Chapter 4

Near-Lossless Compression

4.1 Introduction

Image content analysis is an important research issue in computer vision because applications such as multimedia, image retrieval through Internet, TV broadcast, and storage management etc. require high speed transmission of data and higher compression ratio with high quality. To fulfill these requirements, the size of the data needs to be compressed. Usually, image compression techniques are categorized into lossy and lossless. The lossy compression techniques concentrate on high compression ratio whereas the lossless compression techniques concentrate on high quality. Both the techniques have disadvantage: The lossy does not maintain high quality while the lossless does not maintain high compression ratio. Most of the image compression techniques come under either dictionary-based schemes or statistical-based approaches. The former is computational and time consuming when compared to
the latter. The lossy compression technique is developed based on Discrete Cosine Transform (DCT) and on entropy coding while the lossless compression technique uses predictive coding technique, which also follows entropy coding. The lossless JPEG2000 or JPEG-LS is the current standard for lossless image compression. The JPEG2000 is developed based on wavelet transform, scalar or trellis coded quantizer, and arithmetic coder. In JPEG2000, the input image is transformed by wavelet transform and the transform coefficients are put into two groups. Each group consists of blocks in the wavelet domain. The transform coefficients are quantized with the help of scalar or trellis coded quantizer, and then the arithmetic coder is applied to code the coefficients.

Hewlett-Packard developed a coding scheme LOCO-I [Wein00] during the standardization process, which is the base for the current standard of the lossless compression. LOCO-I utilises the features of simple fixed context model and complex context model for capturing higher-order dependencies, which attains compression ratios similar or superior to that of state-of-the-art schemes based on arithmetic coding. Memon and Wu [Memo97] proposed a scheme, CALIC, which is presented as a competitor for the standard; it gives similar or better performance at the
increased computational complexity when compared to that of LOCO-I. These schemes are based on the same idea of predicting a pixel value on the basis of the values of adjacent pixels. The simplicity-driven schemes like JPEG standard have some limitations in their compression performance by the first order entropy of the prediction residuals, which do not achieve the total decorrelation of the data [Fede94]. In complexity-driven schemes like LOCO-I [Wein00], CALIC [Memo97], and TMW [Meye97], the compression gains are obtained by adjusting the model carefully to the image compression application. The literature reveals the significance of the gap between the complexity-driven schemes and simplicity-driven schemes. The gap between the two schemes is diminished by FELICS algorithm [Howa99]. The FELICS maintains the performance with moderate compression ratio at the expense of low computational complexity.

4.2 Overview of the Proposed Work

In this thesis, a Full Range Gaussian Markov Random Field (FRGMRF) model is proposed for image compression. The proposed compression method performs the compression process at two stages: first, the actual input image is performed using FRGMRF model; second, the residual image is performed based
on the error model discussed in Section 4.3. The proposed family of FRGMRF model is completely free from order determination as discussed in the previous Chapter. An input image \( f_i \) of size \( L \times L \) is assumed to be a Gaussian Markov Random Field (GMRF), which is segregated into various non-overlapping sub-images of equal fixed sizes of \( M \times M \) where \( M < L \). Using the Bayesian methodology, the parameters of the model are estimated for each sub-image as discussed in Chapter 2. Using the estimated parameters \( K, \alpha, \theta \) and \( \phi \), the coefficients \( a_r \) of the model are computed. Based on the model coefficients and seed values, the sub-image is generated. This process is repeated for the entire image. The reconstructed sub-images are synthesised to form the entire image, which is denoted as \( f_r \). The residual image \( f_e \) is computed from the actual input image \( f_i \) and the reconstructed image \( f_r \). With the assumption that the residual image does not have the spatial direction, the proposed FRGMRF model is redefined as an error model as discussed in Section 4.3. The error model is employed to reconstruct the residual image to maintain good quality. The parameters of the error model are estimated by employing the Metropolis-Hastings (M-H) algorithm. Many iterative algorithms, including Gibbs sampler and Expectation Maximization (EM) algorithm are shown to be special
cases of the M-H algorithm. The M-H algorithm needs less than half of the time taken by other iterative techniques [Sidh95]. Based on the estimated parameters of the error model and seed values, the compressed image is decompressed. The residual image is computed from the actual residual image and its reconstructed image. The average value of the residual is calculated. Now, the parameters, seed values and the average are stored for each sub-image. The arithmetic coding [Witt87, Mark96] is used on the images compressed by the FRGMRF model and the error model to obtain furthermore compression. The stored pixel values, average of the residuals and the coefficients of the error model are utilized to reconstruct the sub-images. The reconstructed sub-images are grouped together to form the entire image, which is denoted as $f_{er}$. Again, error image $f_{ee}$ is computed from $f_{er}$ and $f_{e}$. The average is computed on $f_{ee}$ and it is stored together with parameters values and seed values. Based on the stored values, the error image is reconstructed. Now, the reconstructed image of actual input and the residual image are added together, which results in the final reconstructed image. The final reconstructed image is almost error free. The various steps involved in the proposed work are demonstrated in the following block diagram.
The non-causal model given in equation (2.2) is redefined as a causal model, with the combination of two sets of elements as described in equation (4.2a):

$$
\Psi = \Psi_1 + \Psi_2 \quad \ldots (4.1)
$$

where, 
$$
\Psi_1 = \{p, q : 0 \leq p, q \leq 1; p \neq q\} \quad \ldots (4.1a)
$$

$$
\Psi_2 = \{p, q : -1 \leq p \leq 1; q = 1; p \neq 0\} \forall k, l \quad \ldots (4.1b)
$$

Figure 4.1. Block diagram of the proposed work
By using the above constraints in equations (4.1a) and (4.1b), the causal model given in equation (4.2) is obtained from equation (2.2).

\[ X(k,l) = \sum_{p=0}^{1} \sum_{q=0}^{1} \Gamma_r X(k-p,1-q) + \sum_{p=1}^{1} \sum_{q=0}^{1} \Gamma_r X(k-p,1-1) + \epsilon(k,l) \quad \text{...(4.2)} \]

where, \( \Gamma_r = \frac{K \sin(r \theta \cos(r \phi))}{\alpha'} \) and \( r = |p| + |q| \). \quad \text{...(4.2a)}

The model given in equation (4.2) is used to reconstruct the compressed image data at stage one.

**4.3 Error Model**

The residual image \( f_r \) is computed from the actual input image \( f_i \) and the reconstructed image \( f_s \). Since the residual image \( f_r \) has no spatial direction among the pixels, the angle \( \theta \) is set to zero in the proposed model in equation (2.2). Now, the proposed FRGMRF model becomes white noise model as in equation (4.3), because the first term in equation (2.2) becomes zero and the error term, that is, second term only remains.
where $r = |p| + |q|$, $e(k,l)$ represents error term, which follows the Markov process [Coch49] and $\Omega_s$ are the autoregression coefficients.

By applying the constraints used in equations (4.1a) and (4.1b), the model in equation (4.3) is reconstructed as stationary second-order MR(2) model as follows:

$$f_e(k,l) = \sum_{p=-M}^{M} \sum_{q=-M}^{M} \Omega_r f_e(k+p, l+q) + e(k,l) \quad \text{...(4.3)}$$

where, $\Omega_1 = \frac{\zeta_1}{2}$ and $\Omega_2 = \frac{\zeta_2}{2}$.

The Metropolis-Hastings (M-H) algorithm is employed to estimate the coefficients $\zeta_1$ and $\zeta_2$ of the error model for each sub-image. The set of coefficients $\zeta = (\zeta_1, \zeta_2)$ lying in the region $S \subset R^2$ is calculated and that satisfies the following stationarity conditions.

$$\zeta_1 + \zeta_2 < 1; \quad -\zeta_1 + \zeta_2 < 1; \quad \zeta_2 > -1.$$

The computed coefficient values $\zeta_1$ and $\zeta_2$ are substituted in equation (4.4) to obtain the decompressed image.

The following algorithm is employed to estimate the coefficients $\zeta_1$ and $\zeta_2$. 

$$\zeta_1 + \zeta_2 < 1; \quad -\zeta_1 + \zeta_2 < 1; \quad \zeta_2 > -1.$$
M-H Algorithm

Step 1: Check the stationarity conditions for the set of coefficient values \( \zeta = (\zeta_1, \zeta_2) \).

\[ \zeta_1 + \zeta_2 < 1; \quad -\zeta_1 + \zeta_2 < 1; \quad \zeta_2 > -1 \]

Step 2: A sample of draws from the posterior distribution of the parameters \( \pi(\zeta, \sigma^2 / X_n) \) can be obtained by successively sampling from \( \pi(\zeta / X_n, \sigma^2) \), where \( X_n = (x_1, x_2, \ldots, x_n) \).

Step 3: Simulate \( \sigma^2 \) from \( \pi(\sigma^2 / X_n, \zeta) \) using \( \zeta \).

Step 4: Generate candidates from the density function

\[
\int_{\text{nor}} (\zeta / \zeta, \sigma^2 G^{-1}) I(\zeta \in S)
\]

where \( \int_{\text{nor}} \) is the normal density function,

\[ G = \sum_{t=3}^{n} (w_t w_t' ) \text{ and } w_t = (x_{t-1}, x_{t-2}) \]

Step 5: At \( j \)th iteration (the current value \( \sigma^{2(j)} \)) draws a candidate \( a^{(j+1)} \) from a normal density with mean \( \hat{\zeta} \) and covariance \( \sigma^{2(j+1)} G^{-1} \).

(i) if it satisfies stationarity conditions given in Step 1,

then move to this point with probability
\[
\min \left\{ \frac{\psi(\zeta^{(j+1)}, \sigma^2(j))}{\psi(\zeta^{(j)}, \sigma^2(j))}, 1 \right\}
\]

where

\[
\psi(\zeta', \sigma^2) = (\sigma^2)^{-1/2} \left| V^{-1} \right|^{1/2} \exp \left[-\frac{1}{2\sigma^2} X_2 V^{-1} X_2' \right]
\]

(ii) Otherwise set \( \zeta^{(j+1)} = \zeta^{(j)} \).

**Step 6:** Repeat **Step: 2** to **Step: 5** until it satisfies the condition (i) of **Step: 5**.

After estimating the parameters \( \zeta_1 \) and \( \zeta_2 \), the model expressed in equation (4.4) is employed to reconstruct the compressed sub-image.

### 4.4 Measures of Performance

The performance of the proposed FRGMRF model is evaluated by means of Root Mean-Squared Error (RMSE) and Peak Signal to Noise Ratio (PSNR)

\[
RMSE = \sqrt{\frac{1}{N \times N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} [f_i(i,j) - f_r(i,j)]^2}
\]

where \( f_i \) and \( f_r \) are actual input image and final reconstructed image of size 256 \( \times \) 256 respectively.

\[
PSNR = 20 \log_{10} \left( \frac{255}{RMSE} \right)
\]
The compression ratio (CR) is calculated in percentage as follows:

$$CR = \left( 1 - \frac{\text{# bits required to store compressed image}}{\text{# bits required to store original image}} \right) \times 100$$

4.5 Experiments and Results

The proposed FRGMRF model and the error model discussed in Chapter 2 and Section 4.3 of this Chapter are experimented on different types of monochrome images for compression and decompression. For example, two standard textured images D9 (Grass) and D29 (Beach-Sand) and a structured image viz. the Barb image, each of size $256 \times 256$ with pixel values in the range 0 to 255 are given in Figures (4.2a), (4.2b) and (4.2c) respectively. The textured images are taken from standard Brodatz album [Brod66]. The input image is divided into various non-overlapping sub-images of equal size of $8 \times 8$. The parameters of the proposed FRGMRF model are estimated for each sub-image as discussed in Section 4.3. Using the estimated parameters, the coefficients of the model are computed. The coefficients and a few pixel values are stored. Based on the model coefficients and pixel values, the sub-image is generated. This process is repeated for the entire image. All the reconstructed sub-images are grouped together to form the entire image. The reconstructed images by the proposed
method, corresponding to the original images shown in Figure (4.2a), (4.2b) and (4.2c) are presented in Figures (4.2d), (4.2e) and (4.2f) respectively. The proposed image compression scheme is also compared to JPEG2000. The compressed version of the original images for JPEG2000 is obtained with a quality factor 60, by conducting the experiment with Advanced JPEG Compressor V4.8 [Win05] software, and the outputs corresponding to the images shown in Figures (4.2a), (4.2b) and (4.2c) are presented in Figure (4.2g), (4.2h) and (4.2i). The compression ratios along with RMSE and PSNR values by the proposed method and the JPEG2000 are presented in Table-4.1.

<table>
<thead>
<tr>
<th>Images</th>
<th>Proposed Method</th>
<th>JPEG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CR in %</td>
<td>RMSE</td>
</tr>
<tr>
<td>D9 (Grass)</td>
<td>97.89</td>
<td>12.0653</td>
</tr>
<tr>
<td>D29 (Beach Sand)</td>
<td>97.91</td>
<td>13.5233</td>
</tr>
<tr>
<td>Barb</td>
<td>96.78</td>
<td>16.8223</td>
</tr>
</tbody>
</table>

Table-4.1. Compression Results of FRGMRF model and the results obtained by JPEG2000 with Quality Factor 60.
Figure 4.2: (a), (b) and (c) are original images; (d), (e) and (f) are reconstructed images by the proposed model; (g), (h) and (i) are reconstructed images by JPEG with quality factor 60.
Based on the actual input image $f_i$ and the reconstructed image $f_{ir}$, the residual image $f_r$ is computed. The residual images of D9-Grass, D29-Beach-Sand and Barb images are presented in Figures (4.3a), (4.3b) and (4.3c) respectively. As discussed in Section 4.1, the sharp edges are smoothed while reconstructing the compressed images by the FRGMRF model. Smoothed means the loss of information. The information lost (residual image) is extracted by differentiating the images $f_i$ and $f_{ir}$. The error model discussed in Section 4.3 is employed on the residual image to obtain furthermore quality. For more precise estimation of the parameters of the error model, the M-H algorithm discussed in Section 4.3 is used for accurate prediction of the pixels in the residual image. The procedure used for the actual input image (FRGMRF model) is adopted for the residual image (error model). The arithmetic coding is used on image data compressed by the FRGMRF model and the error model to achieve higher compression ratio. The output of the experiment is presented in Figures (4.4d), (4.4e) and (4.4f) and the results obtained are given in Table 4.2. The outputs are obtained by adding the images generated by the FRGMRF model and error model. The final outputs are compared with the outputs obtained by JPEG2000 with the quality factor 95 and presented in Figures (4.4g), (4.4h).
and (4.4i). Conducting the experiment at various levels of quality factors and bpp, the final outputs are obtained. From the experiment, it is observed that the quality factor 95 yields good reconstruction quality at different bit rates. The optimum bpp is fixed at 0.231 for D9-Grass, 0.232 for D29-Beach-Sand and 0.235 for Barb images. The graphical representations of the outputs obtained by the experiments conducted on D9-Grass, D29-Beach-Sand and Barb images at various PSNR vs. bpp are presented in Figures (4.5a), (4.5b) and (4.5c). Next, a comparative study is carried out at different PSNR vs. Quality Factor on the same images. But, due to the space constraint, it is not possible to present all the outputs of the experiment here. The obtained results show that the proposed method requires more or less the same bpp for textured type (Grass and Beach-Sand) images while it produces less PSNR with high bpp for structured (Barb) images. This is illustrated in Figure 4.5. From the experimental results, it is observed that the proposed method gives higher compression ratio with good reconstruction quality for both textured and structured images.
Table 4.2. Final Compression Results of combined FRGMRF and ERROR model, and the results obtained by JPEG2000 with Quality Factor 95.

<table>
<thead>
<tr>
<th>Images</th>
<th>Proposed Method</th>
<th>JPEG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CR in %</td>
<td>RMSE</td>
</tr>
<tr>
<td>D9  (Grass)</td>
<td>88.69</td>
<td>0.9184</td>
</tr>
<tr>
<td>D29 (Beach Sand)</td>
<td>89.56</td>
<td>0.8324</td>
</tr>
<tr>
<td>Barb</td>
<td>87.96</td>
<td>1.4446</td>
</tr>
</tbody>
</table>

Figure 4.3 Residual Image: (a): D9-Grass; (b): D29-Beach Sand; (c): Barb
Figure 4.4. (a), (b) and (c) are original images; (d), (e) and (f) are reconstructed images by the proposed model; (g), (h) and (i) are reconstructed images by JPEG with quality factor 95.
4.6 Conclusions and Future Work

In this thesis, a family of FRGMRF model and an error model are proposed for monochrome continuous-tone still image compression. Since the residual image \( f_r \) has no spatial direction among the pixels, the angle \( \theta \) is set to zero in the proposed model in equation (2.2). Thus, it facilitates to derive the error model from the proposed FRGMRF model. The parameters of the FRGMRF model are estimated using the Bayesian approach. A few pixel values of each sub-image and the coefficients of the model are stored. Using these values, the image is reconstructed and the residual image is computed from the actual input image and the reconstructed image.
The error model is fitted on residual image to compress and decompress. The procedure used to compress the actual input image is used on residual image. The arithmetic coding is subjected to store some pixel values, average values and the coefficients of the model computed on actual input image and the residual image to obtain furthermore compression. The performance of the proposed model is measured with standard measures such as PSNR and RMSE, and the outputs are compared with the outputs obtained by JPEG2000 at different quality factors. The results obtained are comparable with the existing methods.

The proposed model can be applied in data mining for clustering, classification, prediction, and searching. In the proposed method, there is no question of model selection and order determination, as it has infinite structure with a finite number of parameters and so completely avoids the problem of order determination. The number of parameters fixed is only four and the order does not increase the computational complexity, since the estimation of these parameters is the same irrespective of the order of the model, and hence it increases the efficiency of the model. So, the proposed model is a generalized one for most time series (most images) data. Hence, the proposed model can also be extended to colour image compression by restructuring it as a three dimensional model.