7. Numerical study on magneto-convection in a lid-driven cavity with corner heater

7.1 Introduction

From many of the studies in the literature, it can be understood that the analyses have been dealt the fully heated walls much. However, heaters take place over a narrow segment of the vertical walls in many engineering applications. In such cases, the size and location of the heater(s) play an important role on the fluid flow and heat transfer mechanisms. Hence, determining the optimum heater size and their location becomes noteworthy for better utilization of such systems. In fact, the corner heating during the mixed convection in square cavities has not received much attention. On the other hand, increasing the importance of the MHD convection systems requires thermally enhancement analysis. Hence, a numerical investigation involving MHD convection with corner heating would be helpful to enrich the ideology in thermally enhanced design of systems. The main objective of this chapter is to provide the valid, essential and application oriented knowledge about the effects of corner heating on MHD convection. In particular, the proposed work is to examine the heat transfer behavior and flow patterns on mixed convection with corner heating in a lid-driven cavity in the presence of a magnetic field.

7.2 Mathematical formulation

The physical situation is depicted by a schematic diagram of a two-dimensional square cavity of length L in Figure 7.1 in which the origin of the Cartesian coordinate system is kept at the lower left corner of the cavity. It is assumed that the flow is unsteady, laminar, incompressible, and two-dimensional. The velocity components $u$ and $v$ are, respectively, along $x$-direction and $y$-direction. Letting the lid of the cavity to move in its own plane with a constant speed $U_0$, a part of the surfaces along left and bottom walls is maintained at a constant temperature $\theta_h$ and the right sidewall is at a lower temperature $\theta_c$ such that $\theta_h > \theta_c$.

In fact, from the left-bottom corner of the cavity, three different lengths of heater are simultaneously considered along the bottom and left walls. For every length of the heater considered partly along the left wall of the cavity, three different constant lengths of heater are varied along its bottom wall. Thus, nine distinct configurations of corner heating are under investigation to examine the heat transfer characteristics and the flow behavior. The top wall
and the remaining surfaces that are not heated on both left vertical wall and bottom wall are insulated.

The gravity acts in the downward direction. The cavity is filled with an electrically conducting fluid of low Prandtl number like liquid metal. A uniform magnetic field is applied in the horizontal direction with a constant magnitude \( B_0 \). The electric current \( J \) and the electromagnetic force \( F \) are defined by \( J \mathbf{\|} V \mathbf{\|} B \) and \( F \mathbf{\|} V \mathbf{\|} B \mathbf{\|} B \), respectively. The induced magnetic field due to the motion of the electrically conducting fluid is very small compared to the applied magnetic field. Therefore the magnetic Reynolds number is too small and it is neglected. Further, the viscous dissipation and Joule heating are assumed to be negligible.

Figure 7.1 Schematic diagram of the physical configuration and the coordinate system
The unsteady governing equations of motion for incompressible electrically conducting fluid are described as follows:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (7.1) \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (7.2) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \frac{\partial y}{\partial x} - \frac{e}{\sigma} B_0 v^2 \quad (7.3) \\
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} &= \frac{\partial}{\partial x} \left( \rho \frac{\partial y}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho \frac{\partial y}{\partial y} \right) + k \frac{\partial^2 \rho}{\partial x^2} + \frac{\rho c_p}{\rho_o} \frac{\partial^2 \rho}{\partial y^2} \quad (7.4)
\end{align*}
\]

where \( c_p \) is the specific heat, \( g \) is the gravity, \( k \) is the thermal conductivity, \( p \) is the pressure, \( t \) is the time, \( \rho \) is the fluid temperature, \( \nu \) is the kinematic viscosity \( \rho_o \) is the and density.

The appropriate initial and boundary conditions for the problem concerned with the corner heaters can be expressed as follows:

For \( t \neq 0 \):
\[
\begin{align*}
&u \quad v \quad 0 \quad 0 \quad 0 \quad (x, y) \quad L \\
\end{align*}
\]

For \( t = 0 \):
\[
\begin{align*}
&u \quad U_0 \quad v \quad 0 \quad 0 \quad y \quad L \\
&0 \\
&u \quad v \quad 0 \quad 0 \quad \frac{y}{0} \quad 0 \quad x \quad \frac{h x}{x} \quad \frac{0,0 \quad 0 \quad y}{h y} \\
&0 \quad \frac{h}{0} \\
&u \quad v \quad 0 \quad 0 \quad \frac{y}{0} \quad h x \quad x \quad L \\
&0 \quad \frac{y}{0} \\
&u \quad v \quad 0 \quad 0 \quad \frac{x}{L} \\
&0 \quad \frac{c}{x} \\
&u \quad v \quad 0 \quad 0
\end{align*}
\]
\[ x \neq 0, \quad h y \neq y \neq L \]

where \( h x \) & \( h y \) denote the length of heaters along \( x \)- and \( y \)- directions, respectively. By varying heaters of constant length from the left-bottom corner, on left and bottom sidewalls, there arise three different cases of corner heating. As a first case, the lengths of heater along bottom wall are varied as \( L/4 \), \( L/2 \) and \( 3L/4 \) while a constant length \( L/4 \) of heater along left vertical wall is considered. The remaining two cases are set up by changing the lengths along left wall as \( L/2 \) and \( 3L/4 \) for the same heaters on bottom wall taken as in the earlier case. The governing equations (7.1) to (7.5) are transformed into dimensionless form by using the following non-dimensional variables:
\[ X \frac{X}{x}, Y \frac{y}{y}, U \frac{u}{u}, V \frac{v}{v}, Hx \frac{hx}{hx}, Hy \frac{hy}{hy}, T \frac{T}{T}, tU_0, P \frac{p}{p} \]

\[ L \frac{L}{L}, \frac{U_0}{U_0}, \frac{U_0}{U_0}, \frac{L}{L}, \frac{h}{h}, \frac{c}{c}, \frac{L}{L}, \frac{U^2}{U^2}, \]

\[
\begin{align*}
&\text{After non-dimensionalization, the following set of governing equations in vorticity-stream function formulation is obtained.} \\
&\begin{array}{c}
U \frac{U}{X}, V \frac{V}{Y}, 1 \frac{1}{2}, 2 \frac{2}{2}, \frac{\text{Ri}}{\text{Ri}}, \frac{T}{T}, \frac{\text{Ha}^2}{\text{Ha}^2}, \frac{V}{V}
\end{array} \\
&\frac{2}{2}, \frac{\text{Ri}}{\text{Ri}}, \frac{T}{T}, \frac{T}{T}, \frac{2}{2} \frac{T}{T}
\end{align*}
\]

\[ \begin{align*}
&\begin{array}{c}
U \frac{U}{X}, V \frac{V}{Y}, \frac{1}{2}, \frac{\text{PrRe}}{\text{PrRe}}, \frac{X}{X}, \frac{Y}{Y}
\end{array} \\
&\begin{array}{c}
U \frac{U}{Y}, \frac{U}{X}, \frac{X}{X}, \frac{Y}{Y}
\end{array}
\end{align*}
\]

\[ (7.6) \]

\[ (7.7) \]

\[ (7.8) \]

\[ (7.9) \]

The dimensionless parameters in the above equations (7.6) to (7.9) are defined as follows: \( H a \frac{H a}{B_L L} \), the Hartmann number, \( \text{Pr} \frac{\text{Pr}}{v/\alpha} \), the Prandtl number, \( \text{Re} \frac{\text{Re}}{U_0 L} \), the Reynolds number, \( \text{Ri} \frac{\text{Ri}}{Gr/Re^2} \), where \( Gr \frac{Gr}{g TL^3/\alpha^2} \), the Grashof number, and \( \text{Richardson number} \) is a measure to determine the relative strength of the buoyancy-driven convection and forced convection.

The initial and boundary conditions for the considered problem can be expressed in dimensionless form as follows:

\[ \begin{align*}
\text{For } U &\quad V \quad 0 \quad T \quad 0 \quad 0 \quad (X,Y) \quad 1 \\
0 &\quad 0
\end{align*} \]

\[ \begin{align*}
\text{For } U &\quad 1, V \quad 0 \quad T \quad 0 \quad 0 \quad (X,Y) \quad 1 \\
U &\quad 0, V \quad 0 \quad 0 \quad (X,Y) \quad 1 \\
U &\quad 0, V \quad 0 \quad 0 \quad (X,Y) \quad 1
\end{align*} \]

\[ (7.10) \]
The important physical quantity to describe the heat transfer rate across the cavity is the Nusselt number which is a ratio of convective heat transfer and conductive heat transfer. Further, it is a measure to provide the relative strengths between them. The local Nusselt numbers for the heating surfaces along bottom wall and vertical sidewall are respectively given by

\[ \text{Nu}_x \frac{T}{Y}, \quad \text{Nu}_y \frac{T}{X} \] whereas for the cold wall, it is obtained from

\[ \text{Nu}_c \frac{T}{X} \] . The average Nusselt number for overall heat transfer along the cold wall is

\[ \frac{1}{Y} \int_0^Y \text{Nu}_c dY \]. The equations (7.6)-(7.8) with the initial and boundary conditions (7.10) are numerically solved by the finite volume method as given in Chapter 3.

7.3 Results and discussion

A numerical study has been dealt to investigate the effects on mixed convection in a lid-driven square cavity when a uniform magnetic field is present. Both sides of left and bottom surfaces are partly heated from the left-bottom corner of the cavity simultaneously while the right vertical wall is kept at a lower temperature. For a constant length of heater on the left sidewall, three different lengths of heater are considered along bottom wall. Hence, for three different lengths of heater along left wall, three distinct cases with nine configurations occur. The working fluid is chosen as an electrically conducting fluid of low Prandtl number \( Pr \leq 0.054 \). The parameters governing the heat transfer and fluid flow are the lengths of heater \((Hx,Hy)\), the Richardson number \(( Ri \approx Gr/Re^2 )\) and the Hartmann number \(( Ha )\). The lengths of the corner heater along horizontal and vertical directions are \( Hx \) and \( Hy \), respectively, which are taken as 0.25, 0.5, and 0.75. For a fixed Grashof number \( Gr \approx 10^4 \) and the variations of Reynolds number \(( Re )\) from 10 to \( 10^3 \), the range of Richardson number is set as \( 0.01 \leq Ri \leq 100 \). The influence of the magnetic force on the flow field inside the cavity is depending on the magnitudes of the Hartmann number and its values are taken to be 0, 25, and 100.

7.3.1 Effects of various lengths of heater
Figures 7.2(a)-(i) illustrate the isotherms for different lengths of the heater which are varied from left-bottom corner of the cavity along the left sidewall and bottom wall for various Richardson numbers such as $Ri = 0.01, 1$ and 100. The Hartmann number is set constant at 25.
As a first case, keeping the length of heater along the left sidewall as $Hy \equiv 0.25$, the lengths of heater along bottom wall are varied from 0.25 to 0.75; see Figures 7.2(i) (a-c). The second and third cases occur for the vertical lengths of heater 0.5 and 0.75, respectively, while the lengths of heater along bottom wall are varied as earlier. Obviously, for all the three cases, an enhanced temperature distribution is observed around the heated regions along both directions. Due to sharp temperature gradients near the left wall, a strong thermal boundary layer is formed along the heaters when forced convection dominates. Further, the temperature distributions show curl like distribution due to strong convection. The horizontal heated layer remains closer to the bottom wall of the cavity. When the lengths of heater along the bottom wall are 0.5 and 0.75, it seems that a thin boundary layer appears at the right-top corner of the cavity. For further increase in the lengths of heater along left sidewall, it is interesting to see that the formation of the thin boundary layer on the cold wall gradually increases; see Figures 7.2(i) (d-i). Though the lengths of heater are equally considered along both directions, temperature distribution is getting better according to the increase in lengths of the heater. In general, the heat distribution is better when the vertical lengths of heater are increased than that of the increase in the bottom lengths of heater.

The fluid flow of the varied lengths of heater along left and bottom walls for various Richardson numbers is shown in Figures 7.3(a)-(i). In all the three cases of variations in length of heater along both directions, the flow field is described in the form of streamlines which consists of single cell pattern with clockwise rotation. Since the cavity is heated partly along both vertically and horizontally, the heated particles near bottom wall are also raised adjacent to the heater on left sidewall and fall along the opposite cooled wall. Thus, the clockwise rotating cell appears almost in the entire cavity in all the cases. The clustered streamlines near the top wall indicate that steep velocity gradients occur near the top wall of the cavity. Hence, the center of the cell is near the top wall for all variations in the Richardson number. The cell is slightly elongated in all the cases of length variation while natural convection dominates. Particularly, the speed of the flow is almost uniform throughout the cavity and the increase in the lengths of heater shows no significant change in the flow field.

Figures 7.4(a)-(d) demonstrate the local Nusselt number variations for different lengths of heater and for the Richardson numbers $Ri \equiv 0.01$ and 100 when $Ha \equiv 25$. Figure 7.4(a-b) depicts the influence of vertical lengths of heater on heat transfer rate while $Hx$ is kept constant at 0.5. It can be understood that the heat transfer is enhanced well at $Hy \equiv 0.75$ and it becomes low at $Hy \equiv 0.25$ in the dominance of forced convection. But, in the case of natural convection,
high heat transfer occurs at $Hy \cong 0.5$ and heat transfer is observed to be low for $Hy \cong 0.75$. On the other hand, these results turn out opposite when the horizontal lengths of heater are varied while $Hy \cong 0.5$. i.e. highest local Nusselt number occurs at $Hx \cong 0.5$ when $Ri \cong 0.01$ and the length $Hx \cong 0.5$ results the lowest local Nusselt number for $Ri \cong 100$. These can be witnessed in Figures 7.4(c-d).

The overall heat transfer rate of cold wall for various lengths of heater along horizontal and vertical directions against the Richardson numbers with $Ha \cong 25$ is exemplified in Figures 7.5(a)-(c). It can be observed that the average heat transfer rate is strictly increased when the horizontal lengths of heater is increased. Particularly, for all vertical lengths $Hy$, the average Nusselt number gets its highest value for the maximum horizontal length $Hx \cong 0.75$ compared with the others. Moreover, it can be verified that overall heat transfer rate is firmly increased while increasing the vertical lengths of heater. Generally, it can be concluded that the increase in lengths of heater along either of the directions result in the increase of overall heat transfer rate. Also, it is observed that the heater length in the $x$-direction on the heat transfer and on the flow pattern.

### 7.3.2 Effects of the Richardson number

Figures 7.2(a)-2(i) shows the effects of the Richardson numbers $Ri \cong 0.01, 1$ and 100 and for various lengths of heater on temperature distributions with a constant magnetic field parameter $Ha \cong 25$. When $Ri \cong 0.01$, it can be noticed that the concentrated isotherms show steep temperature gradients near the heaters. As a consequence, the temperature gradients near both bottom and left walls lead to the development of thermal boundary layer significantly. The heat energy is transported through convection mode and it can be viewed that the temperature distribution is worse near the cold wall. In the mixed convection regime, the formation of boundary layers along heaters gradually decreases. In the buoyancy-driven convection mode, i.e. for $Ri \cong 100$, temperature gradients occur uniformly throughout the cavity and boundary layers on either side of the vertical walls disappear. Moreover, the isotherms in this natural convection regime are almost vertical in the entire cavity, regardless of the lengths of heater, which shows that the dominant heat transfer mechanism in this regime is almost uniform. The strong thermal boundary layer near the heaters along both directions and the thin boundary layer near cold wall formed in the forced convection regime disappear in the buoyancy-driven convection mode. In a differentially heated cavity, the thermal boundary layer
is formed along the both hot and cold wall. Hence, it is confirmed that corner heating ideology is entirely different from differentially heated cavity.

Figures 7.3(a)-(e) illustrate the fluid flow in the form of streamlines for different Richardson numbers and for various lengths of heater. For $Ri \leq 0.01$, the flow field consists of a single eddy rotating in the clockwise direction. Since the corner heaters are placed along the bottom and left walls, the eddy occupies almost the entire cavity. Obviously, the flow near the upper boundary is dominated by the forced convection due to strong inertia force generated by the moving lid of the cavity. Also, the imposed uniform magnetic field tends to suppress the flow field towards the horizontal top boundary. Since the effect of the applied external magnetic field is poor in this regime, the flow pattern is not much suppressed. Hence, the flow pattern could be governed as desired inside the cavity by suppressing it with the appropriate set up of the moving wall. When mixed convection occurs, the circulating eddy shows no considerable change for any length of heater. When the buoyancy-driven convection dominates, i.e. at $Ri \leq 100$, the eddy is slightly elongated and a similar behavior is noted in the flow field for all the lengths of heater. The core region of the eddy still exists near the top wall.

The effects on the local Nusselt number for various Richardson numbers and lengths of heater are demonstrated by the Figures 7.4 with the Hartmann number 25. By varying the vertical lengths of heater for a constant horizontal length $Hx \leq 0.5$, the variations of local Nusselt numbers at $Ri \leq 0.01$ and $Ri \leq 100$ are exhibited respectively by 7.4(a) and 7.4(b). It is found that the increase in the Richardson numbers decreases the heat transfer rate. On the contrary, even if the horizontal lengths of heater are varied for a fixed vertical length $Hy \leq 0.5$, the local Nusselt number decreases while the Richardson number increases. This means that the heat transfer rate in the forced convection mode is higher than that of in the natural convection mode. This can be viewed in 7.4(c)-(d). The average heat transfer rate on cold wall for various vertical and horizontal lengths of heater against the Richardson numbers is depicted Figure 7.5(a)-(c) for fixed magnetic field strength $Ha \leq 25$. Obviously, the average heat transfer decreases when the Richardson number is increased. However, the overall heat transfer is better and independent of the variations in the Richardson numbers for the maximum heater length 0.75 along both directions.

### 7.3.3 Effects of the Hartmann number

Figures 7.6(a)-(c) exhibit the influence of various Hartmann numbers on temperature distribution and fluid flow for $Ri \leq 0.01$ and $Ri \leq 100$ when the lengths of heater are considered equally on both vertical and horizontal directions. Either $Ha \leq 0$ or $Ha \leq 25$, the
temperature distribution is enhanced through convection mode due to the shear force generated by the moving top wall for the considered equal lengths of heater at $Ri \leq 0.01$. For these values of the Hartmann number, no remarkable changes are noticed in the isotherms. This reveals the fact that still forced convection dominates the heat transfer mechanism and weak magnetic field does not create any impact on the heat distribution remarkably. On the other hand, if the Hartmann number is increased to 100, the isotherms seem to be straightened out as expected. In other words, the energy transportation is changed to conduction mode. Since the convective heat distribution is affected by the strong magnetic field $Ha > 100$, the thermal boundary layers near hot and cold walls vanish. As far as the streamlines considered, during the forced convection dominance, the flow is described by a single cell rotating in the clockwise direction and the cell appears in the entire cavity at $Ha \leq 0$ and 25. The speed of the fluid flow decreases owing to the increase in the magnetic field. A resistive force due to strong magnetic field controls the speed of the fluid particles and results in the suppression of fluid flow. Consequently, the core region of the circulating primary cell is near to the top wall of the cavity and the flow in the lower part of the cavity is almost stagnant.

On the other hand, when the buoyancy-driven convection dominates, i.e. at $Ri \leq 100$, the isotherms are almost parallel to the vertical walls, indicating that most of the heat transfer process is carried out by conduction. No other notable change is observed in the heat transfer though the Hartmann numbers are varied for the considered lengths of heater. But, noteworthy changes can be viewed in the fluid flow. For all the lengths of heater, when $Ha \leq 0$ or 25, a similar behavior in the flow is sighted. But, on imposing a strong magnetic field such as $Ha > 100$, the core region of the eddy is inhibited very close to the top wall of the cavity. Meanwhile, the vortex is elongated vertically for both the lengths 0.25 and 0.75 of the heater along both directions. When the length of heater is $Hx \leq Hy \leq 0.5$, there exists consistency in the flow speed and so is in the flow pattern. The flow becomes broadly stagnated in the lower part of the cavity. It is observed that the convective motion is totally inhibited with the increase of the Hartmann numbers. Further, it is proved that the fluid flow is independent on the lengths of heater since effect of magnetic field is significant in either of the cases, the forced convection and the natural convection.

Figures 7.7 and 7.8 depict the results of variations on local Nusselt numbers along both directions for various lengths $Hx$ and $Hy$ of heater with $Ri \leq 0.01$ and 100, respectively, when $Ha \leq 25$. Generally, in the dominance of forced convection, the heat transfer rate is high in the absence of magnetic field and increasing the Hartmann numbers such as 25 and 100 certainly reduces the heat transfer rate. Indeed, the heat transfer rate increases according to the
increase in lengths of heater and it reacts as stated earlier to the increase of applied magnetic field. From Figures 7.7(b-c), a notorious result is observed that the heat transfer rate is boosted up well instead of decreasing while the Hartmann numbers are increased near the leading edge of the heaters. But, no such results are observed along horizontal heaters. When the heat transfer rate along the horizontal and vertical heaters are compared for the applied transverse magnetic field, it is revealed that it affects the heat transfer rate strongly on vertical heaters. At the same time, it shows a remarkable increase on heat transfer rate for the horizontal heaters. On other hand, when buoyancy-driven convection dominates, i.e. at $\text{Ri} \leq 100$, the influence of the Hartmann number is ignorable. In other words, no significant variations on heat transfer rate are resulted on increasing the Hartmann numbers. Particularly, the local Nusselt numbers are decreased well in the free convection mode compared with forced convection mode. From this, it can be understood that the applied magnetic field creates its remarkable influence during the dominance of buoyancy-driven convection. Moreover, for all the values of the Hartmann number, only slight variations on the heat transfer rate are shown in the free convection regime; see Figures 7.8(a-f). Further, it can be noticed that the heat transfer rate becomes better along the horizontal heaters since the magnetic field affects strongly along vertical heaters.

The changes on overall heat transfer rate along cold wall are shown in Figures 7.9(a)-(e) by plotting the average Nusselt numbers against the Richardson numbers for different lengths of heater and the Hartmann numbers. In the Figures 7.9(a-c), the overall heat transfer rate for the equal lengths of heater such as 0.25, 0.5, and 0.75 on both directions and for the increased values of Hartmann numbers is presented. As a general behavior, it is observed that the average Nusselt numbers are decreased when the Hartmann number is increased. When $Hx \leq Hy \leq 0.25$, the effects for low Hartmann numbers such as 0 and 25 vanish up $\text{Ri} \leq 0.1$ to

and a sudden increase in the average heat transfer rate is shown with the increase in the Richardson numbers. Thereafter, it is gradually decreased while the Richardson numbers are increased; see Figures 7.9(a-c). From forced convection regime to mixed convection regime, the average heat transfer rate is independent of the magnetic field and is not even affected a little for the Hartmann numbers 0, 25 and 100 when $Hy \leq 0.25$ and $Hx \leq 0.75$. It is seen that the overall heat transfer rate is reduced as the Hartmann number increases when $Hy \leq 0.75$ and $Hx \leq 0.25$. The average heat transfer rate is very low for $Ha \leq 100$ and the lengths of heater on bottom and left walls are 0.25.
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Ri= 0.01

![Image](image1.png) ![Image](image2.png) ![Image](image3.png)

Ri=1

![Image](image4.png) ![Image](image5.png) ![Image](image6.png)

Ri=100

![Image](image7.png) ![Image](image8.png) ![Image](image9.png)

Figure 7.2(a-i) Isotherms of different heating regions on both x and y directions with different Richardson numbers and Ha = 25
Figure 7.3(a-i) Streamlines of different heating regions on both $x$ and $y$ directions with different Richardson numbers and $Ha \neq 25$
Figure 7.4 (a-d) Local Nusselt numbers for different heating regions and for different Richardson numbers with $Ha \neq 25$
Figure 7.5(a-c) Average Nusselt number of cold wall for various Richardson numbers and different lengths of heating region along both $x$ and $y$ directions
Figure 7.6 Isotherms and streamlines of different lengths of heating region on both x and y directions, different Hartmann numbers, and Richardson numbers.
Figure 7.7 (a-f) Local Nusselt numbers for different Hartmann numbers and lengths of heating region at $Ri = 0.01$. 

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Figure 7.8 (a-f) Local Nusselt numbers for different Hartmann numbers and lengths of heating region at $Ri = 100$
Figure 7.9 Average Nusselt number of cold wall versus Richardson numbers for different lengths of heating region along both x and y directions and Hartmann numbers.