Preface

The present thesis entitled “Study Of Various Structures On Differentiable Manifolds And Its Submanifolds” is being submitted to the faculty of Science, Department of Mathematics and Astronomy is the outcome of the original research work carried out by the author for the award of Ph.D. degree under the able guidance and supervision of Dr. Shyam Kishor, Assistant Professor in the Department of Mathematics and Astronomy, University of Lucknow, Lucknow-226007, India.

This thesis entitled “Study Of Various Structures On Differentiable Manifolds And Its Submanifolds” consists of nine chapters and each chapter is divided into several sections and subsections. The decimal notation has been used for numbering the equations. The bold decimal notation has been used for numbering the sections subsections, theorems, lemmas, corollaries, propositions, definitions and examples. The numbers written in the square bracket [ ] refer to the references given at the end.

The study of an odd-dimensional manifold was initiated by Boothby and Wang in 1958, and J. W. Gray from the topological point of view and the structure introduced by them is called contact structure. In 1960, Sasaki started to study manifolds with the help of tensor analysis.

We begin with the Chapter 1, which is more or less introductory and open with a brief account of manifolds with structures and some basic definitions having interconnection with my investigations together with a brief account of the recent development in the direction of our work.

In 1924, Friedmann and Schouten [69] introduced the idea of semi-symmetric linear connection on a differentiable manifold. In 1930, H. A. Hayden [79] defined a semi-symmetric metric connection on a Riemannian manifold and this was further developed by K. Yano. He proved that a Riemannian manifold with respect to the
semi-symmetric metric connection has vanishing curvature tensor if and only if it is
conformally flat. This result was generalized for vanishing Ricci tensor of the semi-
symmetric metric connection by T. Imai. Various properties of such connection have
studied by many geometers. Agashe and Chafle introduced a semi-symmetric non-
metric connection on a Riemannian manifold and this was further studied by U. C.
De and D. Kamila, S. C. Biswas and U. C. De, B. B. Chaturvedi and P. N. Pandey,
De and Sengupta defined new type of semi-symmetric non-metric connections on a
Riemannian manifold and studied some geometrical properties with respect to such
connections.

The object of Chapter 2 is to study a semi-symmetric non-metric connection in
an $(e)$-Kenmotsu manifold. We also studied a semi-symmetric non-metric connec-
tion in an $(e)$-Kenmotsu manifold whose projective curvature tensor satisfies certain
curvature conditions and obtained many interesting results.

In Chapter 3, we study generalized Sasakian space forms admitting a semi-
symmetric metric connection with some properties. In this chapter, we recall some
well known basic formulas and properties of generalized Sasakian space form, we give
a brief account of semi-symmetric metric connection, we find the curvature tensor, the
Ricci tensor and the scalar curvature in generalized Sasakian space forms with respect
to the semi-symmetric metric connection. A generalized Sasakian space form $(M, g)$
whose curvature tensor is covariant constant with respect to the semi-symmetric met-
ic connection and $M$ is recurrent with respect to the Levi-Civita connection is stud-
ied. In the last of this chapter, we give the property of $\xi$-conformally flat generalized
Sasakian space forms with respect to the semi-symmetric metric connection.

In Chapter 4, we focus on the study of generalized $\varphi$-recurrency to general-
ized Sasakian-space-forms. We studied generalized $\varphi$-recurrent generalized Sasakian
space forms, generalized concircular $\varphi$-recurrent generalized Sasakian space forms
and obtained a number of results. We also studied generalized Sasakian space forms satisfying the condition $S(X, \xi).R = 0$ is reduced to $\eta$-Einstein.

In 1971, G. P. Pokhariyal and R. S. Mishra [118] defined a new curvature tensor $M$ on a Riemannian manifold known as $M$-projective Curvature Tensor and studied its relativistic significance.

The object of Chapter 5 is to study the certain results on para-Kenmotsu manifolds equipped with $M$-projective curvature tensor. Here we investigate para-Kenmotsu manifolds satisfying some curvature conditions $\tilde{M} \cdot R = 0$, $\tilde{M} \cdot Q = 0$ and $Q \cdot \tilde{M} = 0$, where $R$, $Q$ and $\tilde{M}$, respectively, denote the Riemannian curvature tensor, Ricci operator and $M$-projective curvature tensor.

Pseudo symmetric manifold $(PS)_n$ is introduced by Chaki [32], whose notion is different from that of introduced by Deszcz [63]. Pseudo symmetric spaces, generalized symmetric spaces, were also studied by Mikes [103]. The notion of Ricci soliton which is a natural generalization of an Einstein metric (i.e. the Ricci tensor $S$ is a constant multiple of $g$) was introduced by Hamilton [77] in 1982.

The Chapter 6, deals with the study of Ricci solitons on para-Sasakian manifolds satisfying pseudo-symmetry curvature conditions. First, we investigate Ricci solitons in Ricci-pseudosymmetric para-Sasakian manifolds. Next, we consider Ricci solitons in $W_3$-Ricci-pseudo-symmetric para-Sasakian manifolds. Moreover, we investigate Ricci solitons in Ricci generalized pseudo-symmetric para-Sasakian manifold. Finally, we prove that Ricci solitons in para-Sasakian manifolds satisfying the curvature condition $Q \cdot R = 0$, is expanding.

The concept of $\eta$-Ricci soliton was initiated by Cho and Kimura [39].

In Chapter 7, we study curvature properties of $\eta$-Ricci solitons on para-Kenmotsu manifolds. We obtain some results of $\eta$-Ricci solitons on para-Kenmotsu manifolds satisfying $R(\xi, X).C = 0$, $R(\xi, X).\tilde{M} = 0$, $R(\xi, X).P = 0$, $R(\xi, X).\tilde{C} = 0$ and
\[ R(\xi, X).H = 0, \text{ where } C, \tilde{M}, P, \tilde{C} \text{ and } H \text{ are quasi-conformal curvature tensor; } M-\text{projective curvature tensor; pseudo-projective curvature tensor; concircular curvature tensor and conharmonic curvature tensors.} \]

The purpose of **Chapter 8** is to study \( \eta \)-Ricci solitons on 3-dimensional Kenmotsu manifolds. First, we prove that an \( \eta \)-Ricci soliton on a 3-dimensional Kenmotsu manifold is an \( \eta \)-Einstein manifold. Besides these, we consider an \( \eta \)-Ricci solitons on 3-dimensional Kenmotsu manifolds with Ricci tensor of Codazzi type and cyclic parallel Ricci tensor. Next, we study conformally flat and \( \varphi \)-Ricci symmetric \( \eta \)-Ricci soliton on 3-dimensional Kenmotsu manifolds. Finally, we construct an example to prove the existence of an \( \eta \)-Ricci soliton on 3-dimensional Kenmotsu manifold and verify some results.

In **Chapter 9**, we study some types of an \( \eta \)-Ricci solitons on Lorentzian para-Sasakian manifolds and we give an example of an \( \eta \)-Ricci solitons on a 3-dimensional Lorentzian para-Sasakian manifold. We obtain the conditions of an \( \eta \)-Ricci soliton on \( \varphi \)-conformally flat, \( \varphi \)-conharmonically flat and \( \varphi \)-projectively flat Lorentzian para-Sasakian manifolds. The existence of an \( \eta \)-Ricci solitons implies that \( (M, g) \) is an \( \eta \)-Einstein manifold. In these cases, there is no Ricci soliton on \( M \) with the potential vector field \( \xi \).

In the end, we have given a bibliography arranged alphabetically which by no means is exhaustive on the subject. In fact only those works have been listed which have been referred in the thesis by a serial number in a square bracket.

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