CHAPTER FIVE
5. SIMPLE TECHNIQUE TO ESTIMATE MACRO-BENDING LOSSES IN INDEX-GUIDING PHOTONIC CRYSTAL FIBERS

5.1 INTRODUCTION

Photonic crystal fibers (PCF), a different class of optical fibers, are possible to be fabricated from undoped silica only (Knight J. C. et. al, 1996a). The waveguiding phenomenon in them occurs due to the air-holes in a periodic or regular microstructure, running along the entire length of the fibers, forming the cladding region. When the central hole is missing, it creates a defect, which works as the high index core of the waveguide. This region, where light can be trapped, supports a guided mode. Since the core index is greater than the effective or average index of the cladding, the principle of light guiding is analogous to total internal reflection (TIR) in conventional step index fibers (CSIF) (Birks T. A. et al., 1997). But, because of the distribution of the lower index air-holes in the cladding region, this guiding mechanism is referred to as modified total internal reflection (m-TIR) (Sorensen T. et al., 2001), as stated in the first Chapter. Also, in short wavelength range, a separate waveguiding mechanism, based on photonic band gap (PBG) concept dominates to ensure the single moded propagation (Yablonovitch E., 1993; Broeng J. et al., 1999). Here, the periodically arranged air-holes in PCFs create PBGs, preventing fields from propagating in the cladding region for a fixed wavelength range. PCFs have various attractive and appealing features, such as supercontinuum generation (SCG) (Wadsworth W. J. et al., 2002), endlessly single mode (ESM) merit (Birks T. A. et al., 1997), large or
ultrasmall mode field diameters (MFD) producing corresponding effective areas (Knight J. C. et al., 1998a; Broderick N. G. R. et al., 1999; Mortensen N. A., 2002), ultraflattened dispersions (Ferrando A. et al., 2001), large nonlinearities (Monro T. M. et al., 1999; Broderick N. G. R. et al., 1999) and other prospective features. They are used to provide a better optoelectronic tool in various modern fields, such as imaging, telecommunications, spectroscopy, metrology, and other new emerging areas (Varshney S. K. and Sinha R. K., 2004), as mentioned in the first Chapter. Therefore, these fibers, in near future, are to become, ultimately, reliable transmission media for the propagation of electromagnetic (EM) waves in a suitable context.

In the above stated applications, PCFs are needed to be spliced together and bent in the form of coils of different radii. In a bent fiber, the modal field is distorted, radially, outward in the direction of bend (Baggett J. C. et al., 2003). This results in coupling of the core mode or modes to the radiation and leaky modes, cladding and backward propagating modes and leads to a loss. This loss in these bent fibers is well known as the macro-bending loss, which has an adverse effect in the case of optical transmission. An accurate estimation of macro-bending loss of PCFs is very much important not only for the assessment of PCF properties, but also to help to assess the power budgeting in optical networks involving such losses. This loss characteristic can, also, define the spectral windows for the fibers to be operated in and provide information regarding the propagating modes through the fibers. However, this loss in PCFs is, qualitatively, different from that in step index fibers (SIF) (Baggett J. C. et al., 2003). On one hand, PCFs show bend loss edges at long wavelengths, as do SIFs, since the mode extends more into the cladding. This results in a more feebly guided mode, suffering a greater susceptibility to bending. On the other hand, PCFs also exhibit a short wavelength bend loss edge with critical values of the bend radius, since their unique and distributed cladding structure (Birks T. A. et al., 1997)
makes the effective cladding indices of the PCFs wavelength dependent. This index changes with variation in wavelength and limits the distribution of light in air-holes at shorter wavelength. Thereby, it makes the effective index higher. Consequently, the normalized frequency does not increase appreciably, giving rise to some leakage of the field in the cladding and single moded merit in PCF in contrast to electromagnetics in SIF. Therefore, at short wavelength, it is, also quite likely, to have loss in bent condition of the fiber. In this connection, accurate knowledge of the effective cladding indices for various opto-geometrical parameters is essential.

Although, in the literature, a few investigations on the bending loss properties of PCFs are available with the formula for bending loss coefficient (Sorensen T. et. al, 2001, 2002), they have not provided any effective idea to evaluate the average or effective cladding index over the entire range of opto-geometrical parameters of PCF. As stated above, this index is an important factor in the estimation of bending loss of solid-core PCFs, guiding light by the m-TIR process. Since the effective cladding index is considered, the corresponding formalism is, conveniently, known as the effective index method (EIM) for PCFs. The EIM is used to find different waveguiding parameters, such as the effective normalized frequency $V_{\text{eff}}$, far field radiation pattern, etc. of PCFs (Varshney S. K. and Sinha R. K., 2004). In this method, the PCF is assumed to be analogous to an SIF (Birks T. A. et. al, 1997; Koshiba M. and Saitoh K., 2004), with its core and cladding indices replaced by those of silica and the average or effective cladding index of the PCF. If this cladding index, also called the index of the fundamental space-filling mode (FSM), which is conventionally denoted by $n_{\text{FSM}}$, is easily obtained, various propagation characteristics of PCF, entirely dependent on this index, such as MFD, beam divergence, dispersion, etc.
can be found from the available expressions for them, involving the $n_{FSM}$ (Koshiba M. and Saitoh K., 2004).

In the fourth Chapter, we have presented a simple and elegant equation for the said effective index of the solid-core PCFs (P2 in Section 1.7.2 of first Chapter), valid in the entire single mode region, for which relative air-hole diameter or $d/\Lambda$-values are less than or equal to 0.45 (Ju J. et al., 2004). Our equation is also developed based on the scalar EIM (SEIM), and it, also, enables one to obtain $n_{FSM}$-s, easily and directly, for wide ranges of values of the opto-geometrical parameters of PCFs. Further, it is adaptable in terms of those parameters to fit into various physical situations. This is the real merit of our approach, which is not obvious for the previous SEIMs (Birks T. A. et al., 1997; Sorensen T. et al., 2001, 2002). We have used the equation for easy evaluation of various propagation characteristics of the PCF, as stated above. It would have been much better and advantageous, if the macrobending losses of solid-core PCFs could be estimated using the same equation as typical examples of loss calculations.

In this Chapter, we investigate whether the equation for $n_{FSM}$, which is valid in the entire single mode region of solid-core PCFs and presented in the fourth Chapter, can be used to compute macrobending losses of such fibers for given opto-geometrical parameters, with refinements, if any, with an aim to present a novel and simple scheme [P3, Section 1.7.2, Chapter One]. To show the accuracy of our computed bending losses, we compare our results with those of (Sorensen T. et al., 2001, 2002), obtained by using SEIM (Birks T. A. et al., 1997). Then, we observe our results to match, fairly excellently, with those of (Sorensen T. et al., 2001, 2002), favoring the validity of our method and its acceptance as a simple and suitable alternative to the available rigorous methods. Our approach should find wide attention, for easy use, by system designers.
5.2 ANALYSIS

5.2.1 PREVIEW AND FORMULATION

We take into consideration an all-silica PCF, with a triangular lattice of uniform air-holes of diameter $d$, running along the whole length of the fiber, as in the previous Chapter, which is again shown in Fig. 5.1. The holes are placed, symmetrically, around a central defect or an omitted air-hole, acting as the fiber core. It is made up of solid silica. The air-hole matrix has lattice-constant or hole-pitch $\Lambda$. This region is assumed to act as the cladding of the PCF. The structure remains unchanged in the longitudinal direction. Since the core-index $n_{CO}$ is greater than the effective or average cladding index $n_{FSM}$, the fiber can guide light by modified total internal reflection (m-TIR) for longer wavelength values and by photonic band gap (PBG) mechanism for shorter wavelengths.

The propagation constants $\beta$ of the modes, which are guided through the core of the PCF are given by the equation (Birks T. A. et al., 1997):

$$kn_{CO} > \beta > \beta_{FSM}$$

(5.1)

where $k = 2\pi/\lambda$, $\lambda$ is the operating wavelength, $n_{CO}$ is the index of the core material or silica, $\beta_{FSM}$ is the propagation constant of the FSM, the fundamental mode in the infinite photonic crystal cladding (IPCC) without any core or defect. Hence, $\beta_{FSM}$ is the maximum value of $\beta$ in the cladding region of the concerned PCF.

The effective cladding index or the refractive index of the FSM is given by

$$n_{FSM} = \frac{\beta_{FSM}}{k}$$

(5.2)
Fig. 5.1 The index guiding photonic crystal fiber
The procedure to find the index $n_{FSM}$ of the PCF is same as that, stated in the fourth Chapter. Now, from the analogy with an SIF, the effective cladding index $n_{FSM}$ is used to find the effective normalized frequency or $V$-parameter of the PCF, $V_{eff}$, given as (Birks T. A. et al., 1997; Sorensen T. et al., 2002; Varshney S. K. and Sinha R. K., 2004; Koshiba M. and Saitoh K., 2004):

$$V_{eff} = \frac{2\pi \rho_{eq}}{\lambda} \left( n_{CO}^2 - n_{FSM}^2 \right)^{1/2} = \sqrt{U_{eff}^2 + W_{eff}^2}$$  \hspace{1cm} (5.3)

where $\lambda$ is the wavelength of the light, $n_{CO}$ is the core index and $\rho_{eq}$ is the effective core radius, which we, now, consider to be 0.62 $\Lambda$, in consistence with (Sorensen T. et al., 2001, 2002), with

$$U_{eff} = \frac{2\pi \rho_{eq}}{\lambda} \left( n_{CO}^2 - n_{eff}^2 \right)^{1/2}$$  \hspace{1cm} (5.4)

and

$$W_{eff} = \frac{2\pi \rho_{eq}}{\lambda} \left( n_{eff}^2 - n_{FSM}^2 \right)^{1/2}$$  \hspace{1cm} (5.5)

Here, $n_{eff}$ is the effective index of the fundamental guided mode and the two parameters $U_{eff}$ and $W_{eff}$ are the effective normalized phase and attenuation constants, respectively, of the chosen PCF, similar to an SIF.

It may be relevant to mention that Fig. 5.1 is same as Fig. 4.1, and Eqs. (5.1), and (5.2), are same as Eqs. (4.1), and (4.2), respectively, in this Chapter for continuity and comfort of reading.
5.2.2 APPLICATION TO MACROBENDING LOSS

When a PCF is straight, all the physical properties have translational symmetry along the axis of the fiber. But, as the fiber is bent with a bending radius $R_c$, this symmetry is lost, which results in an effect on the field confinement in the transverse direction. Actually, the field, then, starts to couple to leaky cladding modes, as stated earlier. When the wavelength of the light is short in comparison with the radius $R_c$, the field is weakly perturbed. The effect of macrobending is given by the attenuation constant $\alpha$. From the analogy between a solid-core PCF and an SIF, the equation for the loss coefficient of the PCF due to macrobending losses is obtained from that of an SIF (Sorensen T. et al., 2001, 2002), with the SIF parameters replaced by those of the PCF, given as:

$$\alpha (dB/m) = \frac{\sqrt{\pi} A_e^2}{8P} \frac{\rho_{eq} \exp \left( \frac{-4R_c W_{eff}^2 \delta_{eff}}{3\rho_{eq} V_{eff}^2} \right)}{W_{eff} \sqrt{W_{eff} R_c / \rho_{eq} + V_{eff}^2 / (2\delta_{eff} W_{eff})}}$$  \hspace{1cm} (5.6)

where $R_c$ is the radius of curvature of bend or the bending radius in cm. Here, $V_{eff}$ and $W_{eff}$ are the parameters of the PCF, as mentioned in the previous section and $\delta_{eff}$ is the relative refractive index difference, given by $\delta_{eff} = \frac{n_{CO} - n_{FMM}}{n_{CO}}$. Further, $\rho_{eq}$ is the radius of the solid core, $A_e$ is the amplitude coefficient of the field in the cladding region and $P$ is the power carried by the fundamental mode in the same PCF. For a Gaussian mode, $A_e^2 / P$ in Eq. (5.6) is same as $1/A_{eff}$ (Sorensen T. et al., 2001), where $A_{eff}$ is the effective area. This $A_{eff}$ is simply given by $\pi \omega^2$, where $\omega$ is the spot size or mode field radius (MFR), obtained by using Marcuse equation (Ghatak A. K. and Thyagarajan K., 1998; Koshiba M. and Saitoh K., 2004).
The two PCF parameters $\delta_{\text{eff}}$ and $V_{\text{eff}}$, involved in $\alpha$ of Eq. (5.6), depend on the wavelength of the light propagating through the PCF. They can predict bending losses in PCFs well in the smaller wavelength ranges (Varshney S. K. and Sinha R. K., 2004). Further, it is seen from Eq. (5.6) that although the dependence of $\alpha$ on $\delta_{\text{eff}}$, $V_{\text{eff}}$ and other factors are involved, the role of $\delta_{\text{eff}}$ appears to be more pronounced at smaller $\lambda$. Thereby, at fixed $R_C$, even if $V_{\text{eff}}$ does not change appreciably, $n_{\text{FSM}}$ should be high because of less light in the air-holes. It will make $\delta_{\text{eff}}$ low and $\alpha$ should be greater.

5.3 RESULTS AND DISCUSSIONS

5.3.1 CHOICE OF OPTOGEOMETRICAL PARAMETERS AND PROCEDURE

We consider wide ranges of hole-pitch $\Lambda$ and the normalized hole-diameter or relative air-hole size, $d/\Lambda$-values with variations in the wavelength of light $\lambda$, within the range of 0.2 to 1.6 $\mu$m first. The SIFs are, commonly, used in this region of wavelengths. This is because they show minimum attenuation loss at 1.55 $\mu$m and minimum total dispersion at around 1.3 $\mu$m. Hence, we confine our attention to this region, which can, easily, be extended to longer wavelengths, if required. First, we choose the PCF of (Sorensen T. et al., 2001, 2002), with $\Lambda = 7.8$ $\mu$m and $d = 2.4$ $\mu$m. We obtain the values of $n_{\text{FSM}}$ for the PCF with variations in $\lambda$, using the approach explained in the fourth Chapter. The application of SEIM, based on (Birks T. A. et al., 1997), to present the formulation of the effective cladding index in terms of relevant PCF parameters, is, already, explained in details in the previous Chapter, where we have considered a particular range of these values to fit to the experimental results. We have used this technique in order to interpret a proper
physical property of the PCF, for example, the bending loss as in (Sorensen T. et al., 2001, 2002), where $n_{CO} = 1.444$. To use it, we take, suitably, extended $\Lambda$-values and $\lambda$-s ranging from 0.2 to 1.6 μm and present proper coefficients, as explained in the fourth Chapter. The coefficients are shown in Table 5.1.

Next, with the $n_{FSM}$-values, we find the corresponding effective normalized frequencies, $V_{eff}$-s of the same PCF, by using Eq. (5.3). Here, we take $\rho_{eq} = 0.62 \Lambda$, as in (Sorensen T. et al., 2001, 2002). From the obtained $V_{eff}$-values, we compute the corresponding $U$-values, or $U_{eff}$-s, as roots, by solving the relevant transcendental equation in an SIF (Ghatak A. K. and Thyagarajan K., 1998), as given by Eq. (3.16). Then, we obtain the corresponding $W_{eff}$-s using Eq. (5.3), as given before and substitute all $U_{eff}$ and $W_{eff}$-values in Eq. (5.6).

Further, we use our equation to study and predict the variation of the bending loss with a change in bending radius. Thereby, we consider a group of solid-core PCFs, by varying the hole-pitch $\Lambda$ first, while keeping the air-hole diameter $d$ unchanged for one set, and then, varying the $d$, keeping $\Lambda$ unchanged for another set. This is due to the fact that the number of available theoretical results for variation in bending loss with the bending radius is limited. For the first set, we consider $\Lambda = 7.8$ and 6.5 μm, for $d = 2.4$ μm. For the second one, we consider $d = 2.4$ and 2.0 μm for $\Lambda = 7.8$ μm. In this manner, we obtain different PCFs with different $d/\Lambda$-values, all in the ESM region of solid-core PCFs, but basically we keep $d = 2.4$ and $\Lambda = 7.8$ μm as chosen above for comparison with (Sorensen T. et al., 2001, 2002). For the new PCFs, we again compute all the parameters involved in Eq. (5.6), following the same procedure as before and taking $d = 2.4$ and $\Lambda = 7.8$ μm, as the standard. We like to present the physical insight involved in bending loss of PCF with a
Table 5.1

Values of the coefficients in the formulation for $n_{FSM}$ with $n_{CO} = 1.444$

<table>
<thead>
<tr>
<th></th>
<th>i=0</th>
<th>i=1</th>
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<td></td>
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<td>j=1</td>
<td>j=2</td>
</tr>
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<td>$A_{ij}$</td>
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<td>0.000101</td>
<td>-0.000017</td>
</tr>
<tr>
<td>$B_{ij}$</td>
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<td>0.002167</td>
<td>-0.000129</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>0.006291</td>
<td>-0.001935</td>
<td>0.000139</td>
</tr>
</tbody>
</table>
5.3.2 ESTIMATION OF MACROBENDING LOSS FOR THE CHOSEN PCFS

Now, we find various $\alpha$ -values from Eq. (5.6). First, we plot them in a linear scale with a variation in wavelength $\lambda$, in $\mu$m, for three different values of bending radius $R_c$, as shown by the solid curves in Fig. 5.2. We choose $R_c = 5, 10, \text{ and } 15 \text{ cm}$ as typical examples, in consistence with (Sorensen T. et al., 2001), to determine the $\alpha$ -values. In the same figure, the dotted curves represent the variation of $\alpha$ in dB/m with the wavelength $\lambda$, as given in (Sorensen T. et al., 2001), obtained from the scalar model. From the two different types of curves in Fig. 5.2, it is observed that the two different curves match, excellently, for $R_c = 5 \text{ cm}$ [P3, Section 1.7.2, Chapter One]. But, for other two $R_c$ -values of 10 and 15 cm, though the macro-bending losses obtained from our simple and complete formulation match, reliably, with those in (Sorensen T. et al., 2001), the nature of the variation of bending loss with $\lambda$ in these two cases is very similar.

Again, as stated above, we present the variation of our estimated bending loss, in dB/m, with the bending radius $R_c$, in m, and plot Fig. 5.3 for two PCFs with same $d$ of 2.4 $\mu$m and different $\Lambda$ -values of 7.8 and 6.5 $\mu$m, at the particular wavelength $\lambda = 1.55 \mu$m, where attenuation losses become minimum. From the two curves, it is observed that the bending loss decreases with an increase in the bending radius and for a particular value of the bending radius, the loss is less for a higher value of $d / \Lambda$. In addition, we can comment that as $\Lambda$ decreases with $d$ remaining unchanged, the effective cladding index decreases, which results in an increase in $V_{eff}$, causing a decrease in the bending loss for tighter confinement of the mode in the core, as observed from Fig. 5.3.
Fig. 5.2 Bending loss of the PCF with $\Lambda = 7.8$ $\mu$m and $d = 2.4$ $\mu$m as a function of wavelength.
Fig. 5.3 Bending loss as a function of bending radius for the PCFs with $d = 2.4 \ \mu m$ and $\Lambda = 7.8$ and $6.5 \ \mu m$.

Fig. 5.4 Bending loss as a function of bending radius for the PCFs with $\Lambda = 7.8 \ \mu m$ and $d = 2.4$ and $2.0 \ \mu m$. 
Next, we plot Fig. 5.4 to show the variation of our estimated bending loss, in dB/m, with the bending radius $R_c$, in m, for two PCFs, one previously taken and another new, with same $\Lambda$ and different $d$-values of 2.4 and 2.0 μm, at the same particular wavelength $\lambda = 1.55$ μm. Here also, the bending loss decreases in the same manner with an increase in the bending radius and for a fixed radius, the loss is less for a higher $d/\Lambda$ corresponding to the curve of $d = 2.4$ μm. In this case, it can be stated that as $d$ decreases with $\Lambda$ remaining same, the effective cladding index increases, resulting in a decrease in $V_{eff}$, and hence, an increase in the bending loss due to the less tight confinement of the mode in the core, as seen in Fig. 5.4. Therefore, it is observed from these two sets of curves in Figs. 5.3 and 5.4 that since we have taken $d = 2.4$ and $\Lambda = 7.8$ μm, which are already standardized in Fig. 5.2, our simple equation for $n_{FSM}$, given by Eq. (4.7), works, reliably, for estimation of the macrobending loss in the ESM region of solid-core PCFs and provides useful physical insight.

It is now evident from Fig. 5.2 that our approach for bending loss calculation based on the simple and elegant method, elucidated in the fourth Chapter, estimates the bending loss in solid-core PCFs easily and simply, with much less computation time. Also, this approach can be applied in the entire single mode region of the same PCF, and over a wide wavelength range, since the computation for $n_{FSM}$ based on the formulation in the fourth Chapter is valid for these ranges of $d/\Lambda$ and $\lambda$. Further, with reference to the last two figures, we can predict the results of bending loss for PCFs with appropriate dimensions of hole size and pitch in the single mode region. Experimentalists, system designers and users, preferring simpler and easier approaches for estimation of bending loss of solid-core PCFs, can use this approach, without heavy numerical computations. Further, they can, also,
predict the nature of variation in the bending loss with the wavelength of light and the bending radius for various solid-core PCFs.

5.4 CONCLUSION

Based on an appropriate equation of the effective cladding index, oriented in versatile manner to fit to proper physical situation, we present a simpler and easier analysis to estimate the macro-bending losses in index-guiding photonic crystal fibers (PCF), in terms of their optogeometrical and waveguide parameters. This approach should find wide and ready use by system analysts of PCFs, with less numerical computations.