CHAPTER 3

PROBLEM FORMULATION FOR VOLTAGE STABILITY ENHANCEMENT

3.1 INTRODUCTION

This chapter presents the problem formulation of the multi-objective optimisation problem taking into consideration the minimization of voltage stability index, cost of generator unit, real power loss, load voltage deviation and cost of SVC device. The equality constraints, inequality constraints and fitness function evaluation are also considered.

3.2 PROBLEM FORMULATION

Multi-objective optimization problem is formulated considering five objective functions of minimization of voltage stability index, generator cost, power loss, load voltage deviation and cost of SVC. The optimization problem is subjected to equality and inequality constraints. Power balance constraints are considered as equality constraints. Inequality constraints are considered for the real power output of generating units, generator reactive power, voltages of all PV buses, transformer tap positions, bus voltage magnitudes of all PQ buses, power flow in the transmission line and reactive power rating of SVC. Fitness value is found by satisfying all the constraints. The multi-objective optimization problem can be written mathematically as given in Equation (3.1) (Malakar et al 2010).

Minimize the objective function

$$F(x,u)=\begin{bmatrix} F_1(x,u) \\ \vdots \\ F_k(x,u) \end{bmatrix} \tag{3.1}$$

where k =1,2 …. no. of objectives

Subject to

equality constraints given in Equation (3.2)

$$g(x,u) = 0 \tag{3.2}$$

and inequality constraints given in Equation (3.3)

$$h(x,u) \leq 0 \tag{3.3}$$

x and u are vector of dependent variables and control variables respectively. For example, the dependent variables represent slack bus power, bus voltage angles, load bus voltage magnitudes etc. whereas PV bus voltage magnitude, generated power, tap position of tap changers, shunt compensators etc. are represented by the control variables. g(x,u) denotes the real and reactive power balance equations and h(x,u) represents network or components operational limits. The five objective functions are considered to find optimal location and rating of SVC.

3.2.1 Voltage Stability Index

Voltage stability is vital to electric power system. An indicator L-index is used to evaluate voltage stability at each bus of the system. The indicator value varies between 0 (no load case) and 1 (voltage collapse). L index at load bus j can be expressed as given in Equation (3.4), (Kessel and Glavitsch 1986, Sailaja Kumari et al 2007)

$$L_j=\left|L_j\right|=\left|1-\frac{\sum_{i\in\alpha_G} C_{ji}V_i}{V_j}\right| j\in\alpha_L \tag{3.4}$$

where

α_L is set of load buses

α_G is set of generator buses

V_j is complex voltage at load bus j

V_i is complex voltage at generator bus i

L_j is voltage stability index value of load bus j

C_{ji} is elements of matrix C determined by using Equation (3.5).

$$[C] = -[Y_{LL}]^{-1}[Y_{LG}] \quad (3.5)$$

$[Y_{LL}]$ and $[Y_{LG}]$ are sub matrices of Y bus matrix and they can be found using Equation (3.6).

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \quad (3.6)$$

The objective function considering minimization of voltage stability index can be represented as given in Equation (3.7).

$$F_1 = \text{Voltage Stability Index} = L_{max} \quad (3.7)$$

where $L_{max} = \max (L_j) \quad j \in \alpha_L$

3.2.2 Fuel Cost of Generator Unit

The objective of fuel cost minimization is achieved by allocating best network settings that minimizes overall fuel cost function while satisfying other network constraints. The generator cost curve is a function of generated power output. The objective function considering minimization of generation cost can be represented by the following quadratic Equation (3.8) (Metwally et al 2008, Wood et al 1996).

$$F_2 = F(P_G) = \sum_{i=1}^{nG} a_i P_{Gi}^2 + b_i P_{Gi} + c_i \qquad (3.8)$$

where

nG is the number of generators

P_{Gi} is generated power of i^{th} generator (MW)

a_i is cost coefficient of i^{th} generator (\$/MWh^2)

b_i is cost coefficient of i^{th} generator (\$/MWh)

c_i is cost coefficient of i^{th} generator

3.2.3 Real Power Loss

The objective of real power loss minimization is realized by selecting the best combination of variables, which minimizes the total real power loss of the network, simultaneously satisfying all the network constraints. Mathematically it can be expressed as given in Equation (3.9), (Benabid et al 2009).

$$F_3 = P_{loss} = \sum_{i=1}^{N_L} g_{i,j}(V_i^2 + V_j^2 - 2V_iV_j\cos(\delta_i - \delta_j)) \qquad (3.9)$$

where V_i is the voltage magnitude at bus

$g_{i,j}$ is the conductance of line i-j

δ_i is the voltage angle at bus i

N_L is the total number of transmission lines

3.2.4 Load Voltage Deviation

To have a good voltage performance, the voltage deviation at each load bus must be made as small as possible. The load voltage deviation to be minimized is given in Equation (3.10), (Abdelaziz Laifa et al 2009, Benabid et al 2009).

$$F_4 = VD = \sum_{i=1}^{nPQ}\left(\left|V_i - V_i^{ref}\right|\right)^2 \quad (3.10)$$

where

VD is voltage deviation

nPQ is number of PQ buses

V_i is the voltage magnitude at load bus i.

V_i^{ref} is the prespecified reference value of the voltage magnitude at the i^{th} load bus. V_i^{ref} is usually set to 1.0 p.u.

3.2.5 Cost of SVC

The objective function considering minimization of cost of SVC device can be represented as in Equation (3.11), (Saravanan et al 2005).

$$F_5 = C_{SVC} = 0.0003S^2 - 0.305S + 127.38 \quad (3.11)$$

where

C_{SVC} is cost of SVC in \$/VAR

S is operating range of SVC (MVAR) which can be determined using Equation (3.12)

$$S = |Q_2 - Q_1| \quad (3.12)$$

Q_1 is MVAR flow before placing FACTS device.

Q_2 is MVAR flow after placing FACTS device.

3.3 CONSTRAINTS

The objective function is subjected to equality and inequality constraints. Power balance constraints are considered as equality constraints. Inequality constraints are real power output of generating units, generator reactive power, voltages of all PV buses, transformer tap positions, bus voltage magnitudes of all PQ buses and power flow in the transmission line, reactive power rating of SVC (Malakar et al 2010).

The total power generated by the units must be equal to the sum of total load demand and total real power loss in the transmission lines. Hence the equality constraint Equations are given in (3.13) and (3.14).

$$P_{Gi} - P_{Di} - \sum_{j=1}^{N} |V_i| \left|V_j\right| \left|Y_{ji}\right| \cos\left(\delta_i - \delta_j - \theta_{ij}\right) = 0 \tag{3.13}$$

$$Q_{Gi} - Q_{Di} - \sum_{j=1}^{N} |V_i| \left|V_j\right| \left|Y_{ji}\right| \sin\left(\delta_i - \delta_j - \theta_{ij}\right) = 0 \tag{3.14}$$

where P_{Gi} is the real power generation at bus i

Q_{Gi} is the reactive power generation at bus i

P_{Di} is the real power demand at bus i

Q_{Di} is the reactive power demand at bus i

N is the total number of buses

$\theta_{i,j}$ is the angle of bus admittance element i,j

$Y_{i,j}$ is the magnitude of bus admittance element i,j

$|V_i|$ is the voltage magnitude of bus i

$\left|V_j\right|$ is the voltage magnitude of bus j

The inequality constraints are given in Equations (3.15)-(3.21).

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad \text{for } i=1,2\ldots.nPV \tag{3.15}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad \text{for } i=1,2\ldots.nPV \tag{3.16}$$

$$|V_i^{min}| \leq |V_i| \leq |V_i^{max}| \quad \text{for } i=1,2\ldots.nPV \tag{3.17}$$

$$|V_i^{min}| \leq |V_i| \leq |V_i^{max}| \quad \text{for } i=1,2\ldots.nPQ \tag{3.18}$$

$$T_i^{min} \leq T_i \leq T_i^{max} \quad \text{for } i=1,2\ldots.nT \tag{3.19}$$

$$S_{Li} < S_{Limax} \quad \text{for } i = 1,2 \ldots.NL \tag{3.20}$$

$$Q_{SVC}^{min} \leq Q_{SVC} \leq Q_{SVC}^{max} \tag{3.21}$$

where P_{Gi}^{min}, Q_{Gi}^{min}, $|V_i^{min}|$, T_i^{min} and Q_{SVC}^{min} are minimum limits of real power generation of bus i, reactive power generation of bus i, voltage magnitude of bus i, tap position of transformer i and reactive power of SVC respectively. P_{Gi}^{max}, Q_{Gi}^{max}, $|V_i^{max}|$, T_i^{max}, S_{Limax} and Q_{SVC}^{max} are maximum limitsof real power generation of bus i, reactive power generation of bus i, voltage magnitude of bus i, tap position of transformer i, power flow of the line i and reactive power of SVC respectively.

where T, S_L, nPV, nPQ, nT, NL and Q_{SVC} are the tap position, power flow in the line, number of PV buses, PQ buses, number of tap changing transformers, number of lines and reactive power (lagging or leading) injected into the bus where SVC is placed respectively.

Real power generation of bus, reactive power generation of bus, voltage magnitude of all the buses, tap position of transformers, power flow in the line, and reactive power of SVC should be maintained within their minimum and maximum limits.

3.4 FITNESS FUNCTION EVALUATION

Considering all the objective functions from Equations (3.4)-(3.12) the fitness function is expressed in Equation (3.22) (Jizhong Zhu 2009).

$$\text{Fitness function} = h_1F_1 + h_2 F_2 + h_3 F_3 + h_4 F_4 + h_5 F_5 \qquad (3.22)$$

where $h_{1,}$ h_2 $h_{3,}$ h_4 and h_5 are weighting factors of voltage stability index minimization objective function, fuel cost minimization objective function, real power loss minimization objective function, load voltage deviation minimization objective function and SVC FACTS device cost minimization objective function respectively.

The weighting factors $h_{1,}$ h_2 $h_{3,}$ h_4 and h_5 are constrained by the Equation (3.23) (Nasr Azadani et al 2009)

$$h_1+h_2+h_3+h_4+h_5=1 \qquad (3.23)$$

The optimum values of the weighting coefficients $h_{1,}$ $h_{2,}$ $h_{3,}$ h_4 and h_5 are 0.2, 0.1, 0.5, 0.1 and 0.1 obtained by satisfying Equation (3.23).

3.5 SUMMARY

This Chapter has discussed problem formulation, multi-objective functions: minimization of voltage stability index, cost of generator units, real power loss, load voltage deviation and cost of SVC device. The Chapter discusses further equality constraints, inequality constraints and fitness function evaluation.