefficient vector \(w\). Substituting Equation (2.43) in Equation (2.40) CRLB is given by

\[
\sigma_{\hat{w}_i}^2 \geq \frac{\sigma^2}{\Delta^2_i \left( \sum_{m=1}^{M} c_{mi}^2 \right)} \quad \text{for } i = 1, 2 \cdots M
\]

(2.44)
2.5 RESULTS AND DISCUSSIONS

In this section, mean square error performance of the proposed channel estimation algorithm using BLUE is analyzed and compared with theoretical MSE and Cramer-Rao Lower Bound (CRLB). The channels $g_i$ and $h_i$ are assumed as circularly symmetric Gaussian random variables with unit variances. The noise sequence is also assumed to be circularly symmetric complex Gaussian with zero mean and unit variance. Hence, Signal to Noise Ratio (SNR) is defined as $SNR = \frac{(P_s \times 1)}{N_0} = P_s$, where $P_s$ is the source node power. In the simulations, 1000 independent Monte-Carlo runs are used.

Figure 2.2, shows the MSE performance of proposed algorithm with two relay nodes $M=2$ and length of the optimal training sequence $N=2$ along with the relay powers as $\{0.8P_s, P_s\}$. Further, MSE performance of LS and MMSE channel estimators are also shown for comparison. It is observed that the proposed algorithm achieves the MSE of $10^{-2}$ at 20 dB, whereas conventional LS and MMSE estimators require 23 dB and 21 dB respectively. The proposed channel estimator achieves better MSE performance in comparison to LS and MMSE estimators as it has minimal variance characteristics. The simulation results are validated with the derived theoretical MSE of the BLUE algorithm. The CRLB reaches the MSE of $10^{-2}$ at 15 dB.
Figure 2.2  MSE performance of Channel Estimation Algorithms at M = 2; N = 2

Figure 2.3, shows the MSE performance when the length of the training sequence is increased from N=2 to N=8 with two relay nodes M=2. The proposed algorithm requires 14 dB, to achieve an MSE of $10^{-2}$. The traditional counterparts LS and MMSE estimators require 15 dB for estimation of overall channel coefficients in a SISO AFWRN system. This portrays the importance of increasing the length of the training sequence in comparison to the number of relays as it minimizes implementation cost. It is also observed that the CRLB at 12 dB for an MSE of $10^{-2}$.
Figure 2.3  MSE performance of Channel Estimation Algorithms at 
M = 2; N = 8

Figure 2.4 shows the MSE performance of proposed algorithm by 
increasing the number of relays to M=4, the length of the optimal training 
sequence to N=4 and the relay powers are assumed as \{0.8P_s, P_s, 0.8P_s, P_s\}.

In this scenario, increasing the number of relay nodes and length of the 
training sequence simultaneously reduces the MSE of the proposed estimator 
algorithm to a significant extent. Further, it is also observed that the proposed 
algorithm outperforms the LS and MMSE estimators due to its minimum 
variance characteristics. Moreover, the precoding matrix (Feifei Gao et al 
2008) employed at the relay nodes and usage of training sequence which 
iminizes PAPR also helps in achieving superior performance.
Figure 2.4  MSE performance of Channel Estimation Algorithms at 
M = 4; N = 4

Figure 2.5, shows the MSE performance of proposed algorithm 
with eight relay nodes M=8 and length of optimal training sequence N=8. It is 
observed that the proposed algorithm requires 9 dB only, compared to 15 dB 
when N=4 and M=4, to achieve MSE of $10^{-2}$. The CRLB is at 6 dB for the 
MSE of $10^{-2}$. Although this scenario M=8; N=8, provides reduced MSE in 
comparison to the previous scenarios M=2;N=2,8 and M=4;N=4,8 it is not 
recommended in practice as deployment of more number of relays leads to 
complexities pertaining to implementation aspects.
Figure 2.5  MSE performance of Channel Estimation Algorithms at $M = 8; N = 8$

Figure 2.6, analyzes the significance of increase in the length of training sequence from $N=4$ to $N=8$, with $M=4$ relays. Now, the proposed algorithm requires 11 dB, compared to $N=4$ which requires 15 dB. In this scenario, first two relays have power of $0.8P_s$ and the remaining two relays have the power of $P_s$. As the MSE performance improves by increasing the length of the training sequence the requirement of employing more number of relays for channel estimation can also be minimized.
Figure 2.6  MSE performance of Channel Estimation Algorithms at $M = 4; N = 8$

Table 2.1  SNR requirement of Channel Estimation Algorithms at MSE of $10^{-2}$

<table>
<thead>
<tr>
<th>Channel Estimation Techniques</th>
<th>M=2  N=2</th>
<th>M=4  N=4</th>
<th>M=8  N=8</th>
<th>M=2  N=8</th>
<th>M=4  N=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>23 dB</td>
<td>20 dB</td>
<td>15 dB</td>
<td>15 dB</td>
<td>16 dB</td>
</tr>
<tr>
<td>MMSE</td>
<td>21 dB</td>
<td>17 dB</td>
<td>10 dB</td>
<td>15 dB</td>
<td>14 dB</td>
</tr>
<tr>
<td>BLUE</td>
<td>20 dB</td>
<td>15 dB</td>
<td>9 dB</td>
<td>14 dB</td>
<td>11 dB</td>
</tr>
<tr>
<td>CRLB</td>
<td>15 dB</td>
<td>10 dB</td>
<td>6 dB</td>
<td>12 dB</td>
<td>8 dB</td>
</tr>
</tbody>
</table>

Table 2.1 summarizes MSE performance of the proposed algorithm for various combinations of number of relays and length of training sequence. Further the MSE performance is also compared with conventional LS and MMSE algorithms and CRLB.
Figure 2.7, shows the performance of proposed algorithm assuming that relays have different powers. Three types of power distribution considered here are Unbalance Type 1: represented by \( \{0.8P_s, 0.8P_s, P_s, P_s\} \), Unbalance Type 2: represented by \( \{0.4P_s, 0.8P_s, P_s, 1.4P_s\} \) and Unbalance Type 3: represented by \( \{0.2P_s, 0.4P_s, P_s, 2P_s\} \). It is observed from Figure 2.7, Unbalance Type 1 shows the best MSE performance since two relays have the same power. CRLB for the three types of scenarios are also simulated. CRLB for Unbalance Type 1 provides better MSE performance than Unbalance Type 1 and Unbalance Type 2 justifying two relays with equal power. This unbalanced relay power often happens in practical scenarios as different relays have different powers according to their relaying protocols, category and longevity. Moreover, unbalanced power distribution deteriorates the accuracy of channel estimation.
2.6 SUMMARY

In this chapter, a channel estimation technique based on BLUE algorithm is developed for amplify and forward relay networks. A closed form expression for the MSE of the proposed algorithm is also derived. Further the simulation and theoretical results of the proposed algorithm show better MSE performance than that of the LS and MMSE estimators for the same precoding matrix employed at relays and the same training data sequence broadcasted from the source node in amplify and forward relay networks. The improvement in MSE performance of the proposed algorithm is due to its inherent minimum variance characteristic. Although, BLUE is analogous to the LMMSE estimator, BLUE does not require the knowledge channel covariance matrix, which is difficult to acquire in practice. Furthermore, the BLUE algorithm is simple in practical implementation in comparison to the MMSE estimator, as complete knowledge of the PDF is not essential. The CRLB for the channel estimation in an AFWRN is also derived, which acts as a benchmark for assessing the estimator performance. The results that have been shown in this chapter can be used for selecting the best number of amplify forward relays and length of the training sequence in practice.