CHAPTER 4
TWO-STAGE DIRECT DRIVE WIND ENERGY
CONVERSION SYSTEM WITH Z-SOURCE INVERTER

4.1 INTRODUCTION

The power conversion to obtain variable ac voltage at variable frequency was earlier carried out by using three stages of power conversion. To reduce the power conversion stages and to overcome the limitations of conventional systems, the Z-Source Inverter (ZSI) based Direct Drive Wind Energy Conversion Systems (DDWECS) is introduced. The operating modes of the ZSI with DDWECS are discussed with the support of mathematical equations. Here the distribution of shoot through periods in two PWM control methods with ZSI, such as maximum boost with Third Harmonic Injection (TH1) and Modified Space Vector Pulse Width Modulation (MSVPWM) is carried out. The mathematical equations for boost factor, voltage gain and switching stress are given and explained in detail. Moreover the change of shoot through period in relation to PMG generated voltage is an important requirement and it is described here.

4.2 SHOOT THROUGH DISTRIBUTION

The Z-source inverter is able to provide higher ac voltage related to the dc-link voltage than in conventional Voltage Source Inverter (VSI), possessing embedded property of the boost converter. The output voltage and frequency from Direct Drive Permanent Magnet Generator (DDPMG) is
varied according to the wind velocity. In order to obtain a desired voltage and frequency irrespective of wind velocity, the PMG is coupled with Z-Source Inverter.

The conventional three phase voltage source inverter has eight permissible switching states such as six active states and two zero states. Besides these eight switching states of the inverter, the Z-source inverter has a ninth switching state called shoot through zero state. In this state the load terminals are shorted through both the upper and lower devices of any one phase leg or combination of any two-phase legs or all the three-phase legs. The shoot through zero state provides unique buck boost feature to the inverter (Rajakaruna et al 2010). The switching sequences of three-phase ZSI are given in Table 4.1. The distinct feature of Z-source inverter is its ability to produce ac output voltage of any value between zero and infinity (Peng et al 2003).

Table 4.1 Switching States of Three Phase ZSI

<table>
<thead>
<tr>
<th>States</th>
<th>$S_{ap}$</th>
<th>$S_{ap}$</th>
<th>$S_{bp}$</th>
<th>$S_{bp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active state [100]</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Active state [110]</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Active state [010]</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Active state [011]</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Active state [001]</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Active state [101]</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Null state [000]</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Shoot- Through state $V_1$</td>
<td>1</td>
<td>1</td>
<td>$S_{bp}$</td>
<td>$S_{bp}$</td>
</tr>
<tr>
<td>Shoot- Through state $V_2$</td>
<td>$S_{ap}$</td>
<td>$S_{ap}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Shoot- Through state $V_3$</td>
<td>$S_{ap}$</td>
<td>$S_{ap}$</td>
<td>$S_{bp}$</td>
<td>$S_{bp}$</td>
</tr>
<tr>
<td>Shoot- Through state $V_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Shoot- Through state $V_5$</td>
<td>1</td>
<td>1</td>
<td>$S_{bp}$</td>
<td>$S_{bp}$</td>
</tr>
<tr>
<td>Shoot- Through state $V_6$</td>
<td>$S_{ap}$</td>
<td>$S_{ap}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Shoot- Through state $V_7$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
4.3 OPERATING MODES OF ZSI BASED DDWECS

The complete circuit of proposed WECS with ZSI is illustrated in Figure 4.1. The output of the PMG is fed as input to the diode bridge rectifier through the stator inductances and resistances of PMG (Gholamreza 2006). The diode bridge rectifier acts as the dc source whose output is fed as input to Z-source inverter.

4.3.1 Mode I Zero State

Figure 4.2 shows the equivalent circuit of ZSI for mode I operation. The inverter bridge is operating in one of the two conventional zero states and shorting through either the upper or lower three devices. Hence it acts as an open circuit viewed from Z-source network. The PMG phases a and b are connected to the impedance network through the two diodes $D_1$ and $D_6$ in series with capacitor $C_a$.

![figure 4.1](image_url)

**Figure 4.1 Complete Circuit of PMG with Z-Source Inverter and Rectifier**
4.3.2 Mode II Shoot Through State

Figure 4.3 shows the equivalent circuit of ZSI in shoot through operation. In this mode because of higher voltages in inductors both diodes are OFF, separating dc link from the ac line. Hence PMG is disconnected from the load. This shoot through mode is to be used in every switching cycle during conventional zero state generated by the PWM control. The shoot through time or shoot through duty cycle is determined based on the required boost factor.
From the equivalent circuit of Figure 4.3 and symmetry of the Z-Source network the voltage across inductor and capacitor are equal.

\[ V_{c1} = V_{c2} = V_c, \quad V_{L1} = V_{L2} = V_L \]

When the shoot-through period \( T_0 \) is accommodated within switching period \( T \) during the one switching period \( T \) (Peng 2003) the capacitor and inductor voltages are equal.

\[ V_L = V_c \quad (4.1) \]

The voltage input to the Z-source network is given as

\[ V_{dc} = 2V_c \quad (4.2) \]

The voltage across inverter bridge is

\[ V_i = 0 \quad (4.3) \]

### 4.3.3 Mode III Traditional Active State

Figure 4.4 shows the equivalent circuit of ZSI in conventional active state operation. In this mode, the inverter bridge can be operated in any one of the six conventional active states. During this state, the voltage is impressed across the load. When the active period \( T_1 \) is accommodated within switching period \( T \) the voltage across the inverter is given by

\[ V_i = V_{dc} - V_c \quad (4.4) \]
Figure 4.4 Equivalent Circuit of ZSI for Mode III

The voltage obtained after the Z-source network which is input to inverter bridge is expressed in Equation (4.5).

\[ V_i = 2V_c - V_{dc} \]  \hspace{2cm} (4.5)

The average voltage across the inductor over the one switching period(T) is equal to zero. From Equations (4.2) to (4.5) the relation between capacitor and inductor voltage is given by

\[ \frac{V_c}{V_{dc}} = \frac{T_1}{T_{1-T_0}} \]  \hspace{2cm} (4.6)

The voltage input to inverter bridge given in Equations (4.4) and (4.6) is expressed in Equation (4.8)

\[ V_i = 2V_c - V_{dc} \]

\[ V_i = \frac{T_1}{T_{1-T_0}} V_{dc} \]  \hspace{2cm} (4.7)
\[ V_i = B \frac{V_{dc}}{2} \]  \hspace{1cm} (4.8)

where

\[ B \text{ is Boost Factor, } B = \frac{1}{1 - 2D_0} \]

\[ D_0 \text{ is shoot through duty ratio, } D_0 = \frac{T_0}{T} \]

The rectified output voltage of two-stage WECS is

\[ V_{dc} = \frac{3\sqrt{3}}{\pi} V_m \]  \hspace{1cm} (4.9)

Now the Equation (4.8) becomes

\[ V_i = \frac{3\sqrt{3} V_m}{2\pi} B \]  \hspace{1cm} (4.10)

The rms output voltage of the ZSI can be expressed as,

\[ V_{ac} = \frac{3\sqrt{6}}{2\pi} B M \]  \hspace{1cm} (4.11)

\[ V_{ac} = \frac{3\sqrt{6}}{2\pi} G \]  \hspace{1cm} (4.12)

where

\[ M \text{ is modulation index} \]

\[ G \text{ is voltage gain} \]
4.4 DIFFERENT PWM SCHEMES FOR ZSI

The same Pulse Width Modulation (PWM) methods for VSI can be used to switch the ZSI with slight modifications. The distribution of the shoot-through period in the switching waveforms of the conventional PWM concept is the key factor to control the output voltage of ZSI. The dc link voltage boost, controllable range of ac output voltage, voltage stress across the switching devices and harmonic profile of the ac output parameters are purely based on the method of control algorithm adapted to insert the shoot-through period. There are number of control methods which have been presented in recent years. In this section, maximum boost control (MBC) by Third Harmonic Injection Pulse Width Modulation (THI PWM), and Modified Space Vector Pulse Width Modulation schemes (MSVPWM) are explained elaborately and their performance control characteristics are investigated in many fronts.

4.4.1 Carrier PWM with Third Harmonic Injection Scheme

Figure 4.5 shows the maximum boost control PWM scheme with Third Harmonic Injection (THI) control method. In this PWM scheme, a third harmonic component with $1/6^{th}$ amplitude of the fundamental component is injected into the three phase reference voltages. $V_r$ of the R phase reaches its peak value at $\sqrt{3}/2$ M while $V_y$ of the Y phase reaches its minimum value at-$\sqrt{3}/2$ M at $\pi/3$. Therefore, a unique feature can be obtained through two straight lines, $V_p$ and $V_n$ to control the shoot through time with 16% third harmonic injection. The magnitude and width of the shoot through pulses are varied according to PMG generated voltage variations.
Figure 4.5 Carrier Based PWM Scheme with Third Harmonic Injection

Third harmonics injection is commonly used in three-phase inverter systems to increase the modulation index (M) range. This can also be used here to increase the range of M so as to increase system voltage gain range. In this control, the maximum modulation index can be achieved with the third harmonic injection at \( \frac{1}{6} \) amplitude of fundamental (Shen et al 2005).
To operate ZSI, the shoot through duty ratio is repeated at every cycle. With this control method the boost factor and voltage gain (Shen et al 2005) are given as

\[ B = \frac{\pi}{3\sqrt{3} M - \pi} \]  
\[ G = \frac{\pi M}{3\sqrt{3} M - \pi} \]  

Maximum shoot through duty ratio obtained by this scheme can be written as:

\[ D_0 = \frac{2\pi - 3\sqrt{3}\pi}{2\pi} \]  

Important expressions in this PWM method are given below:

Voltage gain

\[ G = M \frac{1}{1 - 2 D_0} \]  

Modulation index

\[ M = \frac{\pi G}{3\sqrt{3} G - \pi} \]  

Boost factor

\[ B = \frac{3\sqrt{3} G - \pi}{\pi} \]  

The voltage switching stress can be expressed as

\[ V_s = B V_{dc} \]

\[ V_s = \frac{3\sqrt{3} G - \pi}{\pi} V_{dc} \]

\[ V_s = \frac{\pi}{3\sqrt{3} G - \pi} V_{dc} \]
The ratio of voltage across the switch to ac output voltage is

\[ \frac{V_s}{GV_0} = \frac{3\sqrt{3}}{\pi} \frac{1}{G} \]  

(4.24)

The ac output voltage of the ZSI with this scheme is given as

\[ V_{ac} = \frac{9\sqrt{6}}{2\pi} \frac{\sqrt{3} M.B - \pi}{\pi} V_{rms} M \]  

(4.25)

where, \( V_{rms} = \) Per phase PMG generated voltage.

The maximum shoot through duty ratio can be calculated by \((1-\sqrt{3}/2M)\) (Miaosen 2006). When the magnitude of carrier wave is greater than \(V_p\) or smaller than \(V_n\), the inverter works in shoot through state. Figure 4.6 shows the control circuit and pulse generation method of THI PWM based ZSI.

![Figure 4.6 Control Circuit for ZSI Using Third Harmonic Injection Control Scheme](image)

4.4.2 Modified Space Vector Pulse Width Modulation (MSVPWM)

As Z-Source Inverter has shoot through period in addition to six active states and two null states, the shoot through time period is inserted into
the space vector. The insertion of additional shoot through period into the normal space region differentiates this method from the conventional space vector modulation method and hence the name modified space vector modulation. The shoot through period insertion is accomplished without disturbing the active vectors in the space region. It is evenly assigned to each phase within the null or zero state periods (Von Zimmermann et al Piepenbreier 2009). The distribution of shoot-through time period at sector-I is given in Figure 4.7.

The Modified Space Vector Pulse Width Modulation (MSVPWM) is the approximation of the resultant (reference) voltage by vector addition of the inverter output voltage in the given period (Das et al 2009). This helps in producing synchronized and symmetric output voltage which leads to the self balancing of the dc bus voltage over every cycle of the fundamental. Owing to lower current harmonics and wide range of modulation index, the MSVPWM techniques find wide spread applications. The resultant voltage lies between two arbitrary voltage vectors $V_i$ and $V_{i+1}$ (Poh et al 2005 and Tan et al 2005) as shown in Figure 4.8.

Figure 4.7 Switching Sequence of MSVPWM at Sector I
The reference voltage in the space vector is given by,

$$V_{ref} = V_i T_1 + V_{i+1} T_2$$

(4.26)

The modulation index used in the modified space vector modulation method can be found as follows,

$$M = \frac{V_{ref}}{\frac{2}{3} V_{dc}}$$

(4.27)

Time period of the active voltage vector is,

$$T_1 = \sqrt{3} \frac{T_1 |V_{ref}|}{V_{dc}} \sin \left( \frac{\pi}{3} - \alpha + \frac{n-1}{3} \pi \right)$$

(4.28)

and

$$T_2 = \sqrt{3} \frac{T_2 |V_{ref}|}{V_{dc}} \sin(\alpha - \frac{n-1}{3} \pi)$$

(4.29)
Time period of the zero vector is,

\[ T_0 = T_s - T_1 - T_2 \]  \hspace{1cm} (4.30)

Hence the shoot through period is given by,

\[ T_{sh} = \frac{T_0}{3} \]  \hspace{1cm} (4.31)

where,

\[ 0 \leq \alpha \leq 60^\circ \]

\( T_s = \) Total switching period

\( T_{sh} = \) Shoot through period

The placement of shoot through in the switching sequence of ZSI is shown in Figure 4.9.

\[ \text{Figure 4.9 Placement of Shoot Through Period in the Zero State of Sector I in MSVPWM Control Scheme} \]

The major expression for MSVPWM based ZSI are given as follows.
The relation between boost factor and modulation index is given as

\[ B = \frac{1}{1 - 2D_0} \]

The voltage gain is given as

\[ G = \frac{4 \pi M}{7 \sqrt{3} M - \pi} \]  \hspace{1cm} (4.33)

The voltage switching stress is,

\[ V_s = \frac{3\sqrt{3} G - \pi}{\pi} V_{dc} \]  \hspace{1cm} (4.34)

The ratio of voltage across switches to the output ac voltage is expressed as

\[ \frac{V_s}{G V_0} = \frac{19\sqrt{3}}{4 \pi} - \frac{2}{G} \]  \hspace{1cm} (4.35)

The output ac voltage of inverter is

\[ V_{ac} = \frac{V_{rms}}{\sqrt{2 - \sqrt{M - \pi}}} B M \]

where, \( V_{rms} \) = Per phase PMG generated voltage.

Figure 4.9 describes the distribution of shoot through time sectors of MSVPWM. The distribution is achieved without disturbing the active time
period thereby utilizing the zero time period to its maximum value based on varying input voltage. The boost factor is varied with shoot through duty ratio. On the other hand it is changed according to input voltage. Figure 4.10 shows the MSVPWM pulse generation method for ZSI.

![Figure 4.10  Control Circuit for ZSI using MSVPWM Scheme](image)

### 4.5 TRANSFER FUNCTION MODEL

The transfer function model of the proposed controller need to be examined to evaluate the stability of the system. In the conventional controllers the modulation index and shoot through duty ratio are regulated with respect to capacitor voltage and the load terminal voltage. In the proposed system the dc link voltage is boosted to the required level simply by sensing the PMG generated voltage. During the boost mode the generated voltage is regulated to the desired level by adjusting the shoot through duty ratio. During non shoot through mode the shoot through duty ratio is constantly maintained by controlling the modulation index as in the case of
conventional inverters. To obtain the transfer function of the proposed system the terminal voltage of the PMG is represented as a function of shoot through duty ratio.

4.5.1 State Space Averaged Model

State space modeling approach is normally used to model the controller of power converters. The physical state variables are inductor currents and capacitor voltages. The values of the state variables depend on the history of the system at any point of time. The state variables can be written in matrix form. (Gokhan sen 2008, Jingbu Liu et al 2007)

\[
\mathbf{k} \frac{dx(t)}{dt} = \mathbf{A} x(t) + \mathbf{B} u(t) \tag{4.36}
\]

\[
Y(t) = C x(t) + E u(t)
\]

Here, the state vector \(x(t)\) is a vector containing all the state variables, that is, the inductor currents and capacitor voltages. The input vector \(u(t)\) contains the independent inputs to the system, such as the input voltage source. \(\mathbf{k}\) is a matrix containing the values of capacitances and inductances, such that \(\mathbf{k} \frac{dx(t)}{dt}\) is a vector containing the inductor currents and capacitor voltages.

4.5.2 State Space Representation during Shoot Through State

The state space representation of ZSI during the shoot-through state is shown in Figure 4.11. The state space form given in Equation (4.36) is derived as follows.
During shoot through period the rectifier diodes are in off condition.

Hence \( V_{DC} = 0 \)

The voltage input to the ZSI is written as,

\[
L_1 \frac{di_{c1}}{dt} - V_{c1}(t) = 0 \quad (4.39)
\]

From the equivalent circuit of Figure 4.11

\[
i_{L1} + i_{c1} = 0
\]

\[
\text{ie} \quad C_1 \frac{dV_{c1}(t)}{dt} = -i_{L1}(t) \quad (4.40)
\]

The output side load voltage across inductor capacitor combination can be written as,

\[
V_1(t) + R_1 i_1(t) = 0
\]

\[
L_1 \frac{di_1(t)}{dt} = -R_1 i_1(t) \quad (4.41)
\]
Taking \( L_1 = L_2 = L \) and \( C_1 = C_2 = C \)

From Equations (4.39) to (4.41) state space equations for shoot through duty ratio period is given in Equation (4.42)

\[
\begin{bmatrix}
L & 0 & 0 \\
0 & C & 0 \\
0 & 0 & L_e
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} i_L(t) \\
\frac{d}{dt} V_c(t) \\
i_i(t)
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
1 & 0 & -R_e
\end{bmatrix}
\begin{bmatrix}
i_L \\
V_c \\
i_i
\end{bmatrix}
\]

(4.42)

where,

\[
K = \begin{bmatrix}
L & 0 & 0 \\
0 & C & 0 \\
0 & 0 & L_e
\end{bmatrix}
A_1 = \begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & -R_e
\end{bmatrix}
B_1 = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

4.5.3 State Space Representation During Active State

Similarly, the state space representation of ZSI during active state as shown in Figure 4.12 can be written in the form as given in Equation (4.36).

![Figure 4.12 DC Equivalent Circuit of ZSI During Active Period](image-url)
The input voltage to the ZS network is expressed as

\[
V_{DC} = L \frac{di_L(t)}{dt} + V_c(t) \tag{4.43}
\]

\[
i_L = i_c(t) + i_L(t)
\]

\[
i_L(t) = \frac{C}{dt} \frac{dv_c(t)}{dt} + i_L(t)
\]

\[
C \frac{dv_c(t)}{dt} = i_L(t) - i_i(t) \tag{4.44}
\]

The expression for the input voltage to the inverter is given as,

\[
V_{DC} = V_c(t) - R_i i_i(t) - V_i(t)
\]

The voltage across load inductor becomes,

\[
V_i(t) = V_c(t) - R_i i_i(t) - V_{DC}
\]

\[
L_i \frac{di_i(t)}{dt} = V_c(t) - R_i i_i(t) V_{DC} \tag{4.45}
\]

The expressions in Equations (4.43) to (4.45) during active period of ZSI can be put it in the matrix form as

\[
\begin{bmatrix}
L & 0 & 0 \\
0 & C & 0 \\
0 & 0 & L_i
\end{bmatrix}
\begin{bmatrix}
\frac{di_L}{dt} \\
\frac{dv_c}{dt} \\
\frac{di_i}{dt}
\end{bmatrix}
= \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & -1 \\
0 & 1 & -R_i
\end{bmatrix}
\begin{bmatrix}
i_L \\
i_c \\
i_i
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
-1
\end{bmatrix} V_{DC} \tag{4.46}
\]

where,

\[
K = \begin{bmatrix}
L & 0 & 0 \\
0 & C & 0 \\
0 & 0 & L_i
\end{bmatrix}
A = \begin{bmatrix}
0 & -1 & 0 \\
-1 & 0 & -1 \\
0 & 1 & -R_i
\end{bmatrix}
B = \begin{bmatrix}
1 \\
0 \\
-1
\end{bmatrix}
\]
Equation (4.36) can be solved to find the equilibrium state vector. So the equilibrium values (Gokhan Sen 2008) of the state variables can be written as,

\[ 0 = Ax + BU \]  
\[ (4.47) \]

\[ A = DA_1 + D'A_2 \]

where,  
\[ D = \text{Duty ratio during shoot through period} \]
\[ D' = \text{Duty ratio during non shoot through period} \]

\[ A = \begin{bmatrix} 0 & D-D' & 0 \\ D-D' & 0 & -D' \\ 0 & D' & -R_l \end{bmatrix} \]  
\[ (4.48) \]

\[ B = D B_1 + D' B_2 \]

\[ B = \begin{bmatrix} D' \\ 0 \\ -D' \end{bmatrix} \]  
\[ (4.49) \]

Substituting the Equations (4.48) and (4.49) in to (4.47) it is obtained

\[ 0 = (D - D') V_c + D' V_{dc} \]  
\[ (4.50) \]

Equation (4.50) can be solved to find the equilibrium state vector. So the equilibrium values of the state variables can be written as,

\[ 0 = (D - D') V_c + D' V_{dc} \]
\[ 0 = (D' - D) i_c - D' i_L \]
Hence

\[ V_c = \frac{-DV_{DC}}{D - D'} \]

\[ V_c = \frac{D'}{D' - D} V_{DC} \quad (4.51) \]

\[(D' - D) i_L = D' i_i \]

\[ i_L = \frac{D'}{D' - D} i_i \quad (4.52) \]

Also from equation 4.50

\[ 0 = D' V_c - R_i i_i - D' V_{DC} \]

\[ 0 = \frac{D'}{D' - D} D' V_{DC} - R_i i_L - D' V_{DC} \]

\[ i_i = \frac{V_c}{R_i} \quad (4.53) \]

Using Equations (4.50) and (4.51), the state equations of the small signal AC model of the ZSI can be written in open form and written as

\[ V_{DC}(t) = V_{DC} + \dot{V}_{DC}(t) \text{ and } d(t) = D = \dot{d}(t). \]

The resulting small signal perturbations in state variables will be represented as \( x(t) = \dot{x}(t) \). Using Equations (4.50) and (4.51), the state equations of the small signal AC model (Gokhan Sen 2008) of the ZSI can be written in open form,

\[ K \frac{d\dot{x}(t)}{dt} = A \dot{x}(t) + B \dot{u}(t) + \{(A_1 - A_2)X + (B_1 - B_2)U\} \dot{d}(t) \]

\[ \begin{bmatrix} L & 0 & 0 \ D & 0 & 0 \ 0 & C & 0 \end{bmatrix} \begin{bmatrix} \dot{i}_c \ V_c \ \dot{i}_i \end{bmatrix} = \begin{bmatrix} 0 & D - D' & 0 \ D - D & 0 & -D' \ 0 & D - D & -D \end{bmatrix} \begin{bmatrix} i_L \ V_{IC} \ i_i \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \ -2 & 0 & 1 \ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_i(0) \ V_c(0) \ i_i(0) \end{bmatrix} \begin{bmatrix} 1 \ -1 \ 0 \end{bmatrix} \dot{d}(t) \]

\[ \begin{bmatrix} \end{bmatrix} \]

\[ (4.54) \]
\[
A_1 - A_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -R_1 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -R_1 \end{bmatrix}
\]

\[
A = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}
\]

\[
B_1 - B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}
\]

From Equation (4.54)

\[
L \frac{di_L (t)}{dt} = (D - D') V_c (t) + (D - D') V_{dc} (t) + 2 V_c (t) dt - V_{dc} (t) d(t)
\]

\[
C \frac{dV(t)}{dt} = (D - D') i_L - D' i_i + [-2i_L (t) - i_L (t)] d(t)
\]

\[
L \frac{di_L}{dt} = D' V_c - R_i i_L - D' V_{dc} + (-V_c + V_{dc}) d(t)
\]

Replacing \( \frac{d}{dt} \) by \( s \), the above equations can be written in \( s \) domain

\[
s L i_L (s) = (D - D') V_c (s) + D' V_{dc} (s) + (2 V_c - V_{dc}) d(s) \quad (4.55)
\]

\[
s C V_c (s) = (D - D') i_L (s) + (-2I_L + I_i) d(s) - D' i_i (s) \quad (4.56)
\]

\[
s L i_L (s) = 2 D' V_c (s) - D' V_{dc} (s) + (2 V_c - V_{dc}) d(s) - R_i i_L (s) \quad (4.57)
\]

\[
i_L (s) = G_{ig} (s) \hat{V}_{DC} (s) + G_{il} (s) \hat{d}(s)
\]

\[
\hat{V}(s)=G_{vg} (s)\hat{V}_{DC} (s)+G_{sd} (s)\hat{d}(s)
\]

(4.59)
The capacitor voltage with shoot through duty ratio transfer function is given by Equation (4.60).

\[
G_{vd}(s) = \left. \frac{\hat{V}_c(s)}{\hat{d}(s)} \right|_{\hat{v}_{dc}(s)=0}
\]

\[
\frac{(-2I_L + I_i) L_i L_s^3 + \left[(-2I_L + I_i) R_i L + (D' - D) (2V_c - V_{dc}) L_i \right] + L D' (2V_c - V_{dc})}{L_i L_s^3 + R_i LC s^2 + \left[2D'^2 L + L_i (D' - D)^2\right] s + R_i (D - D')^2}
\]

(4.60)

The control-to-output voltage transfer function \( \frac{V_o(s)}{d(s)} \) with different values of shoot through duty ratio is given by,

\[
G_{vd}(s) = \frac{V_o(s)}{d(s)} = \frac{(\frac{D}{2D - 1})^2 \times RV_o}{((R_i L_i C + (\frac{D}{2D - 1}) s^2 + L_i + (\frac{D}{2D - 1})^2 R_i) \times D}
\]

(4.61)

The simulated bode blot of output voltage for duty ratio is shown in Figure 4.13. The bode plot shows the designed system is stable. The system is stable for the voltage range of 90V to 360V (per phase).

![Figure 4.13 Magnitude and Phase Plots of Output Voltage Controlled ZSI](image)
4.6 CONTROLLER FOR ZSI BASED DDWECS

In order to achieve better performance of both DC boost control, ac line voltage control of ZSI, and PMG generated voltage are sensed and compared with the pre-defined reference values. The boost factor is determined based on the shoot through time period and the desired output voltage is obtained. The boost factor is the control variable of the ZSI voltage and the modulation index for that of load voltage. The complete controller block diagram for direct drive wind energy conversion system with PMG and ZSI is shown in Figure 4.14.

![Image of Block Diagram]

**Figure 4.14 Implementation Block diagram of Combined DC Link and Line Voltage Controller**

During fluctuating voltage conditions the ZSI dc link voltage is adjusted in accordance with PMG generated voltage by periodical insertion of
shoot through states. The regulation of dc link voltage is done based on the PMG voltage variations by voltage feedback loop. In this mode the dc link voltage is measured and compared and error is processed to adjust the time period of shoot through state. Figure 4.14 shows the complete block diagram of the controller circuit for ZSI. The two parameters to be changed in order to get the desired output ac voltage in the ZSI are the modulation index and the boost factor which depend on the shoot through time period inserted in the conventional switching waveform.

4.7 SUMMARY

The basic operating modes of Z-Source inverter for direct drive wind energy conversion system with its equivalent circuits were briefly explained. The two PWM methods such as maximum boost by third harmonic injection and modified space vector PWM schemes with shoot through placement for various values of input voltages have been discussed. The mathematical relation between PMG generated voltage, boost factor, shoot through time period, modulation index and voltage gain has been derived. The transfer function and small signal model of ZSI is derived. The stability of the ZSI is checked for shoot through duty ratio from 0.2 to 0.3. The controller for ZSI is to obtain the desired voltage from the variable voltage in direct drive wind energy conversion system. The detailed analysis of these two PWM schemes are compared for different values of PMG generated voltage and discussed in chapter 5.