CHAPTER-3

REST STATE ANALYSIS OF INTERNET TRAFFIC SHARING UNDER A MARKOV CHAIN MODEL

3.1.0 INTRODUCTION:

The increasing demand of Internet access has generated an inherent competition among the service providers, specially in terms of their quality of service (QoS). In India, besides broadband service, a majority of organizations, businessmen and individuals are using modem as a technical device to connect their Internet in the dial-up setup of Public Switched Telecommunications Network (PSTN). In market, a group of companies have owner the network services at the wide area level (called Service Providers) whereas metropolitan area networks are in the control of operators. The immense use of Internet has caused congestion in packet flow in e-network and users have to make repeated attempts before getting their call connected by an operator. Due to other technical problems, the network of an operator fails to provide connectivity even after a large number of efforts and users think of to change the operator to get an early connectivity for surfing. The growth of operators, in high proportion in the market, has generated inherent competition in attracting more and more users for enhancing their customer-base. Traffic distribution, in the network, is usually influence by the market related factors like ‘first choice’ and ‘quality of service’ offered by an operator. The first choice relates to advertising and marketing strategies initiated by service providers to attract the stuff of users. Quality of service associates with the network blocking because of inadequate number of modems,

* A part of this chapter is published in the Proceedings of 2nd National Conference on Computer Science & Information Technology, pp. 46-52, (2008). (See Appendix)
failure of switches, fault in cable connection, instant failure of servers, defects in networking articles etc. Moreover, blocking may occur due to other sources like excess traffic load, packet collision and path congestion. Operators are assumed to invest more capital to keep up a high quality network infrastructure in order to maintain uniformly an adequate low level of blocking probability. Due to competition in business, operators used to provide a variety of attractive offers to consumers which appear in the form of Internet service packages. A users with free Internet access continues to enjoy a low cost service but suffers with a high blocking probability.

The market bears a stress in the form of trade-off among ‘first choice (or initial choice)’, ‘marketing plans’ and ‘service quality’. This opens-up problems to investigate the (a) effect of large number of failed call attempts (b) interrelationship testing and (c) effect of marketing strategies over first choice and final share. Naldi (2002) has a useful contribution for the study of the effect of some of these parameters over traffic distribution among many operators. This Chapter III extends the approach of Naldi (2002) by a focus on the effect of marketing strategies over the same parameters. Strategies are in the form of Hold-On-Plan (HOP) and Pull-Back-Plan (PBP) as shown in fig. 3.1.0. The first one is to attract a tired user to relax

![Diagram of Marketing Plans](image-url)

**Fig 3.1.0 (Marketing plans)**
a bit, after a large number of failed call attempts (in the form of tea-shop, refreshment, game) whereas second is to pull-back the user towards an operator (like lottery, gift voucher, cost free offer) to resume the connecting task. These plans may be implement nearby to the place of work selected by the user. In Chapter III, it is introduced by building a simple stochastic model for user’s behavior with different categories of their attitudes in competitive market. In the beginning part of this text, analysis is performed with only two operators and later extended to the case of N operators. Contributions of Medhi (1976, 1991), Naldi (1999), Yuan and Lygeres (2005) are used as helping tools to design and perform the model based study.

3.1.1 USER’S BEHAVIOR AS A SYSTEM:

Consider following hypotheses for the behavior of user, with rest, blocking, and initial choice parameters, while sharing the traffic between the two operators.

- The competitive market has a café, containing Internet facility of operators $O_1$ and $O_2$.
- A user enters into café with initial choice (first choice) $p$ and $(1-p)$ for $O_1$ and $O_2$ respectively ($0 \leq p \leq 1$).
- The $p$ is affected by advertising, marketing, quality-of-service and past preference (or attractiveness).
- The premise (or nearby) of café has a place for human energy recharge (like, rest, entertainment, games, refreshment etc.) denoted as $R$, with probability $p_R$, which is a Hold-On-Plan (HOP) of internet marketing strategy.
- After each failed call attempt, the user has three choices: he can abandon with probability $p_A$, switch over to other operator for a new attempt or moves for a little rest (on $R$).
- From $R$, user switches to either of operators [with probability $r$ and $(1-r)$] but can not abandon. The $r$ relates to a Pull-Back-Plan (PBP) of marketing introduced by operator.
- Switching among $O_1$, $O_2$ and $R$ is on call-by-call basis depending just on the latest attempt. The physical movement of user is also an attempt.
- During the repeated calls, the blocking probability offered by $O_1$ is $L_1$ and of $O_2$ is $L_2$. The blocking implies situation when call attempt process fails to connect an operator.
3.1.2 USER'S ATTITUDE:

A user may dedicate for an operator or may opportunist which reveals attitude and categorized as:

(a) **Faithful User (FU or \( p_{FU} \)):**

Who is faithful to an operator \( O_i \) \((i = 1, 2)\) only, otherwise, prefers to take rest on \( R \) or abandon, but does not attempt for other competitive operator \( O_j \) \((i \neq j)\).

(b) **Partially Impatient User (PIU or \( p_{PIU} \)):**

Who toggles between \( O_i \) and \( O_j \), \( i \neq j = 1, 2 \), in successive call attempts, or abandons, but never moves to \( R \).

(c) **Completely Impatient User (CIU or \( p_{CIU} \)):**

Who is among \( O_1, O_2, R \) in different call attempts (or physical movements) or prefers to abandon.

The general purpose diagram in fig. 3.1.1 shows this categorization and sharing with \( p = (p_{FU})_i + (p_{PIU})_i + (p_{CIU})_i \), \((i = 1, 2)\).

![Diagram showing categorization and sharing of users among operators and rest option.](attachment:3.1.1_diagram.png)

**Fig. 3.1.1** (Total customers in market shared by \( O_1 \) and \( O_2 \))

Fig. 3.1.2 indicates first choice \( p \) is directly related to blocking probabilities offered by operators to users. It is a kind of picture based model assumed for system study.

![Diagram illustrating the relationship between prior reputation, QoS, marketing strategies, and first choice \( p \).](attachment:3.1.2_diagram.png)

**Fig. 3.1.2** (\( p \) depends on \( L_1, L_2, r, p_R \))
Similarly, fig. 3.1.3 has pictorial representation of Entity-attribute relationship among system variables and their parameters ($\overline{p}_1$, $\overline{p}_2$ are final traffic share probabilities).

3.2.0 MARKOV CHAIN MODEL:

These types models are used by Shukla et. al. (2007), Shukla and Gadewar (2007) in switch architecture analysis in computer network.

Under hypotheses of section 3.1.1, user’s behavior and attitude could be modeled by a five-state discrete-time Markov chain $\{X^{(n)}, n \geq 0\}$ such that $X^{(n)}$ stands for the state of random variable $X$ at $n^{th}$ attempt (call or movement) made by a user over state space $\{O_1,O_2,R,Z,A\}$, where

State $O_1$: First operator
State $O_2$: Second operator
State $R$: Temporary rest for a short time
State $Z$: Success (in connectivity)
State $A$: Abandon the call connectivity attempt process.

The fig. 3.2.1 explains the transition in model and fig. 3.3.1 is a transition probability matrix of order 6X6 of the model.
3.3.0 COMPUTATION OF TRANSITION PROBABILITIES BETWEEN STATES:

(i) The initial probabilities (initial choice) for user to start with from any operators

\[ P[X^{(0)} = O_1] = p \quad P[X^{(0)} = O_2] = (1 - p) \quad \ldots \ (3.3.1) \]

(ii) If in \((n-1)^{th}\) attempt, call for \(O_1\) blocked, and user abandons the process.

\[ P[X^{(n)} = A / X^{(n-1)} = O_1] = P[\text{blocked at } O_1] P[\text{abandon the process}] = L_1p_A \]

\[ \ldots (3.3.2) \]

Similar for \(O_2\),

\[ P[X^{(n)} = A / X^{(n-1)} = O_2] = L_2p_A \]

\[ \ldots (3.3.3) \]

(iii) At \(O_1\) in \(n^{th}\) attempt, call is successful only when call does not block in

\((n-1)^{th}\) and user is at \(Z\) in the next.

\[ P[X^{(n)} = Z / X^{(n-1)} = O_1] = P[\text{not blocked at } O_1] = (1-L_1) \]

\[ \ldots (3.3.4) \]

Similar for \(O_2\),

\[ P[X^{(n)} = Z / X^{(n-1)} = O_2] = (1-L_2) \]

\[ \ldots (3.3.5) \]

(iv) At \(O_1\), when call blocked in \((n-1)^{th}\) attempt, user does not want to abandon, but wants a little rest then,
\[
P\left[ X^{(n)} = R \middle| X^{(n-1)} = O_1 \right] = P[\text{blocked at } O_1] \ P[\text{not abandon}] \ P[\text{a little rest}]
\]
\[
= L_1(1-p_A)p_R
\]  \hspace{1cm} \ldots (3.3.6)

Similar to \( O_2 \), \[
P\left[ X^{(n)} = O_2 \middle| X^{(n-1)} = O_1 \right] = P[\text{blocked at } O_1] \ P[\text{not abandon}] \ P[\text{not rest}]
\]
\[
= L_1(1-p_A)(1-p_R)
\]  \hspace{1cm} - - - (3.3.8)

Similar to \( O_2 \), \[
P\left[ X^{(n)} = O_1 \middle| X^{(n-1)} = O_2 \right] = L_2(1-p_A)(1-p_R)
\]  \hspace{1cm} - - - (3.3.9)

(v) At \( O_1 \), if call is blocked in \((n-1)^{th}\) attempt, user does not want both abandon and rest, then he shifts to \( O_2 \).

\[
P\left[ X^{(n)} = O_2 \middle| X^{(n-1)} = O_1 \right] = P[\text{blocked at } O_1] \ P[\text{not abandon}] \ P[\text{not rest}]
\]
\[
= L_1(1-p_A)(1-p_R)
\]  \hspace{1cm} - - - (3.3.10)

(vi) Also, assume for, \(0 \leq r \leq 1\)

\[
P\left[ X^{(n)} = O_1 \middle| X^{(n-1)} = R \right] = r
\]
\[
P\left[ X^{(n)} = O_2 \middle| X^{(n-1)} = R \right] = 1-r
\]  \hspace{1cm} - - - (3.3.10)

(vii) The A and Z are absorbing states with unit probabilities.

\[
\begin{pmatrix}
O_1 & O_2 & R & Z & A \\
O_1 & 0 & L_1(1-p_A)(1-p_R) & L_1(1-p_A)p_R & 1-L_1 & L_1p_A \\
O_2 & L_2(1-p_A)(1-p_R) & 0 & L_2(1-p_A)p_R & 1-L_2 & L_2p_A \\
X^{(n-1)} & R & r & (1-r) & 0 & 0 & 0 \\
Z & 0 & 0 & 0 & 1 & 0 \\
A & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

\[\text{Fig 3.3.1 (Transition Probability Matrix)}\]
3.4.0 SOME RESULTS FOR $n^{th}$ ATTEMPTS:

**THEOREM 3.4.1:** If user restricts to only $O_1$ and $R$ then $n^{th}$ attempt state probabilities are:

$$P[X^{(2n)} = O_1] = pE^n; \quad P[X^{(2n+1)} = O_1] = 0$$

where $E = B_1 r, B_1 = L_1 (1 - p_A) p_R$

**PROOF:** At $n = 0$, $P[X^{(0)} = O_1] = p$ and since, the transition over $O_2$ from $O_1$ is restricted, then following (3.3.11), the start of attempt is restricted to $O_1$ only therefore, $P[X^{(0)} = R] = 0$

$$
\begin{align*}
\text{n} = 1 & \quad P[X^{(1)} = O_1] = P[X^{(0)} = R] P \left[ \frac{X^{(1)}}{X^{(0)}} = O_1 \right] = 0 \\
\text{n} = 2 & \quad P[X^{(1)} = R] = P[X^{(0)} = O_1] P \left[ \frac{X^{(1)}}{X^{(0)}} = R \right] = pB_1 \\
\text{n} = 3 & \quad P[X^{(2)} = O_1] = P[X^{(1)} = R] P \left[ \frac{X^{(2)}}{X^{(1)}} = O_1 \right] = 0 \\
\text{n} = 4 & \quad P[X^{(2)} = R] = P[X^{(1)} = O_1] P \left[ \frac{X^{(2)}}{X^{(1)}} = R \right] = pB_1^2 r \\
\text{n} = 5 & \quad P[X^{(3)} = O_1] = P[X^{(2)} = R] P \left[ \frac{X^{(3)}}{X^{(2)}} = O_1 \right] = 0 \\
\text{n} = 6 & \quad P[X^{(3)} = R] = P[X^{(2)} = O_1] P \left[ \frac{X^{(3)}}{X^{(2)}} = R \right] = pB_1^3 r^2
\end{align*}
$$

Continuation in similar way provides the proof.

**THEOREM 3.4.2:** If user restricts to only $O_2$ and $R$ then $n^{th}$ attempt state probabilities are:

$$P[X^{(2n)} = O_2] = (1 - p)D^n; \quad P[X^{(2n+1)} = O_2] = 0$$

where, $D = B_2 (1 - r), B_2 = L_2 (1 - p_A) p_R$
PROOF: $P[X^{(0)} = O_2] = (1 - p)$ and following (3.3.11), with restriction over
starting attempt to $O_2, P[X^{(0)} = R] = 0$

\[
\begin{align*}
n = 1 & \quad \begin{cases}
P[X^{(1)} = O_2] = P[X^{(0)} = R] P \left[ X^{(1)} = O_2 / X^{(0)} = R \right] = 0 \\
P[X^{(1)} = R] = P[X^{(0)} = O_2] P \left[ X^{(1)} = R / X^{(0)} = O_2 \right] = (1 - p) B_2 \\
\end{cases} \\

n = 2 & \quad \begin{cases}
P[X^{(2)} = O_2] = P[X^{(1)} = R] P \left[ X^{(2)} = O_2 / X^{(1)} = R \right] = (1 - p) B_2 (1 - r) \\
P[X^{(2)} = R] = P[X^{(1)} = O_2] P \left[ X^{(2)} = R / X^{(1)} = O_2 \right] = 0 \\
\end{cases} \\

n = 3 & \quad \begin{cases}
P[X^{(3)} = O_2] = P[X^{(2)} = R] P \left[ X^{(3)} = O_2 / X^{(2)} = R \right] = (1 - p) B_2^2 (1 - r) \\
P[X^{(3)} = R] = P[X^{(2)} = O_2] P \left[ X^{(3)} = R / X^{(2)} = O_2 \right] = 0 \\
\end{cases}
\end{align*}
\]

For $n^{th}$, one can get required result with similar continuation of steps.

THEOREM 3.4.3: If user restricts to only between $O_1$ and $O_2$, not interested for
$R$ then $P[X^{(2n)} = O_1] = pC^n$ ; $P[X^{(2n+1)} = O_1] = (1 - p) A_2 C^n$

\[
\begin{align*}
P[X^{(2n)} = O_1] = (1 - p) C^n & \quad ; \quad P[X^{(2n+1)} = O_2] = p A_1 C^n \\
\end{align*}
\]

where $C = A_1 A_2$ , $A_1 = L_4 (1 - p_A) (1 - p_b)$ , $A_2 = L_2 (1 - p_A) (1 - p_b)$

PROOF: At $n = 0$, $P[X^{(0)} = O_1] = p$, $P[X^{(0)} = O_2] = (1 - p)$. Following (3.3.11),
the start may either from $O_1$ or $O_2$,

\[
\begin{align*}
n = 1 & \quad \begin{cases}
P[X^{(1)} = O_1] = P[X^{(0)} = O_2] P \left[ X^{(1)} = O_1 / X^{(0)} = O_2 \right] = (1 - p) A_2 \\
P[X^{(1)} = O_2] = P[X^{(0)} = O_1] P \left[ X^{(1)} = O_2 / X^{(0)} = O_1 \right] = p A_1 \\
\end{cases} \\

n = 2 & \quad \begin{cases}
P[X^{(2)} = O_1] = P[X^{(1)} = O_2] P \left[ X^{(2)} = O_1 / X^{(1)} = O_2 \right] = p A_1 A_2 \\
P[X^{(2)} = O_2] = P[X^{(1)} = O_1] P \left[ X^{(2)} = O_2 / X^{(1)} = O_1 \right] = (1 - p) A_1 A_2 \\
\end{cases} \\

n = 3 & \quad \begin{cases}
P[X^{(3)} = O_1] = P[X^{(2)} = O_2] P \left[ X^{(3)} = O_1 / X^{(2)} = O_2 \right] = (1 - p) A_1 A_2^2 \\
P[X^{(3)} = O_2] = P[X^{(2)} = O_1] P \left[ X^{(3)} = O_2 / X^{(2)} = O_1 \right] = p A_1^2 A_2 \\
\end{cases}
\end{align*}
\]

68
The continuation of above process provides the proof of theorem.

**THEOREM 3.4.4:** If call attempt is among \( O_1, O_2 \) and \( R \) only then for \( n^{th} \) state probability the approximate expressions are:

\[
\begin{align*}
P[X^{(2n)} = O_1] &= p(C + E)^n \\
P[X^{(2n+1)} = O_1] &= (1 - p)A_2(C + D + E)^n \\
P[X^{(2n)} = O_2] &= (1 - p)(C + D)^n \\
P[X^{(2n+1)} = O_2] &= pA_1(C + D + E)^n
\end{align*}
\]

**PROOF:** \( P[X^{(0)} = O_1] = p, P[X^{(0)} = O_2] = (1 - p), P[X^{(0)} = R] = P[X^{(0)} = R] = 0 \) (when from \( O_i \) to \( R \)) and using (3.3.11) among \( O_1, O_2, R \)

\[
\begin{align*}
P[X^{(1)} = O_1] &= P[X^{(0)} = O_2]P[X^{(1)} = O_1/X^{(0)} = O_2] + P[X^{(0)} = R]P[X^{(1)} = O_1/X^{(0)} = R] \\
&= (1 - p)A_2 \\
P[X^{(1)} = O_2] &= P[X^{(0)} = O_1]P[X^{(1)} = O_2/X^{(0)} = O_1] + P[X^{(0)} = R]P[X^{(1)} = O_2/X^{(0)} = R] \\
&= pA_1 \\
P[X^{(1)} = R] = P[X^{(0)} = O_1]P[X^{(1)} = R/X^{(0)} = O_1] = pB_1 \\
P[X^{(1)} = R] = P[X^{(0)} = O_2]P[X^{(1)} = R/X^{(0)} = O_2] = (1 - p)B_2
\end{align*}
\]
\[
\begin{align*}
\text{for } n = 2
\end{align*}
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(2)} = O_2] = \mathbb{P}[X^{(1)} = O_2] \mathbb{P}\left[ X^{(2)} = O_2 / X^{(1)} = O_2 \right] + \mathbb{P}[X^{(1)} = R] \mathbb{P}\left[ X^{(2)} = O_2 / X^{(1)} = R \right] \\
\quad = pA_1A_2 + pB_2 r = p(C + E) \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(2)} = O_2] = \mathbb{P}[X^{(1)} = O_2] \mathbb{P}\left[ X^{(2)} = O_2 / X^{(1)} = O_2 \right] + \mathbb{P}[X^{(1)} = R] \mathbb{P}\left[ X^{(2)} = O_2 / X^{(1)} = R \right] \\
\quad = (1 - p)A_2(AA_1 + B_2) (1 - r) = (1 - p)(C + D) \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(2)} = R|O_1] = \mathbb{P}[X^{(1)} = O_1] \mathbb{P}\left[ X^{(2)} = R / X^{(1)} = O_1 \right] + \mathbb{P}[X^{(1)} = R] \mathbb{P}\left[ X^{(2)} = R / X^{(1)} = R \right] \\
\quad = (1 - p)A_2(AA_2 + B_2) r = pA_1 A_2 B_2 + pB_2 r \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(2)} = R|O_2] = \mathbb{P}[X^{(1)} = O_2] \mathbb{P}\left[ X^{(2)} = R / X^{(1)} = O_2 \right] \\
\quad = (1 - p)A_2(AA_2 + B_2)(1 - r) \\
\end{array} \right.
\]

\[
\begin{align*}
\text{for } n = 3
\end{align*}
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(3)} = O_1] = \mathbb{P}[X^{(2)} = O_2] \mathbb{P}\left[ X^{(3)} = O_2 / X^{(2)} = O_2 \right] + \mathbb{P}[X^{(2)} = R] \mathbb{P}\left[ X^{(3)} = O_2 / X^{(2)} = R \right] \\
\quad = (1 - p)A_3[A_1A_2 + B_3] (1 - r) = (1 - p)(C + D + E) \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(3)} = O_2] = \mathbb{P}[X^{(2)} = O_2] \mathbb{P}\left[ X^{(3)} = O_2 / X^{(2)} = O_2 \right] + \mathbb{P}[X^{(2)} = R] \mathbb{P}\left[ X^{(3)} = O_2 / X^{(2)} = R \right] \\
\quad = pA_1[A_2 + B_3] r = pA_1 A_2 B_3 + pB_3 r \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(3)} = R|O_1] = \mathbb{P}[X^{(2)} = O_1] \mathbb{P}\left[ X^{(3)} = R / X^{(2)} = O_1 \right] \\
\quad = (1 - p)A_2(AA_2 + B_3)(1 - r) \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(3)} = R|O_2] = \mathbb{P}[X^{(2)} = O_2] \mathbb{P}\left[ X^{(3)} = R / X^{(2)} = O_2 \right] \\
\quad = (1 - p)A_3[A_2B_2](1 - r) \\
\end{array} \right.
\]

\[
\begin{align*}
\text{for } n = 4
\end{align*}
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(4)} = O_1] = \mathbb{P}[X^{(3)} = O_2] \mathbb{P}\left[ X^{(4)} = O_2 / X^{(3)} = O_2 \right] + \mathbb{P}[X^{(3)} = R] \mathbb{P}\left[ X^{(4)} = O_2 / X^{(3)} = R \right] \\
\quad = p[A_1^2A_2^2 + A_1A_2B_2(1 - r) + AA_2B_2 r + AA_2B_2 r + B_3 r^2] \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(4)} = O_2] = \mathbb{P}[X^{(3)} = O_2] \mathbb{P}\left[ X^{(4)} = O_2 / X^{(3)} = O_2 \right] + \mathbb{P}[X^{(3)} = R] \mathbb{P}\left[ X^{(4)} = O_2 / X^{(3)} = R \right] \\
\quad = p(C^2 + E^2 + 2CE + CD) = p[(C + E)^2 + CD] \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(4)} = R|O_1] = \mathbb{P}[X^{(3)} = O_1] \mathbb{P}\left[ X^{(4)} = R / X^{(3)} = O_1 \right] \\
\quad = p[A_1^2A_2^2 + A_1A_2B_2(1 - r) + AA_2B_2 r + AA_2B_2 (1 - r) + B_3 r^2] \\
\end{array} \right.
\]

\[
\begin{align*}
\left| \begin{array}{c}
\mathbb{P}[X^{(4)} = R|O_2] = \mathbb{P}[X^{(3)} = O_2] \mathbb{P}\left[ X^{(4)} = R / X^{(3)} = O_2 \right] \\
\quad = p(C^2 + D^2 + 2CD + CE) = p[(C + D)^2 + CE] \\
\end{array} \right.
\]

Some expressions for \( n > 4 \) are given in table 3.4.1.
The terms in expressions for \( n > 4 \), contain high degree powers on \( C, D \) and \( E \) which are negligible when \( n \) is large. The table 3.4.2 shows a numerical justification in support to this fact.
TABLE 3.4.2 (Value of neglecting terms)

<table>
<thead>
<tr>
<th>Attempt</th>
<th>Term</th>
<th>Neglecting Expression</th>
<th>Numerical Value</th>
<th>Term</th>
<th>Neglecting Expression</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>n =4</td>
<td>$[X^{(4)} = O_1]$</td>
<td>$CD$</td>
<td>0.0009922</td>
<td>$[X^{(4)} = O_2]$</td>
<td>$CE$</td>
<td>0.0004961</td>
</tr>
<tr>
<td>n =5</td>
<td>$[X^{(5)} = O_1]$</td>
<td>$-DE$</td>
<td>-0.0010125</td>
<td>$[X^{(5)} = O_2]$</td>
<td>$-DE$</td>
<td>-0.0010125</td>
</tr>
<tr>
<td>n =6</td>
<td>$[X^{(6)} = O_1]$</td>
<td>$2CD(C+E) + CD^2$</td>
<td>0.000133</td>
<td>$[X^{(6)} = O_2]$</td>
<td>$2CE(C+D) + CE^2$</td>
<td>0.0000776</td>
</tr>
<tr>
<td>n =7</td>
<td>$[X^{(7)} = O_1]$</td>
<td>$-2DE (C+D+E)$</td>
<td>-0.000204</td>
<td>$[X^{(7)} = O_2]$</td>
<td>$-2DE (C+D+E)$</td>
<td>-0.000204</td>
</tr>
<tr>
<td>n =8</td>
<td>$[X^{(8)} = O_1]$</td>
<td>$3CD (C+E)^2 + 2CD (C+E) + CD^2 (C+D)$</td>
<td>0.0000121</td>
<td>$[X^{(8)} = O_2]$</td>
<td>$3CE(C+D)^2 + 2CE (C+D) + CE^2(C+D)$</td>
<td>0.0000735</td>
</tr>
<tr>
<td>n =9</td>
<td>$[X^{(9)} = O_1]$</td>
<td>$3DE (C+D+E)^2 - 2C^2DE - D^2E^2$</td>
<td>-0.0000257</td>
<td>$[X^{(9)} = O_2]$</td>
<td>$3DE (C+D+E)^2 - 2C^2DE - D^2E^2$</td>
<td>-0.0000257</td>
</tr>
<tr>
<td>n =10</td>
<td>$[X^{(10)} = O_1]$</td>
<td>$4CD(C+E)^3 + 3CD^2 (C+E)^2 + 3C^2D^2 (C+E) + 2CD^3 (C+E) + CD^2(C+D) + C^2D^3$</td>
<td>0.0000008</td>
<td>$[X^{(10)} = O_2]$</td>
<td>$4CE(C+D)^3 + 3CE^2 (C+D)^2 + 3C^2E^2 (C+D) + 2CE^3 (C+D) + CE^2(C+D)$</td>
<td>0.0000006</td>
</tr>
</tbody>
</table>

By table 3.4.1 and 3.4.2, after neglecting terms when $n$ is large, one can obtain the core part of general expression as given in the theorem 3.4.4.
3.5.0 \hspace{1cm} \textbf{AVERAGE BLOCKING PROBABILITY EXPERIENCE BY USERS:}

The user experiences varying average blocking probability, at \( n^{\text{th}} \) attempt, described as:

\[
B_i^{(n)} = \frac{P[X^{(n-1)} = O_1]L_1 + P[X^{(n-1)} = O_2]L_2}{P[X^{(n-1)} = O_1] + P[X^{(n-1)} = O_2]}
\]  
[See Naldi (2002)]

In case of faithful user, by using theorem 3.4.1 and 3.4.2.

\[
\begin{align*}
[B_i^{(n)}]_{FU} &= \frac{pE^{(n-1)}L_1 + (1-p)D^{(n-1)}L_2}{pE^{(n-1)} + (1-p)D^{(n-1)}} \quad \text{when } n \text{ even,} \\
[B_i^{(n)}]_{FU} &= 0 \quad \text{when } n \text{ odd}
\end{align*}
\]  
\[-\ -\ (3.5.1)\]

For Partially Impatient User (PIU), using theorem 3.4.3

\[
\begin{align*}
[B_i^{(n)}]_{PIU} &= \frac{(1-p)A_2 L_1 + pA_1 L_2}{(1-p)A_2 + pA_1} \quad \text{for odd } n
\end{align*}
\]  
\[-\ -\ (3.5.2)\]

For Completely Impatient Users (CIU), by theorem 3.4.4

\[
\begin{align*}
[B_i^{(n)}]_{CIU} &= \frac{p(C+E)^n L_1 + (1-p)(C+D)^n L_2}{p(C+E)^n + (1-p)(C+D)^n} \quad \text{for even } n \\
[B_i^{(n)}]_{CIU} &= \frac{(1-p)A_2 L_1 + pA_1 L_2}{(1-p)A_2 + pA_1} \quad \text{for odd } n
\end{align*}
\]  
\[-\ -\ (3.5.3)\]

3.6.0 \hspace{1cm} \textbf{COMPARISONS AMONG USERS:}

(a) \([B_i^{(n)}]_{PIU} < [B_i^{(n)}]_{FU}\) when \( p < 1 \) along with \( L_1 > L_2 \)

(b) \([B_i^{(n)}]_{CIU} < [B_i^{(n)}]_{FU}\) when \( p < 1 \) along with \( L_1 > L_2 \)  
\[-\ -\ (3.6.1)\]

(c) \([B_i^{(n)}]_{CIU} < [B_i^{(n)}]_{PIU}\) when \( p < 1 \) along with \( L_1 > L_2 \)
3.7.0 TRAFFIC SHARE AND CALL CONNECTION:

The traffic is shared between O₁ and O₂ and call connection is made in \( n \)th attempt with \( O_i \) (\( i = 1, 2 \)).

\[
\overline{P}_i^{(n)} = P[\text{call completes in } n^{th} \text{ attempt with operator } O_i] \\
= P[ (n-1)^{th} \text{ attempt is on } O_i ] P[ n^{th} \text{ attempt on } Z \text{ when was at } O_i \text{ in } (n-1)^{th} ] \\
= P[X^{(n-1)} = O_i] \frac{X^{(n)}}{X^{(n-1)} = O_i} = P[X^{(n-1)} = O_i] (1 - L_i) \\
= \left\{ \sum_{i=0}^{n-1} P[X^{(i)} = O_i] \right\} (1 - L_i) = (1 - L_i) \left[ \sum_{i=\text{even}}^{n-1} P[X^{(i)} = O_i] + \sum_{i=\text{odd}}^{n-1} P[X^{(i)} = O_i] \right] \quad - (3.7.1)
\]

The probabilities \( \overline{P}_i \) (\( i = 1, 2 \)) reflect the proportion by which the overall traffic offered by a user is distributed between the two operators. At the same time they tell us how the actual traffic share of the two operators modified with respect to the initial choices \( p \) and \( (1-p) \) because of the blocking probability offered in QoS. If user is FU, PIU or CIU then this division differs as per attitude categorization.

**CASE I: BY FU:**

\[
\overline{R}_1^{(2n)} \bigg|_{FU} = (1 - L_1) \left( 1 - \frac{E^n}{1 - E} \right) \\
\overline{R}_1^{(2n+1)} \bigg|_{FU} = (1 - L_1) \left( 1 - \frac{E^n + 1}{1 - E} \right) \quad - (3.7.2)
\]

Using expression (3.7.1) and theorem 3.4.1, 3.4.2, for \( O_1 \),

\[
\overline{R}_2^{(2n)} \bigg|_{FU} = (1 - L_2) \left( 1 - \frac{D^n}{1 - D} \right) \\
\overline{R}_2^{(2n+1)} \bigg|_{FU} = (1 - L_2) \left( 1 - \frac{D^n + 1}{1 - D} \right) \quad - (3.7.3)
\]

Similar for \( O_2 \):
CASE II: BY PIU:

\[
\left[ \overline{P}_1^{(n)} \right]_{PIU} = (1 - L_1) \left[ \sum_{i=0}^{n-1} P[X^{(i)} = O_1]_{PIU} + \sum_{i=0}^{n-1} P[X^{(i)} = O_1]_{PIU} \right]
\]

Using (3.7.1) and theorem 3.4.3, for O₁,

\[
\left[ \overline{P}_1^{(2n)} \right]_{PIU} = (1 - L_1) \left[ p + (1 - p)A_2 \frac{1-C^n}{1-C} \right]
\]

\[
\left[ \overline{P}_1^{(2n+1)} \right]_{PIU} = (1 - L_1) \left[ \frac{p(1-C^n+1) + (1 - p)A_2(1-C^n)}{1-C} \right] \quad -- (3.7.4)
\]

For O₂:

\[
\left[ \overline{P}_2^{(2n)} \right]_{PIU} = (1 - L_2) \left[ (1 - p) + pA_1 \frac{1-C^n}{1-C} \right]
\]

\[
\left[ \overline{P}_2^{(2n+1)} \right]_{PIU} = (1 - L_2) \left[ \frac{(1 - p)(1-C^n+1) + pA_1(1-C^n)}{1-C} \right] \quad -- (3.7.5)
\]

CASE III: BY CIU:

\[
\left[ \overline{P}_1^{(n)} \right]_{CIU} = (1 - L_1) \left[ \sum_{i=0}^{n-1} P[X^{(i)} = O_1]_{CIU} + \sum_{i=0}^{n-1} P[X^{(i)} = O_1]_{CIU} \right]
\]

Using (3.7.1) and theorem 3.4.4, for O₁,

\[
\left[ \overline{P}_1^{(2n)} \right]_{CIU} = (1 - L_1) \left[ p\left\{ \frac{1-(C+E)^n}{1-(C+E)} \right\} + (1 - p)A_2 \left\{ \frac{1-(C+D+E)^n}{1-(C+D+E)} \right\} \right]
\]

\[
\left[ \overline{P}_1^{(2n+1)} \right]_{CIU} = (1 - L_1) \left[ p\left\{ \frac{1-(C+E)^{n+1}}{1-(C+E)} \right\} + (1 - p)A_2 \left\{ \frac{1-(C+D+E)^{n+1}}{1-(C+D+E)} \right\} \right] \quad -- (3.7.6)
\]
For $O_2$:

$$\left[ P_2^{(2n)} \right]_{CIU} = (1 - L_2) \left[ (1 - p) \left\{ \frac{1-(C+D)^n}{1-(C+D)} \right\} + pA_1 \left\{ \frac{1-(C+D+E)^n}{1-(C+D+E)} \right\} \right] $$

$$\left[ P_2^{(2n+1)} \right]_{CIU} = (1 - L_2) \left[ (1 - p) \left\{ \frac{1-(C+D)^{n+1}}{1-(C+D)} \right\} + pA_1 \left\{ \frac{1-(C+D+E)^{n+1}}{1-(C+D+E)} \right\} \right] $$

- (3.7.7)

3.8.0 **TRAFFIC SHARE OVER LARGE NUMBER OF ATTEMPTS:**

Suppose $n$ is large, then $\bar{P}_i = \left[ \lim_{n \to \infty} P_i^{(n)} \right]$, $i = 1, 2$ and

$$\left[ \bar{P}_1^{(n)} \right]_{FU} = \frac{(1-L_4)p}{1-E} ; \quad \left[ \bar{P}_2^{(n)} \right]_{FU} = \frac{(1-L_2)(1-p)}{1-D} $$

- (3.8.1)

$$\left[ \bar{P}_1^{(n)} \right]_{PIU} = (1-L_4) \left[ \frac{p + (1-p)A_2}{1-C} \right] ; \quad \left[ \bar{P}_2^{(n)} \right]_{PIU} = (1-L_2) \left[ \frac{(1-p) + pA_1}{1-C} \right] $$

- (3.8.2)

$$\left[ \bar{P}_1^{(n)} \right]_{CIU} = (1-L_4) \left[ \frac{p}{1-(C+E)} + \frac{(1-p)A_2}{1-(C+D+E)} \right] $$

$$\left[ \bar{P}_2^{(n)} \right]_{CIU} = (1-L_2) \left[ \frac{1-p}{1-(C+D)} + \frac{pA_1}{1-(C+D+E)} \right] $$

- (3.8.3)

3.9.0 **COMPARISON:**

For $O_1$:

$$[\bar{P}_1]_{FU} < [\bar{P}_1]_{PIU} \quad \text{if} \quad p < \frac{1}{1 + \frac{1}{A_2} \left\{ \frac{E-C}{1-E} \right\}} $$

- (3.9.1)

$$[\bar{P}_1]_{FU} < [\bar{P}_1]_{CIU} \quad \text{if} \quad p < \frac{1}{1 - \frac{C}{A_2} \left\{ \frac{1}{1-E} \right\} \left\{ \frac{D}{1-C-E} \right\}} $$

- (3.9.2)

$$[\bar{P}_1]_{PIU} < [\bar{P}_1]_{CIU} \quad \text{if} \quad p < \frac{1}{1 + \frac{E}{A_2} \left\{ \frac{1}{D+E} \right\} \left\{ \frac{D}{1-C-E} \right\}} $$

- (3.9.3)
Similarly for O₂:

\[ \frac{P_2}{FU} < \frac{P_2}{PIU} \quad \text{if} \quad p < \frac{1}{1 + \frac{A_1(D-1)}{C-D}} \]  
- - - (3.9.4)

\[ \frac{P_2}{FU} < \frac{P_2}{P_{I,U}} \quad \text{if} \quad p < \frac{1}{1 - \frac{A_1(1-D)}{C} \left\{1 + \frac{E}{1 - C - D - E}\right\}} \]  
- - - (3.9.5)

\[ \frac{P_2}{PIU} < \frac{P_2}{P_{I,U}} \quad \text{if} \quad p < \frac{1}{1 - \frac{A_1(D+E)}{D} \left\{1 + \frac{E}{1 - C - D - E}\right\}} \]  
- - - (3.9.6)

### 3.10.0 SIMULATION:

While comparing the FU and PIU using (3.9.1), relevant to fig. 3.10.1 and 3.10.2 that with the increment in blocking probability \( L_1 \) for O₁ a constant loss appears in initial share of faithful users. But, if the blocking probability of competitor O₂ also increases, a gain happens for O₁ in terms of \( p \). As special, from fig. 3.10.1

![Graph showing comparison between FU and PIU]

Fig. 3.10.1 ( \( p_A = 0.7 \), \( p_R = 0.6 \), \( r = 0.3 \) )  
Fig. 3.10.2 ( \( p_A = 0.7 \), \( p_R = 0.6 \), \( r = 0.3 \) )  

(Between FU and PIU over varying \( L_1 \) and \( L_2 \))

when \( L_1 = L_2 = 0.5 \) then \( 0.5 \leq p \leq 0.6 \) which indicates that 50% call blocking for both leads to nearly 50% the initial share.

Define \( \Delta L_1 = (L_1)_{a+h} - (L_1)_a \), \( \Delta L_2 = (L_2)_{a+h} - (L_2)_a \), where \( a \) is initial value and \( h \) is interval of differencing, and define \( (BD)_p = \text{Blocking Derivative} = \left( \frac{\Delta L_1}{\Delta L_1} \right)_{p=\text{constant}} \)

From fig. 3.10.1, when \( p = 0.8 \), one can find that \( (BD)_{p=0.8} \approx 0.3 \), But in fig. 3.10.2, when \( p = 0.8 \), \( (BD)_{p=0.8} = 2.0 \). It shows the rate of variation of \( L_2 \) with respect to \( L_1 \)
reduces with the increase of $L_1$ and $L_2$ for a fixed $p$. The competitor’s blocking probability has a strong impact for improving the proportion of faithful users of operator $O_1$ after a large number of attempts.

Comparison between FU and CIU as per condition (3.9.2), shown in from fig. 3.10.3, 3.10.4 and 3.10.5. The graphical pattern shows that, $[\overline{P}_1]_{FU} < [\overline{P}_1]_{CIU}$ when $p < 1$ for $O_1$ which is always true. It means $p$ does not play any role in between FU and CIU. The completely impatient users will always have larger initial traffic share than FU irrespective of whatever may the values of $L_2$, $r$ and $p_R$ after large attempts. To note that even if probability $r$ decreases, the initial traffic share of faithful user remain unchanged. Both the marketing plans HOP and PBP are not effective to alter the initial shares.

![Graphical pattern showing](image)

Fig. 3.10.3 ( $p_A = 0.7, p_R = 0.6, r = 0.3$ )

Fig. 3.10.4 ( $p_A = 0.7, p_R = 0.6, L_2 = 0.5$ )

Fig. 3.10.5 ( $p_A = 0.7, L_2 = 0.6, r = 0.3$ )

(Between FU and CIU over varying $L_2$, $r$, $p_R$)

While PIU and CIU using (3.9.3), the proportion of partially impatient users is lesser than the completely impatient users when opponent’s blocking probability is small. But increases gradually as the $L_2$ increases. This pattern is observed in fig. 3.10.6 – 3.10.7. The increment in user’s proportion leads to a gain in business in terms of higher initial selection probability. As special, if $L_1 = 0.3$, $L_2 = 0.3$, then after large attempts the initial traffic share of $O_1$ is nearly 10%. When $L_1 = 0.05$, $L_2 = 0.3$, 

78
the share is 20% along with high proportion of PIU. By fig 3.10.6, table 3.10.0 is created using Lower Zone blocking (0 < L₂ ≤ 0.5), with p = 0.2, one can calculate (BD)ₚ=0.2 ≈ 1.66, But in fig. 3.10.7, when p = 0.2, (BD)ₚ=0.2 = 0.80 and when p = 0.4, (BD)ₚ=0.4 = 3.33

TABLE 3.10.0 : Using fig. 3.10.6 Blocking Derivative Computation
(when L₂ in Lower Zone , 0 < L₂ ≤ 0.5 )

<table>
<thead>
<tr>
<th>p</th>
<th>(L₂)ₐ</th>
<th>(L₂)ₐ+h</th>
<th>ΔL₂</th>
<th>(L₁)ₐ</th>
<th>(L₁)ₐ+h</th>
<th>ΔL₁</th>
<th>(BD)ₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.30</td>
<td>0.40</td>
<td>0.10</td>
<td>0.27</td>
<td>0.47</td>
<td>0.20</td>
<td>0.50</td>
</tr>
<tr>
<td>0.2</td>
<td>0.30</td>
<td>0.50</td>
<td>0.20</td>
<td>0.03</td>
<td>0.16</td>
<td>0.12</td>
<td>1.66</td>
</tr>
<tr>
<td>0.3</td>
<td>0.40</td>
<td>0.50</td>
<td>0.10</td>
<td>0.03</td>
<td>0.07</td>
<td>0.03</td>
<td>2.63</td>
</tr>
</tbody>
</table>

TABLE 3.10.1 : Using fig. 3.10.7 Blocking Derivative Computation
(when L₂ in Higher Zone , 0.5 < L₂ < 1 )

<table>
<thead>
<tr>
<th>p</th>
<th>(L₂)ₐ</th>
<th>(L₂)ₐ+h</th>
<th>ΔL₂</th>
<th>(L₁)ₐ</th>
<th>(L₁)ₐ+h</th>
<th>ΔL₁</th>
<th>(BD)ₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.60</td>
<td>0.80</td>
<td>0.20</td>
<td>0.22</td>
<td>0.47</td>
<td>0.25</td>
<td>0.80</td>
</tr>
<tr>
<td>0.3</td>
<td>0.60</td>
<td>0.80</td>
<td>0.20</td>
<td>0.12</td>
<td>0.27</td>
<td>0.15</td>
<td>1.33</td>
</tr>
<tr>
<td>0.4</td>
<td>0.60</td>
<td>0.80</td>
<td>0.20</td>
<td>0.06</td>
<td>0.12</td>
<td>0.06</td>
<td>3.33</td>
</tr>
<tr>
<td>0.5</td>
<td>0.60</td>
<td>0.80</td>
<td>0.20</td>
<td>0.02</td>
<td>0.07</td>
<td>0.05</td>
<td>4.00</td>
</tr>
</tbody>
</table>
It is observed from Table 3.10.0, that

(a) Blocking derivative is directly proportional to the blocking value \( L_1 \) of operator \( O_1 \) and could be expressed in a differential equation

\[
\frac{\partial L_2}{\partial L_1} = s L_1,
\]

where \( s \) is a constant.

(b) Blocking derivative are also directly proportional to the initial share (\( p \)) and could be expressed

\[
\frac{\partial L_2}{\partial L_1} = tp,
\]

with \( t \) a constant.

(c) When opponents blocking probability is in Lower Zone (\( 0 < L_2 \leq 0.5 \)) then in order to maintain the same 20% level of initial traffic share \( p \), the operator \( O_1 \) has to reduce his call blocking by \( 1/1.66 = 0.6 \) for every unit reduction by \( O_2 \) in \( L_2 \). Similarly, to maintain 30% initial share \( p \) for unit reduction of opponents blocking, \( O_1 \) has to reduce his blocking proportion by \( 1/2.63 = 0.38 \) which leads to 30%.

(d) When \( L_2 \) is in Higher Zone (\( 0.5 < L_2 < 1 \)), for maintaining 20% share in initial traffic, for unit change in blocking by \( O_2 \) in \( L_2 \), the \( O_1 \) has to put reduction level of blocking more by 1.25 times than competitor. In case of 30% initial value, \( O_1 \) has to reduce blocking by 0.75 times only; therefore, a significant effect is observed of the blocking zone level of \( L_2 \) to the initial traffic distribution under large number of attempts.

3.11.0 REST STATE ANALYSES OF USERS:

This is in view to an effect of newly introduces marketing strategies HOP and PBP in the form of \( R \) over the traffic distribution (initial and final both) while large connecting attempts are made. The analytical approach contains (a) to examine \( p \) over varying \( r \) (b) to examine \( p \) over \( p_R \) (c) examine \( p \) over joint variation of \( (r, p_R) \).

3.11.1 PULL-BACK-PLAN ANALYSIS:

The probability term \( r \) relates like a parameter to PBP and express the transition probability from rest state \( R \) to operator \( O_1 \). When \( r \) is high, chance of transition
(Between FU and PIU over varying \( r \))

Towards \( O_1 \) is also high. The fig. 3.11.1 and fig. 3.11.2 show a comparative variation of initial probability \( p \) over \( r \) when \( p_R = 0.6 \). These figures are iso-resters due to constant \( p_R \) indicating gain of \( O_1 \) in initial share when \( r \) is high. For example, in fig. 3.11.1, \( p \approx 0.8 \) when \( L_1 = 0.1, r = 0.5 \), in fig. 3.11.2, \( p \approx 0.9 \) when \( L_1 = 0.1, r = 0.8 \). The higher \( p \) for higher \( r \) leads to success of Pull-back marketing plan because sub-group of faithful users increases with this marketing strategy. The FU is a committed group to an operator whose growth is a pleasant event. With increasing self blocking probability \( L_1 \) of \( O_1 \), a proportion of faithful users move to the rest state \( R \). When \( r = 0.3 \) made doubled (\( r = 0.6 \)), the PIU proportion becomes half, showing proportional decline over increasing \( r \).

By fig. 3.11.3 – 3.11.4, the increment in \( r \) grows the proportion of completely impatient user in a fast way. For example, in fig. 3.11.1, \( p \approx 0.22 \) when \( L_1 = 0.1, r = \ldots \)
0.3 and \( p \approx 0.11 \) when \( L_1 = 0.1, r = 0.6 \). This show, due to high \( r \), \( O_1 \) gains his \( FU \) and \( CIU \) more and loses proportion of \( PIU \). This is important feature of marketing strategy \( R \).

### 3.11.2 HOLD-ON-PLAN ANALYSIS:

The probability parameter \( p_R \) relates to \( HOP \) and when it is high, it attracts more and more users to joint relax offer and participate in the marketing offer. Fig. 3.11.5 – 3.11.8 reflect the effect of \( p_R \) on initial share \( p \) under large attempts. The \( O_1 \) looses a big proportion of \( FU \) and initial share \( p \) both when \( p_R \) is high. For example, if

\[
\begin{align*}
\text{Fig. 3.11.5 (} p_A = 0.7, r = 0.6, L_2 = 0.3 \text{)} \\
\text{Fig. 3.11.6 (} p_A = 0.7, r = 0.6, L_2 = 0.3 \text{)}
\end{align*}
\]

\[\text{(Between FU and PIU over varying} \ p_R \text{)}\]

\( p \approx 0.83 \) when \( L_1 = 0.2, p_R = 0.5 \) and \( p \approx 0.51 \) when \( L_1 = 0.2, p_R = 0.8 \). It is because the marketing plan \( HOP \) stops them from abandoning the attempt process. Therefore, \( FU \) join the impatient group. The Pull-back-plan together with \( HOP \) reimburse this \( FU \) loss.

\[
\begin{align*}
\text{Fig. 3.11.7 (} p_A = 0.7, r = 0.6, L_2 = 0.3 \text{)} \\
\text{Fig. 3.11.8 (} p_A = 0.7, r = 0.6, L_2 = 0.3 \text{)}
\end{align*}
\]

\[\text{(Between PIU and CIU over varying} \ p_R \text{)}\]
A comparative look over of PIU and CIU shows a slow decay of PIU proportion [fig. 3.11.7-3.11.8]. The pR probability has significant effect on the reduction of PIU just to provide a gain to O₁ because his CIU proportion lifts up. The Hold-on-plan helps to increase CIU for O₁ even at the cost of loss of FU. At this point, Pull-back plan plays important role to attract CIU towards O₁. The increment in self-blocking reduces initial share constantly.

3.11.3 JOINT ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.1 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.2 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.3 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.4 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.5 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.6 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.7 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.8 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.9 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.10 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.11 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.12 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.13 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.14 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03

3.11.3.15 JOIN ANALYSIS (HOP and PBP)

When pR and r parameters both increase simultaneously then a faster decline in FU appears along with first choice p. For example, from fig. 3.11.9, if pR = 0.1, r = 0.2 then p = 0.95

r = 0.5 then p = 0.90
r = 0.8 then p = 0.80

when L₁ = 0.06, L₂ = 0.03
At this stage over the rest state R, a new market strategy required to be adopted (eg. gifts, coupons, prize distributions by O₁) by which users may pulls–back to O₁ from R, because with higher r the O₁ gains the initial share p due to partially impatient users. The recommended strategy between r and pₐ is r < pₐ but pₐ < 0.25 and r < 0.2 is a good choice (under small L₁, L₂). In response to fig. 3.11.10, when self blocking probability of O₁ is high along with pₐ, the initial share of O₁ falls down rapidly as he looses major proportion of his faithful stuff of customers. This is rather unfortunate and the rest state involvement with marketing plans does not pay much to restore the loss.

When L₂ is high along with increasing pₐ [fig 3.11.11] the operator O₁ gains the traffic. More importantly, the stuff of FU for O₁ is high which a positive sign for business upliftment. In this, even when the r is small, which means without any imposition of pull-back market plan (PBP), the operator O₁ gains the traffic (fig. 3.11.12).

Fig. 3.11.13 - 3.11.16, reveals comparison between FU and CIU. The initial share p is found independent of variation of parameter L₁, L₂, pₐ and r. It also shows the composition of FU and CIU in p do not depend on these parameters.

**Fig. 3.11.13** (L₁ = 0.03, L₂ = 0.06, pₐ = 0.5)  
**Fig. 3.11.14** (r = 0.5, L₂ = 0.06, pₐ = 0.5)  
**Fig. 3.11.15** (L₁ = 0.03, r = 0.5, pₐ = 0.5)  
**Fig. 3.11.16** (pₐ = 0.5, L₁ = 0.06, pₐ = 0.5)

(Between FU and CIU varying L₁, L₂, pₐ, r)
A view of PIU and CIU (from fig. 3.11.17 - 3.11.20) shows the fall in $p$ due to increasing $r$ and $p_R$ simultaneously. But the stuff of PIU reduces over CIU for higher $r$. Actually the operator $O_1$ gains CIU traffic for high $r$. While increasing opponent blocking probability, the stuff of PIU is high for $O_1$ and he is also benefited in terms of initial traffic share. The $r$ with increasing $L_2$ shows $O_1$ gains initial share ($p$) rapidly due to high growth of PIU over small $r$. The parameter $r$ has useful contribution in improving the CIU and PIU stuff for operator $O_1$. This reveals Pull-back plan (PBP) helps to upgrade the level of initial share $p$ over $r$ small and high $L_2$. Even at low $L_1$,

...(Graphs and figures are not transcribed as they are not directly relevant to the text interpretation.)

small $r$ and small $p_R$ justify the importance and impact of marketing plans to improve upon customer-base.

### 3.12.0 EFFECT OF CALL ATTEMPTS:

The expressions of traffic share for operator $O_1$, $O_2$ are in equation (3.7.2)-(3.7.7). From fig. 3.12.1 – 3.12.3, one can observe the final traffic share remains
almost same over large n when parameters $L_1$, $L_2$, $p_R$, $p_A$ and $r$ are fixed. The final traffic share of $O_1$ falls a little in the case of PIU over specific attempts $n = 2$ and $n = 3$. The FU group reduces as the $n$ is high but there is a marginal gain to PIU and CIU. Clearly, multiple attempts for call connectivity leads to poor reputation for first choice in the mind of committed users (FU).

### 3.12.1 ATTEMPT ANALYSIS OVER MARKETING PLANS:

One can think of to examine the effect of increasing $n$ over different groups of users individually by inspiring through results of section 3.12.0. The effect of marketing plans in light of $n$ is a matter of interest to know about.

#### 3.12.1.1 BY FU:

In context to fig. 3.12.4 – 3.12.7 for $O_1$ and fig. 3.12.8 – 3.12.11 for $O_2$ this is to observe that at first and second attempt, the downfall in faithful user traffic...
Fig. 3.12.4 (L₁ = 0.3, pₐ = 0.7, p = 0.8, pᵣ = 0.2)
Fig. 3.12.5 (L₁ = 0.3, pₐ = 0.7, p = 0.8, pᵣ = 0.2)

(Traffic share by FU during call attempt process for O₁ varying r, pᵣ)

Fig. 3.12.6 (L₁ = 0.3, pₐ = 0.7, p = 0.8, pᵣ = 0.8)
Fig. 3.12.7 (L₁ = 0.3, pₐ = 0.7, p = 0.8, pᵣ = 0.8)

Fig. 3.12.8 (L₂ = 0.6, pₐ = 0.7, p = 0.8, pᵣ = 0.2)
Fig. 3.12.9 (L₂ = 0.6, pₐ = 0.7, p = 0.8, pᵣ = 0.2)

Fig. 3.12.10 (L₂ = 0.6, pₐ = 0.7, p = 0.8, pᵣ = 0.8)
Fig. 3.12.11 (L₂ = 0.6, pₐ = 0.7, p = 0.8, pᵣ = 0.8)

(Traffic share by FU during call attempt process for O₂ varying r,pᵣ)
share occurs for \( O_1 \) and \( O_2 \) both. The sharing due to faithful users increases in little proportion for \( O_1 \) over large \( n \), when the pull-back probability \( r \) has a constant increase. But for \( O_2 \), the converse observed as increase in \( r \) (for \( n > 3 \)) shows a reduction in traffic share. The increment in \( p_R \) uplifts the traffic share of faithful user in a little amount for both operators. The Hold-on strategy does not have much impact but Pull-back-plan has effect for growing share.

3.12.1.2 BY PIU:

![Traffic share by PIU during call attempt process \( O_1 \) for varying \( p_R \)](image)

![Traffic share by PIU during call attempt process \( O_2 \) for varying \( p_R \)](image)

While looking at fig. 3.12.12 – 3.12.15, the PIU also looses the proportion during first and second attempt but picks-up thereafter. But, overall the PIU reduces on increasing \( p_R \). This seems Hod-on-plan reduces the proportion of PIU over the large number of attempts.
3.12.1.3 BY CIU:

From fig. 3.12.16 – 3.12.19 and fig. 3.12.20 – 3.12.23, it is clear that the traffic share of both operators has a downfall during first and second attempts but patterns gets stability on large n. The change in r has little effect over proportion of CIU of O₃ when pᵣ is very small (pᵣ = 0.2 ). But high r provides better traffic which means Pull-back-plan generate more CIU customers. Share of CIU also falls down in initial attempts but gradually picks-up the level. The traffic share of operator O₂ is

![Graph showing traffic share changes](image1)

![Graph showing traffic share changes](image2)

![Graph showing traffic share changes](image3)

![Graph showing traffic share changes](image4)

![Graph showing traffic share changes](image5)

![Graph showing traffic share changes](image6)

(Traffic share by CIU during the call attempt process for O₃ varying r,pᵣ)
constantly lower than operator $O_1$ because of $L_2 > L_1$. For fixed value of $p_R$, the Pull-back-plan probability $r$ has a little impact on traffic sharing. When $r$ is fixed, the variation of $p_R$ has also not much effect over sharing. But, when both are high ($p_R = 0.8, r = 0.8$) a sudden growth of traffic of by CIU is found. So, both marketing plans, HOP and PBP, are effective in attracting customers.

3.13.0 TRAFFIC SHARE ANALYSIS BY FU, PIU AND CIU:

From fig 3.13.1, It is found that $r = 0.8$ of $O_1$ overlaps with $r = 0.2$ of $O_2$ in the upper curve similarly $r = 0.8$ of $O_2$ overlaps with $r = 0.2$ of $O_1$ in lower curve. For a fix $p_R$, the traffic of $O_1$ at $r = 0.8$ is higher than at $r = 0.2$. This remains maintain for increasing $p_R$, therefore Hold-on-plan probability $p_R$, and Pull-back-plan probability $r$ has strong contribution in increasing the traffic share of $O_1$. The FU of $O_1$ increases with the joint increment of $r$ and $p_R$. 
In view to fig 3.13.2, with increase of r, pR, the PIU increases upto when pR = 0.1 and then reduces thereafter. To note that PIU affected by only Hold-on plan and r independent of Pull-back plan. The recommended value of Hold-on probability is 0.1 in view to PIU. While to consider CIU (fig 3.13.2), the effect of variation r and pR together provides benefits to other operator for increasing probability of Pull-back plan. The O1 bears a little loss of CIU for the joint increment of r and pR. The overall view indicates that the joint variation of probability due to both marketing plans, there is a gain in traffic due to faithful user to O1 but little loss occur due to PIU and CIU proportion of user.

3.14.0 MODEL IN CASE OF MULTI-OPERATORS:

The case of two operators could be extended to the case of N operators under following assumptions:

(a) Let O_i (i = 1, 2, 3, . . . N) be N operators in a competitive market.

(b) There are total (N + 3) states as O_i (i = 1, 2, 3, . . . N), R, Z and A.

(c) \{ X^{(n)}, n \geq 0 \} is a Markov chain having transitions over state space.

(d) User in the café chooses any i\textsuperscript{th} operator O_i with probability p_i \left( \sum_{i=1}^{N} p_i = 1 \right).

(e) The blocking probability of O_i is L_i.
(f) The attempt by user is on call-by-call basis. If the call for $O_i$ is blocked in $k^{th}$ attempt ($k > 0$) then in $(k + 1)^{th}$, the user shifts to $O_j$ ($i \neq j$). If this also fails, user switches to $O_t$ ($t \neq i \neq j = 1, 2, 3, \ldots, N$) and so on.

(g) Whenever call connects through either of $O_i$ or $O_j$ ($i \neq j$), the system reaches to the state of success $Z$.

(h) The user can terminate the process marked as system to the abandon state $Z$ with state probability $p_A$.

(i) During attempts between $O_i$ and $O_j$ ($j \neq i = 1, 2, \ldots, n$), user is allow to reach to a state $R$, which is a temporary rest due to failed call-by-call attempts with probability $p_R$. It also relates to Hold-On Plan of marketing.

(j) If reaches to $R$ from $O_i$ in $n^{th}$ attempts, then in $(n + 1)^{th}$, he with be on $O_j$ ($j \neq i$). with probability $r_j \left( \sum_{j=1}^{N} r_j = 1 \right)$. This relates to Pull-Back marketing plan.

(k) From $R$, user cannot move to state $Z$ and $A$. The transition diagram shown in the fig (3.14.1)
The generalized transition matrix is in fig. 3.14.2

The initial probability for choosing $i^{th}$ operator is, $P[X^{(0)} = O_i] = p_i$, $\sum p_i = 1$

For **Faithful User (FU)**

$$P[X^{(i)} = O_i] = \sum_{i=1}^n P[X^{(0)} = R] P\left[\frac{X^{(i)} = O_i}{X^{(0)} = R}\right]$$

For **Partially Impatient User (PIU):**

$$P[X^{(i)} = O_i] = \sum_{i=1}^n P[X^{(0)} = O_i] P\left[\frac{X^{(i)} = O_i}{X^{(0)} = O_i}\right]$$

For **Completely Impatient User (CIU):**

$$P[X^{(i)} = O_i] = \sum_{i=1}^n \left\{ P[X^{(0)} = O_i] P\left[\frac{X^{(i)} = O_i}{X^{(0)} = O_i}\right] \right\} + n P[X^{(0)} = R] P\left[\frac{X^{(i)} = O_i}{X^{(0)} = R}\right]$$

### 3.15.0 CONCLUDING REMARKS:

The proposed Markov chain model for user's behavior explains the traffic sharing pattern between two operators in presence of rest state as a marketing strategy. This state has a significant role in uplifting the customer-base proportion of an operator. The initial market group $p$ of an operator highly depends on network blocking probabilities $L_1$ and $L_2$ both along with rest state probabilities $p_R$ and $r$. Final traffic share also depends on blocking probability parameters along with marketing.
plans HOP and PBP who have shown their important effect. All the three categories FU, PIU, CIU have displayed variations due to implementation of these plans.

The competitor's blocking probability has a strong impact over improving proportion of faithful users of operator O_1. The completely impatient user always have to keep larger initial traffic share than FU irrespective of whatever may the values of L_2, r and p_R. The proportion of partially impatient user is lesser than the completely impatient user when opponent's blocking probability is small, but increases gradually as the L_2 increases. The blocking derivative is directly proportional to the blocking value L_1 and initial share (p).

The Pull-back plan of marketing is successful because with the increases of r the sub-group of faithful user increases in proportion with this marketing strategy. Since FU is a committed group to an operator therefore, the growth is a positive sign in support of impact of PBP. Due to this actually the PIU convert into CIU for an operator which contributes for gain in the level of traffic. The HOP provides loss in FU and p both for high p_R but increment in CIU proportion. The joint variation of r and p_R has positive growing effect. The recommended marketing strategy between HOP and PBP is p_R < 0.25, r < 0.5 and r < p_R along with small value of L_1 and L_1 < L_2. When L_2 is high, the operator O_1 gains the traffic specially due to high proportion of FU and CIU. Both the marketing plans contribute more on CIU traffic to Pull-on towards an specific operator.

It is recommended for internet service providers to open-up entertainment and refreshment facilities in the form of Hold-on marketing plan in and around to their Internet cafe. Apart from this the gift vouchers, free time offer like Pull-back marketing plans are also required to grow up the high customer-base. Both the plans, HOP and PBP if implemented together at moderate level have significant role to improve upon the traffic share of an operator. One more important suggestion is that Internet operator always has to keep his network blocking probability lower than to the competitor (L_1 < L_2). While competitor's blocking is high, the operator does not have to bother much about his service quality offered.