FAVoured Disconnectivity Analysis in Internet Traffic Sharing

5.1.0 INTRODUCTION:

We have discussion so far for the effect of marketing plans and cyber-crime facilities to improve upon traffic sharing between two operators. Basically these are marketing strategies which help to grow-up the traffic proportion of an operator. If similar setup of two operators is assumed, one can think of for additional service parameters affecting the sharing amount. The disconnectivity is one such parameter which directly relates to the network efficiency. When a call attempt is successful through dialup setup, the probability of disconnectivity starts and user feels the fear due to this event. The customer, at this stage, has option to choose to other operator (or service provider) as his favourate if disconnectivity is frequent. This parameter could be divided into two types as shown in fig. 5.1.0(b).

![Fig. 5.1.0(a)](network-blocking-diagram)

![Fig. 5.1.0(b)](disconnectivity-diagram)

* A part of this chapter is published in the Electronic Proceedings of International Conference on Mathematics & Computer Science (ICMCS-08), July 2008, pp. 278-287. (See Appendix)
The favoured disconnectivity is that even when user is dedicated to re-start call attempt through his earlier and favourate operator. In contrary, when user leaves to favourate before re-start of call attempt, it is non-favoured. Suppose \( L_1, \ L_2 \) are network blocking probabilities [fig. 5.1.0(a)] and \( s_1, \ s_2 \) are favoured disconnectivity probabilities then according to Naldi (2002) the quality of service (QoS) is a function of network blocking parameters.

\[
\text{QoS} = f(L_1, L_2)
\]

We consider a modified form of this function in light of favoured disconnectivity as

\[
\text{QoS} = f(L_1, L_2, s_1, s_2)
\]

A model of traffic distribution is drawn in fig 5.1.1 and 5.1.2 where first choice (p) and Internet traffic share both are assumed related through disconnectivity parameters.

![Fig. 5.1.1](image)

![Fig. 5.1.2](image)

Shukla et al. (2007) produced an analysis of Space Division switches using Markov chains. Shukla and Gadewar (2007) used Markov chain model for Knockout Switch explaining the packet flow phenomenon. Shukla and Jain (2007) applied the Markov chain model for explaining the transition mechanism in job scheduling process. Deriving motivation from all these, this chapter V presents an analysis of traffic distribution in light of QoS offered by operators with special reference to the favoured disconnectivity event. A Markov chain model is used to explain the system as user behavior and to derive the mathematical expressions of traffic sharing.
5.1.1 USER’S BEHAVIOR AS A SYSTEM:

Consider following hypotheses for the behavior of a user, with network blocking, disconnectivity and initial choice as parameters.

- A competitive market has a café, containing Internet facility of two operators \( O_1 \) and \( O_2 \).
- A user enters into café with initial choice in mind (first choice) as \( p \) for \( O_1 \) and \( (1-p) \) for \( O_2 \).
- During the repeated connectivity call attempts, the blocking probability of \( O_1 \) is \( L_1 \) and of \( O_2 \) is \( L_2 \). Blocking implies the event when call attempt process fails to connect.
- After each fail call attempt, the user has two choices: he can abandon with probability \( p_A \) or switch over to other operator for a new attempt.
- Switching between \( O_1 \) and \( O_2 \) is on a call-by-call basis depending just on the latest attempt. The two successive call attempts are not on same operator.
- When call connects, the success appears and user performs Internet surfing through connected operator. Suddenly, due to lack of quality of service, the call disconnects with probability \( s_1 \) for \( O_1 \) and \( s_2 \) for \( O_2 \).
- For re-connectivity, the user bears a favoured attitude for earlier operator and retries with the same. The results into favoured disconnectivity with probability parameter \( s_i \) \((i = 1, 2)\).

5.2.0 MARKOV CHAIN MODEL:

These models are used by Shukla et. al. (2007), Shukla and Gadewar (2007) in switch system analysis of computer network.

Under hypotheses of section 5.1.1, the user’s behavior is modeled through a four-state discrete-time Markov chain \( \{D^{(n)}, n \geq 0\} \) such that \( D^{(n)} \) stands for the state of random variable \( D \) at \( n^{th} \) call attempt made by user over state space \( \{O_1, O_2, Z, A\} \), where

- State \( O_1 \): First operator
- State \( O_2 \): Second operator
- State \( Z \): Success obtained in call connection
- State \( A \): User leaving (abandon) the call attempt process
Some other assumptions and their interpretations related to model are:

(a) The connectivity attempts of user between operators are on call-by-call basis, which means if the call for $O_1$ is blocked in $k^{th}$ attempt ($k > 0$), and does not abandon, then in $(k + 1)^{th}$ user shifts to $O_2$. If this also fails, user switches to $O_i$ (or abandon).

(b) Whenever call connects through either of $O_i$ ($i=1,2$), we say system reaches to the state of success after $n$ attempts.

(c) If connected with operator $O_i$ successfully and disconnected thereafter, the user is back to $O_i$ again for a retry, which is faithfulness.

(d) From state Z user can not move to the abandon state A.

(e) State A is absorbing state.

\[ \text{Fig. 5.1.3 [Transition Diagram of Model]} \]

5.3.0 COMPUTATION OF TRANSITION PROBABILITIES:

(i) The initial probabilities for user, before the first call attempt, selecting any one of operators are

\[ P[D^{(0)} = O_1] = p; \quad P[D^{(0)} = O_2] = (1 - p); \quad P[D^{(0)} = Z] = 0; \quad P[D^{(0)} = A] = 0 \]

(ii) If in the $(n - 1)^{th}$ attempt, call for $O_1$ is blocked, the user may abandon the process in the $n^{th}$ attempts. Therefore,

\[ P[D^{(n)} = A/\text{blocked at } O_1] = P[\text{abandon the process}] = L_1p_A \quad \ldots (5.3.1) \]
Similar for $O_2$, \( P \left[ \frac{D^{(n)}}{D^{(n-1)}} = O_2 \right] = L_2 P_A \) \(\ldots (5.3.2)\)

(iii) At $O_1$ in $n^{th}$ attempt call may successful and system reaches to state $Z$ from $O_1$.
\[
P \left[ \frac{D^{(n)}}{D^{(n-1)}} = Z \mid D^{(n-1)} = O_1 \right] = P[\text{not blocked at } O_1] = 1 - L_1 \quad \ldots (5.3.3)
\]

Similar for $O_2$, \[
P \left[ \frac{D^{(n)}}{D^{(n-1)}} = Z \mid D^{(n-1)} = O_2 \right] = 1 - L_2 \quad \ldots (5.3.4)
\]

(iv) The user shifts from $O_1$ to $O_2$ in $n^{th}$ attempt, this probability is
\[
P \left[ \frac{D^{(n)}}{D^{(n-1)}} = O_2 \mid D^{(n-1)} = O_1 \right] = P [\text{blocked at } O_1] P[\text{not abandon }]
\]
\[
= L_1 (1 - p_A) \quad \ldots (5.3.5)
\]

Similarly, \[
P \left[ \frac{D^{(n)}}{D^{(n-1)}} = O_2 \mid D^{(n-1)} = O_2 \right] = L_2 (1 - p_A) \quad \ldots (5.3.6)
\]

(v) Disconnectivity occurs when success state $Z$ achieved either through $O_1$ or $O_2$. After this, user return to state $O_1$ with probability $s_1$ and to $O_2$ with $s_2$.
\[
P \left[ \frac{D^{(n)}}{D^{(n-1)}} = Z \right] = s_1 ; \quad P \left[ \frac{D^{(n)}}{D^{(n-1)}} = Z \right] = s_2 \quad \ldots (5.3.7)
\]

Incorporating above all, the transition probability matrix is in the form in fig. 5.1.4

Fig. 5.1.4 [Transition Probability Matrix]
5.4.0 RESULTS FOR \( n^{th} \) ATTEMPTS.

The probabilities of ultimate state \( n^{th} \) attempt are derived in the following theorem.

**THEOREM 5.4.1:** If user attempts between \( O_1 \) and \( O_2 \), then the \( n^{th} \) step state probability, for \( n = 1, 2, 3 \ldots \), is:

\[
P[D^{(2n)} = O_1] = p [(L_4 L_2)^n (1 - p_A)^{2n} + (L_4 L_2)^n (1 - p_A)^{2(n-1)} (1 - L_2)s_1]
\]

\[
P[D^{(2n+1)} = O_1] = (1 - p) [(L_4 L_2)^n (1 - p_A)^{2n+1} + (L_4 L_2)^n (1 - p_A)^{2(n-1)} (1 - L_2)s_1]
\]

\[
P[D^{(2n)} = O_2] = (1 - p) [(L_4 L_2)^n (1 - p_A)^{2n} + (L_4 L_2)^n (1 - p_A)^{2(n-1)} (1 - L_2)s_2]
\]

\[
P[D^{(2n+1)} = O_2] = p L_1 [(L_4 L_2)^n (1 - p_A)^{2n+1} + (L_4 L_2)^n (1 - p_A)^{2(n-1)} (1 - L_2)s_2]
\]

**PROOF:** At \( n = 0 \), we have

\[
P[D^{(0)} = O_1] = p; \quad P[D^{(0)} = O_1] = (1 - p), \text{ following (5.3.8), the start may either from } O_1 \text{ and } O_2, \text{ and we have ,}
\]

For \( n = 1 \)

\[
P[D^{(1)} = O_1] = P[D^{(0)} = O_2]P[D^{(1)} = O_1/D^{(0)} = O_2] = (1 - p)L_2(1 - p_A)
\]

\[
P[D^{(1)} = O_2] = P[D^{(0)} = O_1]P[D^{(1)} = O_2/D^{(0)} = O_1] = p L_1(1 - p_A)
\]

\[
P[D^{(1)} = Z] = P[D^{(0)} = O_1]P[D^{(1)} = Z/D^{(0)} = O_1] = p(1 - L_1)
\]

\[
P[D^{(1)} = Z] = P[D^{(0)} = O_2]P[D^{(1)} = Z/D^{(0)} = O_2] = (1 - p)(1 - L_2)
\]

For \( n = 2 \)

\[
P[D^{(2)} = O_1] = P[D^{(1)} = O_2]P[D^{(2)} = O_1/D^{(1)} = O_2] + P[D^{(1)} = Z]P[D^{(2)} = O_1/D^{(1)} = Z]
\]

\[
= p[(L_4 L_2)^n (1 - p_A)^{2n} + (1 - L_2)s_1]
\]

\[
P[D^{(2)} = O_2] = P[D^{(1)} = O_1]P[D^{(2)} = O_2/D^{(1)} = O_1] + P[D^{(1)} = Z]P[D^{(2)} = O_2/D^{(1)} = Z]
\]

\[
= (1 - p) [(L_4 L_2)^n (1 - p_A)^{2n+1} + (1 - L_2)s_1]
\]

\[
P[D^{(2)} = Z] = P[D^{(1)} = O_1]P[D^{(2)} = Z/D^{(1)} = O_1] = (1 - p)L_2(1 - p_A)(1 - L_2)
\]

\[
P[D^{(2)} = Z] = P[D^{(1)} = O_2]P[D^{(2)} = Z/D^{(1)} = O_2] = (1 - p)L_1(1 - p_A)^2
\]

\[
P[D^{(2)} = Z] = P[D^{(1)} = O_2]P[D^{(2)} = Z/D^{(1)} = O_2] = p L_1 (1 - p_A)(1 - L_2)
\]

121
\[ P[D^{(2)} = O_1] = P[D^{(1)} = O_2] P\left[ D^{(2)} = O_1 \bigg/ D^{(1)} = O_2 \right] = pL_1L_2(1 - p_A)^2 \]

For \( n = 3 \)

\[ P[D^{(3)} = O_1] = P[D^{(2)} = O_2] P\left[ D^{(3)} = O_1 \bigg/ D^{(2)} = O_2 \right] + P[D^{(2)} = Z] P\left[ D^{(3)} = O_1 \bigg/ D^{(2)} = Z \right] = (1 - p)L_2(1 - p_A)(L_1L_2(1 - p_A)^2 + (1 - L_1)s_1) \]

\[ P[D^{(3)} = O_2] = P[D^{(2)} = O_2] P\left[ D^{(3)} = O_2 \bigg/ D^{(2)} = O_2 \right] + P[D^{(2)} = Z] P\left[ D^{(3)} = O_2 \bigg/ D^{(2)} = Z \right] = pL_2(1 - p_A)^2(1 - L_1) \]

\[ P[D^{(3)} = Z] = P[D^{(2)} = O_2] P\left[ D^{(3)} = Z \bigg/ D^{(2)} = O_2 \right] = pL_2L_2(1 - p_A)^2(1 - L_2) \]

For \( n = 4 \)

\[ P[D^{(4)} = O_1] = P[D^{(3)} = O_2] P\left[ D^{(4)} = O_1 \bigg/ D^{(3)} = O_2 \right] + P[D^{(3)} = Z] P\left[ D^{(4)} = O_1 \bigg/ D^{(3)} = Z \right] = pL_1L_2(1 - p_A)^3 \]

\[ P[D^{(4)} = O_2] = P[D^{(3)} = O_2] P\left[ D^{(4)} = O_2 \bigg/ D^{(3)} = O_2 \right] + P[D^{(3)} = Z] P\left[ D^{(4)} = O_2 \bigg/ D^{(3)} = Z \right] = (1 - p)L_2(1 - p_A)^2(1 - L_2) \]

On continuation in similar way, the theorem exits

**Remarks 5.4.1:** At \( s_1 = 0 \), we get results as described by Naldi [2002]

\[ P[D^{(2n)} = O_1] = (L_1L_2)^n(1 - p_A)^{2n} \]

\[ P[D^{(2n+1)} = O_1] = (1 - p)L_2((L_1L_2)^n(1 - p_A)^{2n+1}) \]

\[ P[D^{(2n)} = O_2] = (1 - p)((L_1L_2)^n(1 - p_A)^{2n}) \]

\[ P[D^{(2n+1)} = O_2] = pL_2((L_1L_2)^n(1 - p_A)^{2n+1}) \]
5.5.0 COMPUTATION OF TRAFFIC SHARE AND CALL CONNECTION:

The traffic is shared between $O_1$ and $O_2$ and we wants to calculate the probability of call connection achieved in $n^{th}$ attempt with $O_i$. The symbol $\overline{P}_i^{(n)}$ denotes the traffic division after $n^{th}$ successful call attempt when initial share at $n = 0$ was $p$ [or $(1-p)$].

$$\overline{P}_i^{(n)} = P[\text{call completes in n^{th} attempt with operator } O_i]$$

$$\overline{P}_i^{(n)} = P[\text{at (n-1)^{th} attempt on } O_i \text{ P[Z at n^{th} attempt when was at } O_i \text{ in (n-1)^{th} attempt}]$$

$$= P[X^{(n-1)} = O_i]P\left[\frac{X^{(n)}}{X^{(n-1)}} = O_i\right]$$

$$\overline{P}_i^{(n)} = (1 - L_i) \left\{ \sum_{i=0}^{n-1} P[X^{(i)} = O_i] \right\}$$

$$\overline{P}_i^{(n)} = (1 - L_i) \left[ \sum_{i=0 \text{ } i \text{ even}}^{n-1} P[X^{(i)} = O_i] + \sum_{i=0 \text{ } i \text{ odd}}^{n-1} P[X^{(i)} = O_i] \right] \quad \cdots (5.5.1)$$

$$\overline{P}_i^{(n)} = (1 - L_i) \left\{ \sum_{i=0 \text{ } i \text{ even}}^{n-1} \left\{ (L_1L_2)^{i} (1 - p_A)^{2i} + (L_1L_2)^{i-1} (1 - p_A)^{2(i-1)} (1 - L_A)s_i \right\} \right\}$$

$$+ \left\{ (1 - p)L_2 \sum_{i=0 \text{ } i \text{ odd}}^{n-1} \left\{ (L_1L_2)^{i} (1 - p_A)^{2i+1} + (L_1L_2)^{i-1} (1 - p_A)^{2i-1} (1 - L_A)s_i \right\} \right\} \quad \cdots (5.5.2)$$

$$\overline{P}_i^{(2n)} = (1 - L_i) \left\{ p + (1 - p)L_2(1 - p_A) \right\}\frac{1 - [L_1L_2(1 - p_A)^2]^n}{1 - L_1L_2(1 - p_A)^2}$$

$$+ (1 - L_A)s_i \left\{ p + (1 - p)L_2(1 - p_A) \right\}\frac{1 - [L_1L_2(1 - p_A)^2]^{n-1}}{1 - L_1L_2(1 - p_A)^2} \quad \cdots (5.5.3)$$

$$\overline{P}_i^{(2n+1)} = (1 - L_i) \left\{ \frac{p\left[1 - (1 - p)L_2(1 - p_A)^2 \right]^{n+1} + (1 - p)L_2(1 - p_A)\left[1 - (1 - p)L_2(1 - p_A)^2 \right]^{n+1}}{1 - L_1L_2(1 - p_A)^2} \right\}$$

$$+ (1 - L_A)s_i \left\{ \frac{p\left[1 - (1 - p)L_2(1 - p_A)^2 \right]^{n+1} + (1 - p)L_2(1 - p_A)\left[1 - (1 - p)L_2(1 - p_A)^2 \right]^{n+1}}{1 - L_1L_2(1 - p_A)^2} \right\} \quad \cdots (5.5.4)$$

123
Similar for $O_2$

$$
\overline{P}_2^{(2n)} = (1 - L_2) \left[ \left( 1 - p + pL_4(1 - p_A) \right) \frac{1}{} \left[ \begin{array}{c}
(1 - p + pL_4(1 - p_A)) \frac{1 - [L_4L_2(1 - p_A)^2]^n}{1 - L_4L_2(1 - p_A)^2} \\
(1 - L_2)s_2 \left( 1 - p + pL_4(1 - p_A) \right) \frac{1}{} \left[ \begin{array}{c}
1 - [L_4L_2(1 - p_A)^2]^{n-1} \\
1 - L_4L_2(1 - p_A)^2 
\end{array} \right] \right] \right] \quad \ldots(5.5.5)
$$

$$
\overline{P}_2^{(2n+1)} = (1 - L_2) \left[ \left( 1 - p + pL_4(1 - p_A) \right) \frac{1}{} \left[ \begin{array}{c}
1 - [L_4L_2(1 - p_A)^2]^{n+1} \\
1 - L_4L_2(1 - p_A)^2 
\end{array} \right] \right] \quad \ldots(5.5.6)
$$

**Remark 5.5.1:** when $s_1 = 0$, $s_2 = 0$ the following expressions as per Naldi [2002].

$$
\overline{P}_1^{(2n)} \| N = (1 - L_1) \left[ (p + (1 - p)L_2(1 - p_A)) \frac{1 - [L_4L_2(1 - p_A)^2]^{n}}{1 - L_4L_2(1 - p_A)^2} \right] \\
\overline{P}_1^{(2n+1)} \| N = (1 - L_1) \left[ (p + (1 - p)L_2(1 - p_A)) \frac{1 - [L_4L_2(1 - p_A)^2]^{n+1}}{1 - L_4L_2(1 - p_A)^2} \right] \\
\overline{P}_2^{(2n)} \| N = (1 - L_2) \left[ (1 - p + pL_4(1 - p_A)) \frac{1 - [L_4L_2(1 - p_A)^2]^{n}}{1 - L_4L_2(1 - p_A)^2} \right] \\
\overline{P}_2^{(2n+1)} \| N = (1 - L_2) \left[ (1 - p + pL_4(1 - p_A)) \frac{1 - [L_4L_2(1 - p_A)^2]^{n+1}}{1 - L_4L_2(1 - p_A)^2} \right]
$$

### 5.6.0 COMPUTATION OF TRAFFIC SHARE OVER LARGE ATTEMPTS:

When number of attempts are infinitely large before getting call connected, then

$$
\overline{P}_1 = \lim_{n \to \infty} \overline{P}_1^{(2n)} = \lim_{n \to \infty} \overline{P}_1^{(2n+1)} ; \quad \overline{P}_2 = \lim_{n \to \infty} \overline{P}_2^{(2n)} = \lim_{n \to \infty} \overline{P}_2^{(2n+1)} \quad \text{which provides}
$$

$$
\overline{P}_1 = (1 - L_1) \left[ p + (1 - p)L_2(1 - p_A) + (1 - L_4)s_1 \left( p + (1 - p)L_2(1 - p_A) \right) \right] \quad \ldots(5.6.1)
$$

$$
\overline{P}_2 = (1 - L_2) \left[ (1 - p) + pL_4(1 - p_A) + (1 - L_2)s_2 \left( 1 - p + pL_4(1 - p_A) \right) \right] \quad \ldots(5.6.2)
$$

**Remark 5.6.1** When $s_1 = 0$, $s_2 = 0$, we have

$$
\overline{P}_1 | N = (1 - L_1) \left[ p + (1 - p)L_2(1 - p_A) \right] \\
\overline{P}_2 | N = (1 - L_2) \left[ (1 - p) + pL_4(1 - p_A) \right]
$$
5.7.0 ISO-SHARE EQUATIONS:

The expression (5.6.1) and (5.6.2) establish a trade-off between the investments needed to acquire the first choice rank \( p \) and those on the network infrastructure needed to maintain an adequate blocking and disconnectivity probability \( (L_1, L_2, s_1, s_2) \). The same value of final share can be reached through different choices as to these parameters. The couples of points \((p, L_1, L_2, s_1, s_2)\) that lead to the prefixed final share \( \bar{P}_1 \) could be obtained by rewriting expression (5.6.1) so as to get \( p \) as

\[
p = \frac{L_1 L_2 (1 - p_A) [1 - \bar{P}_1 (1 - p_A)] + \bar{P}_1 - L_2 (1 - p_A) - (1 - L_2) s_1 (1 - p_A)}{L_1 [L_2 (1 - p_A) - 1] + L_2 (1 - L_1) s_1 (1 - L_1) + 1 - L_2 (1 - p_A) + (1 - L_2) s_1 (1 - p_A)} \quad \cdots (5.7.1)
\]

Remark 5.7.1: When \( s_1 = 0 \), we get

\[
[p] = \frac{L_1 L_2 (1 - p_A) [1 - \bar{P}_1 (1 - p_A)] + \bar{P}_1 - L_2 (1 - p_A)}{L_1 [L_2 (1 - p_A) - 1] + 1 - L_2 (1 - p_A)}
\]

5.8.0 SHARE LOSS:

Share loss relate to the imbalance between the blocking and favoured disconnectivity probabilities offered to user by the two operators. Defining the share loss \( \Delta p \) as the difference between the initial share of \( O_1 \) and the final share, recovered by expressions (5.6.1), for \( O_1 \)

\[
\Delta p = p - \bar{P}_1
\]

\[
\Delta p = p - (1 - L_1) \left[ \frac{p + (1 - p_A) + (1 - L_2) s_1 [p + (1 - p) L_2 (1 - p_A)]}{1 - L_1 L_2 (1 - p_A)^2} \right] \quad \cdots (5.8.1)
\]

if \( \Delta p \) is negative, means the operator \( O_1 \) has actually benefited from the repeated call attempts and has increased its traffic share beyond its initial expectation \( p \).

NOTE: From (5.8.1), the share loss is proportional to the initial share, so that popular and reputed operators have to concentrate more for the quality of service they offer. However, one cannot jump to the immediate conclusion that the worse QoS operator always losses traffic.

By the manipulation of expression (5.8.1), one can derive the conditions on the initial share and the blocking probability for \( O_1 \) under which a traffic loss occur (i.e. \( \Delta p > 0 \)).
\[ p > p_{\text{lim}} = \frac{L_2(1-p_A) - L_1L_2(1-p_A) + s_1L_2(1-p_A) + s_1L_1^2L_2(1-p_A) - 2s_1L_1L_2(1-p_A)}{L_1 - L_1L_2(1-p_A)^2 + L_2(1-p_A) - L_2L_2(1-p_A) - s_1 + s_1L_2(1-p_A) - s_1L_1^2} \]

\[ + s_1L_1^2L_2(1-p_A) + 2s_1L_1 - 2s_1L_1L_2(1-p_A) \]

\[ \ldots (5.8.2) \]

**Remark 5.8.1:** When \( s_1 = 0 \), we get

\[ p > p_{\text{lim}} = \frac{L_2(1-L_2)(1-p_A)}{L_2 + L_2(1-p_A)[1 - L_2(2-p_A)]} \]

### 5.9.0 SIMULATION STUDY:

The study provides that there is positive effect of probability \( s_1 \) on the traffic share by the first operator. The fig. 5.9.1 shows higher values of \( s_1 \) increases the traffic share of \( O_1 \). From fig. 5.9.2-5.9.5, while blocking probability of operator \( O_1 \) increases, the traffic share reduces for the operator \( O_1 \). But even with the increase of \( L_1 \), if \( s_1 \) is high the \( O_1 \) gains the traffic share. The \( s_1 \) parameter of favoured disconnectivity has significant affect on upliftment of traffic share level of first operator. The traffic loss due to high \( L_1 \) is reimbursed by high \( s_1 \).

**Fig 5.9.1** (\( p = 0.5, p_A = 0.2, L_2 = 0.4, L_2 = 0.6 \))

(Traffic share with transitions (n) over varying \( s_1 \))

**Fig 5.9.2** (\( p = 0.5, p_A = 0.2, L_2 = 0.6, s_1 = 0 \))

**Fig 5.9.3** (\( p = 0.5, p_A = 0.2, L_2 = 0.6, s_1 = 0.2 \))
When fig. 5.9.6-5.9.9 are in consideration, the own blocking probability of operator O₁ must be lower than the blocking probability of opponent (O₂) in order to have high share of traffic but if s₁ parameter of O₁ increases then the operator O₁ gains in the traffic. For instance, if s₁ from 0.2 increases to 0.8, the traffic share jumps from 0.7 - 0.9 when L₂ = 0.8, L₂ = 0.6. When L₁ and L₂ parameters both operator increase in same proportional, the stability level of the traffic share occurs.

(Traffic share with transitions (n) over varying s₁ and L₁)

(Traffic share with transitions (n) over varying s₁ and L₂)
at relatively low value (0.5, 0.6). But when the favoured disconnectivity parameter $s_1$ increases than this stability level of traffic share improves considerably. The fig.5.9.10-5.9.13 reflect that overall stability occur for $n > 4$ attempts.

Fig. 5.9.10 (p = 0.5, $p_A = 0.2$, $s_1 = 0$)

Fig. 5.9.11 (p = 0.5, $p_A = 0.2$, $s_1 = 0.2$)

Fig. 5.9.12 (p = 0.5, $p_A = 0.2$, $s_1 = 0.5$)

Fig. 5.9.13 (p = 0.5, $p_A = 0.2$, $s_1 = 0.8$)

(Traffic share with transitions (n) over varying $s_1$ and $L_1, L_2$)

Fig. 5.9.14, contains fixed value of $L_1 = 0.4$, $L_2 = 0.6$ and increasing $s_1$ has an growing impact over the level of traffic share since both are directly proportional as well as linearly related. But, when $s_1$ and $L_1$ both simultaneously high than decrease of traffic sharing is observed (fig. 5.9.15). Even then, compare to $s_1 = 0$, the $s_1 = 0.8$ indicates for better share. In contrary, when $s_1$ and $L_2$ have joint increment, the traffic share has almost a linearly increasing trend. Therefore, higher opponents blocking provides gain in traffic to the first operator $O_1$ (in fig. 5.9.16).

Fig. 5.9.14 (p = 0.5, $p_A = 0.2$, $L_2 = 0.4$, $L_2 = 0.6$)

(Traffic share without transitions (n) over varying $s_1$ and fixed $L_1, L_2$)
5.10.0 SIMULATION OF ISO-SHARE EQUATIONS:

Looking into fig. 5.10.1-5.10.5, we observed that curves are iso-share for paired

Traffic share without transitions (n) over varying $s_1$ and fixed $L_1, L_2$
input \((p, L_1)\) on X-axis and initial share \(p\) of operator \(O_1\) at Y-axis. The iso-share curves are slightly concaved upward showing that in order to achieve the final prefixed share level, the operator has to compensate his initial share in the market. If the blocking \(L_1\) is high then operator \(O_1\) has to raise-up his initial share more in order to achieve the pre-desired level of final share. The increasing favoured disconnectivity \(s_1\) plays a significant role in reducing this like compensation. If \(s_1 = 0.02\), the initial level be 0.72 to get final output 0.70. But if \(s_1 = 0.8\), the initial level needed only 0.45 to get pre-fixed final output 0.70 while fixed \(L_1 = 0.1\) in both cases.

Therefore, the increase in \(s_1\) parameter which is favoured disconnectivity has significant effect in controlling the initial share on iso-share curves. Looking over to fig. 5.10.6 - 5.10.8, one can arrive at the same conclusion.

5.11.0 SIMULATION OF SHARE LOSS:

The fig. 5.11.1-5.11.3, are showing the curves of initial share and blocking probability of operator \(O_1\) subject to condition that \(L_2\) and \(s_1\) parameters are varying.
Although, O₁ has blocking probability 0.5 but he gains the traffic if opponents blocking is > 0.5. If s₁ parameter increases then his share loss is lower than earlier. For example, if s₁ = 0.02, then up to 0 < L₁ < 0.5 not much loss in initial share for O₁. But when s₁ = 0.10, the operator O₁ can bear the reduced blocking level 0 < L₁ < 0.1, in order to maintain the same amount of initial traffic share level. Therefore, the increasing s₁ probability provides a useful gain to operator O₁ in terms of traffic share even if his blocking probability L₁ is slightly high.

5.12.0 CONCLUDING REMARKS:

The proposed Markov chain model explains the user behaviour in presence of two new parameters s₁, s₂ related to quality-of-service offered by an operator. The event of call disconnectivity, although unfortunate in a network but has a positive aspect in the form of 'favoured' disconnectivity which is found a hidden and useful parameter affecting the traffic distribution. By virtue of marketing plans and service quality together this parameter of the system could be utilized just to have a better traffic share.
The $s_1$ of favoured disconnectivity has significant affect on the upliftment of traffic share level of an operator. Even if the traffic loss caused due to high self blocking $L_1$ is reimbursed by higher $s_1$. The number of attempts if more than four ($n > 4$) the curves of all types have a stability pattern. It seems increasing number of failed attempts do not contribute much over traffic distribution. When $s_1$ and $L_1$ both are high there is loss of traffic for an operator but favoured disconnectivity, when more high fulfils this loss in substantial amount. The operator must has to keep his blocking probability lower to the competitor ($L_1 < L_2$) in order to gain a better share along with high $s_1$.

Iso-share curves prove this fact that favoured disconnectivity has strong effect in controlling and reducing the initial share $p$ of customers in the market in order to achieve a prefixed final share. It is observed further that the favoured disconnectivity controls traffic share loss when self blocking in the network of an operator is high.

It is therefore recommended for Internet service providers (or operators) to incorporate favoured disconnectivity as factor in their quality-of-service package and develop related marketing plans to uplift the value of this parameter to get better traffic share in the competitive market. If an operator offers a small time slot free of cost for Internet access for each call drop then it results into favoured disconnectivity and certainly leads to better share.