CHAPTER V

NEUTROSOPHIC FUZZY MODELS WITH RANKING TECHNIQUE USING DISTANCE OF MULTIPLE ATTRIBUTE GROUP DECISION MAKING

Introduction:

Since in the real world there is vague information about different applications, fuzzy set theory has long been introduced to handle inexact and imprecise data (Zadeh,1965). In fuzzy set theory, each object $u \in U$ is assigned a single real value, called the grade of membership, between zero and one. (Here U is a classical set of objects, called the universe of discourse).

Section 5.1 - Previous studies and Literatures:

Gau & Buehrer(1994) pointed out that the drawback of using the single membership value in fuzzy set theory is that the evidence for $u \in U$ and the evidence against $u \in U$ are in fact mixed together. In order to tackle this problem, Gau & Buehrer(1994) proposed the notion of allowing interval-based membership instead of using point-based membership as in fuzzy sets. The interval-based membership generalization in vague sets is more expressive in capturing vagueness of data. However, vague sets are shown to be equivalent to that of Intuitionistic Fuzzy Sets (IFSs) (Bustince & Burillo,1996). For this reason, the interesting features for handling vague data that are unique to vague sets are largely ignored. We find that there are many interesting features of vague sets from a data modelling point of view.

Atanassov (1986,1989) introduced the concept of intuitionistic fuzzy sets, which is also a generalization of the concept of fuzzy set. Gau & Buehrer(1994) introduced the concept of vague set.

But Bustince & Burillo(1996) showed that vague sets are intuitionistic fuzzy sets. But later, in the literature Lu, A & Ng, W(2004, 2005) pointed out the differences between intuitionistic fuzzy sets and vague sets, and concluded in their work that, only vague sets can handle vague information better than intuitionistic fuzzy sets.

Essentially, due to the fact that a vague set corresponds to a more intuitive graphical view of data sets, it is much easier to define and visualize the relationship of vague data objects.

Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. The MCDM problems may be divided into two kinds. One is the classical MCDM problems (Hwang & Yoon,1981), among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems, among which the ratings and the weights of criteria evaluated on imprecision and vagueness are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers.

As the vague set (Bustince & Burillo,1996) took the membership degree, non-membership degree and hesitancy degree into account, and has more ability to deal with uncertain information than traditional fuzzy set, lots of scholars pay attentions to the research of vague sets. Atanassov & Gargov (1989) extended the intuition vague set and proposed the concept of interval intuition vague set, also named interval vague set. Interval vague set has more ability to express vagueness and uncertainty.

Section 5.2 - Assumptions:

Roughly speaking, a fuzzy set is a class with fuzzy boundaries. The fuzzy set A in the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, is a set of ordered pairs $\{(u_1, \mu_A(u_1)), (u_2, \mu_A(u_2)), \ldots, (u_n, \mu_A(u_n))\}$, where μ_A is the membership function of the fuzzy set A, $\mu_A : U \to [0, 1]$, and $\mu_A(u_i)$ indicates the grade of membership of u_i in A. When the universe of discourse U is a finite set, then the fuzzy set A can be represented by $A = \mu_A(u_1)/u_1 + \mu_A(u_2)/u_2 + \ldots + \mu_A(u_n)/u_n$. Gau & Buehrer (1994) pointed out that this single value combines the evidence for uiinU and the evidence against uiinU, without indicating how much there is of each. They also pointed out that the single number tells us nothing about its accuracy. Thus Gau & Buehrer (1994) presented the concepts of vague sets. They used a truth-membership function t_A and false-membership function f_A to characterize the lower bound on μ_A. These lower bounds are used to create a subinterval on $[0, 1]$, namely $[t_A(u_i), 1 - f_A(u_i)]$, to generalize the $\mu_A(u_i)$ of fuzzy sets, where $t_A(u_i) \leq \mu_A(u_i) \leq 1 - f_A(u_i)$. For example, let A be a vague set with truth-membership function t_A and false-membership function f_A, respectively. If $[t_A(u_i),\ 1 - f_A(u_i)] = [0.5, 0.8]$, then we can see that $t_A(u_i) = 0.5;\ 1 - f_A(u_i) = 0.8;\ f_A(u_i) = 0.2$. It can be interpreted as,the vote for resolution is 5 in favor, 2 against, and 3 abstentions.

A multiple attribute group decision making(MAGDM) problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In the process of MAGDM problems with vague fuzzy information, sometimes, the attribute values take the form of vague-fuzzy number. The information about attribute weights may sometimes be known or partially known or sometimes completely unknown.

MAGDM problems are assumed to have a predetermined, limited number of decision alternatives. Solving a MAGDM problem involves sorting and ranking, and can be viewed as alternative methods for combining the information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking or selection from among the alternatives.

Besides the information contained in the decision matrix, all but the simplest MAGDM techniques require additional information from the decision matrix to arrive at a final ranking or selection.

Chen &Tan (1994) presented new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. Li(2005), Szmidt & Kacprzyk (2002) and Wei,G(2010) contributed novel approaches to the field of fuzzy decision making.

Also Hong & Choi (2000) provided a new technique for handling multiple attribute fuzzy decision making problems based on vague set theory. MAGDM problems with the application of correlation coefficient of triangular intuitionistic fuzzy sets was proposed byRobinson & Amirtharaj(2011a).

By correlation analysis, the joint relationship of two variables can be examined with the aid of a measure of interdependence of the two variables. Fuzzy correlation has captured the attention of many researchers in recent days. Wei, Get al.(2011) have applied fuzzy correlation in decision making analysis. Park et al.(2009) have applied correlation coefficient of interval valued intuitionistic fuzzy sets to decision making problems.

Bustince & Burillo (1995),Zeng & Li (2007), and Robinson & Amirtharaj(2011a, 2011b, 2012a, 2012b) have contributed to the literature of Intuitionistic Fuzzy correlation analysis. Xu (2005), Xu & Yager(2006, 2008) developed some geometric aggregation operators and some arithmetic aggregation operatorsfor decision making problems.

Wei,G(2010) developed a MAGDM model for intuitionistic fuzzy sets and trapezoidal intuitionistic fuzzy sets. Shakib & Fazli (2012) have presented a MAGDM model combining Fuzzy AHP and TOPSIS. In this paper, the IVOWA operator for vague sets is proposed, and a MAGDM method is proposed based on the IVOWA and VWA operators, where the attribute values are all vague values. Firstly, according to the operation rules of the interval vague sets, the weighted calculations are done by the interval vague attribute values. Then using the correlation coefficient of the interval vague values, the closeness between each project and the ideal solutions are calculated. Then the order of the projects is confirmed according to the correlation coefficients obtained.

Group Decision Making problems can be roughly defined as decision situations where two or more decision makers or experts try to achieve a common solution to a decision problem, which consists of a set of possible solutions or alternatives (Kacprzyk, 1986, 1987).

Experts must express their individual opinion on each of the alternatives. Two different methods in addition to the proposed model are presented in this paper to support and handle some critical situations in MAGDM problems. However, in many situations a problem arises when some experts consider that their individual opinions have not been sufficiently taken into account, and therefore they disagree with the solution achieved.

This may lead to either a lack of implication in future group decision making problems or a behavior against the solution obtained. For this reason, the need for making relevant decisions under consensus is becoming increasingly common in a variety of social situations.

Consensus processes imply that experts achieve an agreement about the problem before making the decision, thus yielding a more accepted solution by the organization, society or themselves. Different consensus approaches have been proposed ranging from rigid methods to flexible approaches (Kacprzyk, 1986, 1987).In these approaches, it is crucial to establish a consensus measure to calculate the level of agreement.

Consensus measures are indicators to evaluate how far a group of experts' opinions are from unanimity. Mohanty & Bhasker (2005) have applied the concepts of Linguistic Quantifiers in the product classifications by the customer preference in Internet-Business. The RIM-Linguistic Quantifiers (Yager, 1988) based on the Ordered Weighted Averaging (OWA) operators are used to derive the weights of the experts and Gaussian Distribution based method proposed by Xu (2005) is also used for the purpose. It was organized as follows: Preliminaries of vague sets and Correlation coefficient of vague sets is presented in the next section. Various classes of Ordered Weighted Averaging (OWA) operators are presented. Two different methods of determining the weights, when the weights of the decision makers are not given, are presented. An algorithm for the proposed model of MAGDM is presented. Numerical illustration was presented, dealing in three different ways of approaching an MAGDM problem when the weights are known and when the weights are unknown. Comparison of the proposed model with other existing methods are also presented. Finally, some discussions and conclusions are made based on the models presented.

Section 5.2-Preliminaries and Definitions:

PRELIMINARIES OF VAGUE SETS

In this section, we introduce some basic concepts related to vague sets. Let U be a classical set of objects, called the universe of discourse, where an element of U is denoted by u.

Definition 5.2.1 : (Vague Set)

A vague set A in a universe of discourse U is characterized by a true membership function, t_A, and a false membership function, f_A , as follows(Chen & Tan, 1994): $t_A : U \to [0,1]$, $f_A : U \to [0,1]$, and $t_A(u) + f_A(u) \leq 1$, where $t_A(u)$ is a lower bound on the grade of membership of u derived from the evidence for u, and $f_A(u)$ is a lower bound on the grade of membership of the negation of u derived from the evidence against u.

Definition 5.2.2 :

Suppose $U = \{u_1, u_2, \ldots, u_n\}$. Let the vague set A of the universe of discourse U can be represented by $A = \sum_{i=1}^{n} [t(u_i),\ 1 - f(u_i)] / u_i$, $0 \leq t(u_i) \leq 1 - f(u_i) \leq 1$, $1 \leq i \leq n$.In other words, the grade of membership of u_i is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of $[0,1]$. Thus, vague sets are a generalization of Fuzzy sets, since the grade of membership $\mu A(u)$ of u in the above definition may be inexact in a vague set.

Basic definitions and Operations in vague sets

Let $x,\ y$ be the two vague values in the universe of discourse U, $x = [t_x, 1 - f_x],\ y = [t_y, 1 - f_y]$, where $t_x,\ f_x,\ t_y,\ f_y \in [0, 1]$ and $t_x + f_x \leq 1,\ t_y + f_y \leq 1$; the operation and relationship between vague values is defined as follows(Lu, A & Ng, W, 2005):

Definition 5.2.3:

The minimum operation of vague values x and y is defined by

$$\begin{aligned} x \wedge y &= [min(t_x,\ t_y),\ min(1 - f_x,\ 1 - f_y)] \\ &= [min(t_x,\ t_y),\ 1 - max(1 - f_x,\ 1 - f_y)] \end{aligned}$$

Definition5.2.4 :

The maximum operation of vague values x and y is defined by

$$\begin{aligned} x \vee y &= [max(t_x,\ t_y), max(1 - f_x,\ 1 - f_y)] \\ &= [max(t_x,\ t_y),\ 1 - min(1 - f_x,\ 1 - f_y)] \end{aligned}$$

Definition5.2.5:

The complement of vague value x is defined by $\bar{x} = [f_x,\ 1 - t_x]$

Let A, B be two vague sets in the universe of discourse $U = \{u_1, u_2, \ldots, u_n\}$, $A = [t_A(u_i),\ 1 - f_A(u_i)]/u_i$ and $B = [t_B(u_i),\ 1 - f_B(u_i)]/u_i$, then the operations between vague sets are defined as follows:

Definition 5.2.6:

The intersection of vague sets A and B is defined by

$A \cap B = \sum_{i=1}^{n} \{[t_A(u_i),\ 1 - f_A(u_i)] \wedge [t_B(u_i),\ 1 - f_B(u_i)]\} / u_i$.

The union of vague sets A and B is defined by

$A \cup B = \{[t_A(u_i),\ 1 - f_A(u_i)] \vee [t_B(u_i),\ 1 - f_B(u_i)]\} / u_i$. The complement of vague set A is defined by

$\bar{A} = \sum_{i=1}^{n} [f_A(u_i),\ 1 - t_A(u_i)]/u_i$.

Definition 5.2.7:

For vague value $x = [t_x,\ 1 - f_x]$, define the defuzzification function to get the precise value as follows:

$Dfzz(x) = t_x/(t_x + f_x)$.

Section 5.3-Proposed model of decision making:

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of alternatives, $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weighting vector of the attribute G_j, $j = 1, 2, \ldots, n$, where $\omega_j \in [0, 1]$, such that $\sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, \ldots, D_n\}$ be the set of decision makers, $V = (V_1, V_2, \ldots, V_n)^T$ be the weighting vector of the decision makers, with $V_k \in [0, 1]$, $\sum_{k=1}^{t} V_k = 1$.

Section 5.3.1 :The developed model of MAGDM is given as follows:

Step 1:

Let $R_i = (r)_{ij}^{(k)} = \left(T_{ij}^{(k)},\ I_{ij}^{(k)},\ F_{ij}^{(k)}\right)$ be the neutrosophic fuzzy decision matrix, where$(T)_{ij}^{(k)}$ is the degree of the truth membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k, $(I)_{ij}^{(k)}$ is the degree of the indeterminic membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $(F)_{ij}^{(k)}$ is the degree of false membership value for the alternative A_i, where $T_{ij}^{(k)},\ I_{ij}^{(k)},\ F_{ij}^{(k)}$ are in $[0,\ 1]$, $= 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, t$.

Step 2:

Fix $= 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$

$$\mu_{ij}^{(k)} = max_k \left\{T_{ij}^{(k)} :: k = 1, 2, \ldots, t\right\}.$$
$$\gamma_{ij}^{(k)} = min \left\{I_{ij}^{(k)} :: k = 1, 2, \ldots, t\right\}.$$
$$\lambda_{in}^{(k)} = min \left\{F_{ij}^{(k)} :: k = 1, 2, \ldots, t\right\}.$$

Step 3:

Assume weight $w = (0.28,\ 0.24,\ 0.20,\ 0.16,\ 0.12)$ is determined from standard normal distribution about 5 variables.

Step 4:

Find reduced matrix $R = (R_{ij}) = (T_{ij}\, I_{ij},\ F_{ij})$ from the given k decision making matrices as follows:

$$T_{ij}, = \sum_{k=1}^{t} T_{ij}^{(k)} \mu_{ij}^{(k)}; \quad I_{ij}, = \sum_{k=1}^{t} I_{ij}^{(k)} \gamma_{ij}^{(k)}; \quad F_{ij}, = \sum_{k=1}^{t} F_{ij}^{(k)} \lambda_{ij}^{(k)}$$

Step 5:

Taking $r = (1, 0, 0)$ as decision row is fixed.

Step 6:

The distance $4d(r, r_i) = \sqrt{(\frac{1}{2}\left[\prod_{j=1}^{n} T_{ij} + \prod_{j=1}^{n} (1 - I_{ij}) + \prod_{j=1}^{n} (1 - F_{ij})\right]}$ where i varies from 1 to m.

Step 7:

Select the alternative $max_i d(r, r_i)$, say s.

Step 8:

Identity the best alternative is A_s.

Section 5.4 NUMERICAL ILLUSTRATION

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money;

A_1 is an IT company,

A_2 is a multinational company,

A_3 is a tools company,

A_4 is an airlines company and

A_5 is an automobile company. The investment company must take a decision according to the four following attributes; G_1 is the risk analysis, G_2 is the growth analysis, G_3 is the political impact analysis and G_4 is the environmental impact analysis. The decision matrices of vague values $R_k = \left(\tilde{r}_y^{(k)}\right)_{5\times 4}$, $k = 1, 2, 3$ are given as:

$$R^1 = \begin{bmatrix} <0.25, 0.54, 0.8> & <0.3, 0.4, 0.9> & <0.7, 0.35, 0.5> & <0.9, 0.2, 0.8> \\ <0.6, 0.5, 0.5> & <0.6, 0.2, 0.3> & <0.2.0.4.0.9> & <0.6, 0.23, 0.7> \\ <0.3, 0.45, 0.9> & <0.7, 0.1, 0.4> & <0.6, 0.5, 0.5> & <0.4, 0.2, 0.9> \\ <0.45, 0.38, 0.27> & <0.37, 0.68, 0.16> & <0.6, 0.25, 0.3> & <0.1, 0.4, 0.8> \end{bmatrix}$$

$$R^2 = \begin{bmatrix} <0.1, 0.3, 0.7> & <0.6, 0.6, 0.5> & <0.4, 0.2, 0.1> & <0.3, 0.7, 0.6> \\ <0.3, 0.55, 0.37> & <0.75, 0.42, 0.1> & <0.32, 0.67, 0.56> & <0.35, 0.56, 0.72> \\ <0.5, 0.4, 0.32> & <0.65, 0.25, 0.32> & <0.6, 0.3, 0.1> & <0.75, 0.25, 0.55> \\ <0.27, 0.9, 0.81> & <0.31, 0.4, 0.6> & <0.75, 0.65, 0.55> & <0.3, 0.7, 0.9> \end{bmatrix}$$

$$R^3 = \begin{bmatrix} <0.32, 0.47, 0.6> & <0.9, 0.1, 0.3> & <0.6, 0.4, 0.5> & <0.3, 0.5, 0.7> \\ <0.12, 0.32, 0.52> & <0.17, 0.81, 0.9> & <0.5, 0.3, 0.1> & <0.45, 0.65, 0.27> \\ <0.50, 0.6, 0.23> & <0.56, 0.52, 0.23> & <0.3, 0.6, 0.1> & <0.57, 0.52, 0.55> \\ <0.54, 0.83, 0.72> & <0.73, 0.86, 0.61> & <0.5, 0.52, 0.4> & <0.6, 0.4, 0.2> \end{bmatrix}$$

$$R^4 = \begin{bmatrix} <0.7, 0.3, 0.1> & <0.5, 0.4, 0.4> & <0.2, 0.1, 0.6> & <0.7, 0.9, 0.6> \\ <0.3, 0.56, 0.73> & <0.57, 0.24, 0.1> & <0.23, 0.76, 0.65> & <0.53, 0.65, 0.27> \\ <0.32, 0.32, 0.6> & <0.56, 0.52, 0.32> & <0.1, 0.3, 0.9> & <0.57, 0.52, 0.55> \\ <0.72, 0.5, 0.18> & <0.13, 0.6, 0.4> & <0.55, 0.56, 0.78> & <0.7, 0.1, 0.6> \end{bmatrix}$$

$$R^5 = \begin{bmatrix} <0.52, 0.45, 0.1> & <0.57, 0.37, 0.1> & <0.76, 0.65, 0.23> & <0.57, 0.52, 0.55> \\ <0.3, 0.6, 0.7> & <0.7, 0.4, 0.1> & <0.3, 0.7, 0.6> & <0.5, 0.4, 0.6> \\ <0.2, 0.3, 0.2> & <0.6, 0.2, 0.5> & <0.1, 0.6, 0.65> & <0.3, 0.9, 0.7> \\ <0.27, 0.5, 0.81> & <0.75, 0.25, 0.32> & <0.32, 0.67, 0.56> & <0.35, 0.56, 0.72> \end{bmatrix}$$

$W = \{0.28,\ 0.24,\ 0.2,\ 0.16,\ 0.12\}$

Step 2:

NEW REDUCED MATRIX

$$R = \left[\begin{matrix} <0.0465, 0.0252, 0.0108> & <0.1043, 0.0077, 0.0102> & <0.0814, 0.0064, 0.0078> \\ <0.0418, 0.0318, 0.0395> & <0.0836, 0.0162, 0.0063> & <0.0306, 0.0323, 0.0116> \\ <0.0379, 0.0258, 0.0198> & <0.0876, 0.0060, 0.0159> & <0.0480, 0.0271, 0.0081> \\ <0.0643, 0.0478, 0.0194> & <0.0652, 0.0277, 0.0099> & <0.0862, 0.0250, 0.0293> \end{matrix} \right.$$

$$\left.\begin{array}{c} < 0.1016, 0.212, 0.0737 > \\ < 0.0584, 0.0221, 0.0291 > \\ < 0.0800, 0.0179, 0.0733 > \\ < 0.0524, 0.0089, 0.0265 > \end{array}\right]$$

Step 3: $d(r, r_1) = 0.9594 = A_1$

$d(r, r_2) = 0.9635 = A_2$

$d(r, r_3) = 0.9518 = A_3$

$d(r, r_4) = 0.9586 = A_4$

Step 4: $A_2 > A_1 > A_4 > A_3$

Step 5: A_2 is best alternative

5.5 Discussion and remarks:

Having to use crisp values is one of the problematic points in the crisp evaluation process. As some attributes are difficult to measure by crisp values, they are usually neglected during the evaluation. Another reason is about mathematical models that are based on crisp values. These methods cannot deal with decision makers' ambiguities, uncertainties and vagueness that cannot be handled by crisp values. The use of fuzzy set theory, especially vague set theory allows us to incorporate unquantifiable information, incomplete information, non-obtainable information and partially ignorant facts into the decision model.

Decisions to be made in complex contexts, characterized by the presence of multiple evaluation aspects, are normally affected by uncertainty, which is essentially from the insufficient or imprecise nature of input data as well as the subjective and evaluative preferences of the decision maker.In an age of extensive competition among organizations, managers search for efficiency and excellence in solving decision problems.

An effective group decision mechanism would enhance the quality of the group decision making process, and thus improve the performance of the organization. It proposes a group decision model which differs from the traditional ones, and considers different aspects of attribute weights, alternative priorities, and group ideal solutions to be taken into the construction. Therefore, the proposed model would result in a decision which is more realistic and acceptable for decision makers, because it includes the vague entries for the correlation coefficient which is used for ranking the preferences.

Due to the development of e-democracy and information technology, decision makers are now able to conclude a group decision without a face-to- face meeting. In such cases, decision makers only need to rank alternatives using any multiple attribute decision making technique by the aggregation of their preferences, which could be attained by the proposed model.

As a result, and without much time consumption, the problems of aggregation of preferences can be solved and the managerial operations of the organization can be enhanced. Note that the proposed model is based on the assumption that the decision makers will always honestly report their preferences. Even if they donot, the method of generating weights when the expert weights are not given explicitly can be utilized.

Moreover, varieties of methods on MAGDM have been proposed in recent years, such as problems in linguistic variables, weight elicitation and distance measurement. Further study may thus utilize some existing revised MAGDM approaches to extend the proposed model. In the proposed method, the correlation coefficient of vague sets is used as a tool for ranking the alternatives. Three different methods were proposed for the same decision making situation dealing with known weights and unknown weights of the experts and a comparison is made between the proposed models and the existing models in the literature.

Conclusion

A new approach of MAGDM is developed based on the IVOWA operator and correlation coefficient of vague sets. The IVOWA operator is introduced and utilized for aggregating the vague information. Here, both the attribute weights and the expert weights are in the form of real numbers, and the attribute values are in the form of vague numbers. Operators, namely the IVOWA operator, VOWA operator and the VWA operator were developed for aggregating the vague information. The method of correlation coefficient of vague sets was used in this MAGDM process for finding the relative closeness between the project values and the ideal solution. Linguistic Quantifiers were used for determining weights when the weights of the decision makers were completely unknown. Finally, a numerical example was presented to demonstrate all the three proposed methods, with almost same kind of rankings, giving A_4 as the best alternative (refer Figure-3). A comparison is also made between the proposed models (Methods-1, 2, 3) and other existing methods(Hung & Yang , 2004; Wang & Xin, 2005; Szmidt & Kacprzyk, 2000). From the comparison study (Table 1 and 2) it is observed that using correlation coefficient as a tool for ranking the decision alternatives plays a vital role in choosing the best alternative when compared to other methods, because correlation coefficient gives the linear relationship between the decision variables in the study. The effectiveness of the proposed methods is evident from the comparison. We shall continue to work in the extension and application of the developed method in some of the complicated linguistic fuzzy domains also.

A new approach for multiple attribute group decision making (MAGDM) problems where the attribute weights and the expert weights are real numbers and the attribute values take the form of vague values, is presented in this paper. Since families of ordered weighted averaging (OWA) operators are available in the literature, and only a few available for vague sets, the vague ordered weighted averaging (VOWA) operator and the induced vague ordered weighted averaging (IVOWA) operator are introduced in this paper and utilized for aggregating the vague information. The correlation coefficient for vague sets is used for ranking the alternatives and a new MAGDM model is developed based on the IVOWA operator and the vague weighted averaging (VWA) operator. In addition to the proposed model, two different models are proposed based on Linguistic Quantifiers for the situation when the expert weights are completely unknown. An illustrative example is given and a comparison is made between the models to demonstrate the applicability of the proposed approach of MAGDM.