5. Numerical study on magneto-convection in a lid-driven cavity with discrete heating

5.1 Introduction

The main aim of this chapter is to examine the effects of various lengths and different locations of heater on the left sidewall in a square lid-driven cavity. In the literature, the fully heated vertical walls are mostly emphasized in mixed convection flow and heat transfer in enclosures. The influence of discrete heat sources on the natural convection heat transfer performance has been investigated by many researchers. Those investigations lead to the miniaturization / cooling of electronic devices in electronic technology. Hence, a detailed study of the convective flow and heat transfer in a discretely heated cavity with magnetic field is very helpful to understand the complex phenomena of MHD convection in practical applications.

5.2 Mathematical formulation

Consider unsteady, laminar, incompressible combined convective flow and heat transfer in a two-dimensional square cavity of size L as shown in Figure 5.1. A heater with full length or lesser length is used to keep the left sidewall at a higher temperature $T_h$ whereas the right sidewall is kept at a lower temperature $T_c$, such that $T_h > T_c$. Three different lengths of heater ($L_h$) from the bottom and top of the cavity and different locations of the heater namely, bottom, middle, and top on the left sidewall of the cavity are considered to examine the effects of heater location and size along the left wall. The regions exempted from heating as well as the horizontal walls of the cavity are adiabatic. The origin of the Cartesian coordinate system is at the lower left corner of the cavity. The direction of the x-axis is along the length of the cavity and that of y-axis is along the height of the cavity. The velocity components $u$ and $v$ are taken in x- and y-directions, respectively. The top wall of the cavity is moving with constant speed $U_o$ in its own plane.

The cavity is filled with an electrically conducting fluid of low Prandtl number. The gravitational acceleration acts downward. The uniform magnetic field with a constant magnitude $B_o$ is applied in the horizontal direction. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. The electric current $J$ and the electromagnetic force $F$ are defined by
\[ J \cdot \nabla \cdot \mathbf{V} = B \text{ and } F \cdot \nabla \times \mathbf{V} = \mathbf{B} \times \mathbf{B}, \] respectively. It is assumed that the viscous dissipation and Joule heating is neglected.

\[ L_H = \frac{L}{3} \]
\[ I = \frac{3L}{6} \]

Figure 5.1 Schematic diagram of physical configuration and coordinate system

The governing equations of motion for incompressible electrically conducting fluid are written as follows:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (5.1) \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + B_0 v^2 \quad (5.2) \]

\[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = \frac{1}{\rho} \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) - g \quad (5.3) \]

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = 0 \quad (5.4) \]

where \( c_p \) is the specific heat, \( g \) is the gravity, \( k \) is the thermal conductivity, \( p \) is the pressure, \( t \) is the time, \( \theta \) is the fluid temperature and \( \rho_0 \) is the density.
Since heating on the left sidewall is performed in three different ways there arise three cases and hence the appropriate initial and boundary conditions are written as follows:
For \( t \neq 0 \): \[ u \otimes v \otimes \quad \otimes \otimes \quad 0 \otimes (x,y) \otimes L \],

For \( t = 0 \): \[ u \otimes v \otimes 0 \quad \otimes \otimes \quad y \otimes 0 \]

\[ u \otimes U_0; v \otimes 0 \quad \otimes \otimes \quad y \otimes L \]

\[ u \otimes v \otimes 0 \quad \otimes \otimes \quad x \otimes L \]

\[ u \otimes v \otimes 0 \quad \otimes \otimes \quad x \otimes 0 \]

Case 1: (Increasing heater length from bottom)

\[ x \otimes 0 , \ 0 \otimes y \otimes \frac{L}{6} \]

\[ \frac{L}{6} \otimes y \]

Case 2: (Increasing heater length from top)

\[ x \otimes 0 , \ \frac{L}{6} \otimes y \otimes L \]

\[ x \otimes 0 , \ 0 \otimes y \otimes \frac{L}{6} \]

Case 3: (Heater placement)

\[ x \otimes 0 , \ \frac{L}{6} \otimes y \otimes \frac{L}{6} \]

\[ \frac{L}{6} \otimes y \]

where \( \frac{L}{6} = L/6, 3L/6 \) and \( 5L/6 \), denotes the center of the heating portion from the bottom of the cavity. Case 1 is a set, consisting of three configurations, that occurs when the normalized length, \( L_H \), of the heater is extended from the left-bottom of the cavity. In this case, \( L_H \) is varied as \( L/3, 2L/3, L \) on the cavity height. Case 2 represents a set consisting of three configurations which are obtained when the normalized length, \( L_H \), of the heat source is extended from the left top of the cavity. In this case, length of the heater is varied analogous to case 1 i.e. \( L/3, 2L/3, L \) on the cavity height. In case 3, \( L_H \) is kept constant at \( L/3 \), but the placement is varied as first at the bottom, second at middle, and finally at the top of the left sidewall so that \( \frac{L}{6} \) is \( L/6, 3L/6, 5L/6 \) from the lower left corner. However, heater length \( L_H = 75 \)
L/3 at bottom in case 1 and placement of heater when $\theta_1 = L/6$ at the bottom in case 3 are same. Similarly, length of the heater $L_h = L/3$ at top in case 2 and placement of heater when $\theta_1 = 5L/6$ at the top in case 3 are identical. The complete length of the heater extended fully from left-bottom in case 1 and total length of the heater increased from the left-top in case 2 are alike. Thus, out of nine configurations, there are six distinct configurations among three cases.
The non-dimensionalization of equations (5.1) to (5.5) has been performed using the following dimensionless variables

\[ X \equiv \frac{x}{L}, \ Y \equiv \frac{y}{L}, \ \frac{U}{U_0}, \ \frac{V}{U_0}, \ \frac{T}{T_r}, \ \frac{P}{P_r}, \ \frac{L}{L} \]

and \( \frac{\sqrt{\text{Pr} \, \text{Re} \, \text{Gr} \, \text{Ha}}}{U_0} \).

After non-dimensionalization, the governing equations in vorticity-stream function formulation are written as follows:

\[ \frac{U}{X}, \ \frac{V}{Y}, \ \frac{T}{X^2 \, \text{Re}}, \ \frac{\text{Ri}}{X^2}, \ \frac{\text{Ha}^2 \, V}{X} \]  

\[ \frac{U}{X \, \text{Re}}, \ \frac{V}{Y \, \text{Pr} \, \text{Re}}, \ \frac{1}{\text{Re}^2}, \ \frac{T}{X^2 \, \text{Re}^2} \]  

\[ U \equiv \frac{U}{U_0}, \ V \equiv \frac{V}{U_0} \]  

The non-dimensional parameters in the above equations are defined as follows: \( \text{Pr} \equiv \frac{v}{\alpha} \), the Prandtl number, \( \text{Gr} \equiv \frac{g \, x \, T^3}{\nu^2} \), the Grashof number, \( \text{Ha} \equiv \frac{B_0 L}{\mu} \), the Hartmann number, \( \text{Re} \equiv \frac{U_0 L}{\nu} \), the Reynolds number, \( \text{Ri} \equiv \frac{Gr}{Re^2} \), the Richardson number where this number, ratio is used to indicate the relative strengths of the two modes of convection in a mixed convection.

The non-dimensional initial and boundary conditions of the considered problem are written as follows:

For \( \text{X} \leq 0 \):
\[ U = V = T = 0 \]  

For \( \text{X} > 0 \):
\[ U = V = 0 \]  

\[ T = 0 \]  

\[ U = 1, \ V = 0 \]  

\[ T = 0 \]  

0
\[ Y \square 0 \quad Y \square 1 \]

\[
\begin{array}{ccc}
U \square V & T \square 0 & X \square 1 \\
0 & & X \square 0 \\
\end{array}
\]

Case 1:

\[
\begin{array}{ccc}
U \square V & T \square 1 & X \square 0, 0 \square Y \square \frac{1}{2} 1/6 \\
0 & & \end{array}
\]

\[
\begin{array}{l}
\square T \\
\square X
\end{array}
\]

\[ X \square 0, \frac{1}{2} 1/6 \square Y \square 1 \]

(5.10)
Case 2: \[ T \mathbin{T} 1 \quad X \mathbin{X} 0, \quad x_2 \mathbin{x} 1/6 \quad y \mathbin{y} 1 \]
\[
\frac{\partial T}{\partial x} \mathbin{X} 0 \quad X \mathbin{X} 0, \quad 0 \mathbin{0} y \mathbin{y} 16 \quad x_2 \mathbin{x} 2
\]

Case 3: \[ T \mathbin{T} 1 \quad X \mathbin{X} 0, \quad x_2 \mathbin{x} 1/6 \quad Y \mathbin{Y} 2 \mathbin{2} 1/6 \]
\[
\frac{\partial T}{\partial x} \mathbin{X} 0 \quad X \mathbin{X} 0, \quad 0 \mathbin{0} Y \mathbin{y} 16 \quad x_2 \mathbin{x} 2, \quad 1/6 \quad Y \mathbin{Y} 1 \quad x_2 \mathbin{x} 2
\]

where \( x_2 \mathbin{x} 1/6, 3/6 \) and \( 5/6 \).

It is obvious that the most important characteristic of the problem which describes the rate of heat transfer across the enclosure is the Nusselt number. In this present problem, the local Nusselt number along the left wall is defined as 
\[
\overline{\text{Nu}} \mathbin{\text{Nu}} \frac{1}{L_{hu}} \int_{0}^{L_{hu}} \text{Nu} \, dY
\]

and

(5.11)

where \( L_{hu} \) is the length of the heater. The numerical solutions are obtained by solving the non-dimensional equations (5.6)-(5.8) along with the boundary conditions (5.10) by finite volume method as discussed in Chapter 3.

5.3 Results and discussion

A numerical approach has been made out to analyze the effects of size and location of heater on the mixed convection flow in the presence of a magnetic field within an electrically conducting fluid filled square cavity. A fixed value of the Prandtl number \( Pr \mathbin{Pr} 0.054 \) is chosen in this analysis entirely. The parameters involved to control the heat transfer rate and the fluid flow in this study are Richardson number (\( Ri \mathbin{Ri} Gr/Re^2 \)) and Hartmann number (\( Ha \)). Setting the Reynolds number constantly at \( Re \mathbin{Re} 100 \), the computations have been made under a range \( 0.01 \mathbin{0.01} Ri \mathbin{Ri} 100 \) of the Richardson number by varying the Grashof numbers from \( 10^2 \) to \( 10^6 \) (\( Gr \)). The value of the Richardson number provides a measure of the importance of buoyancy-driven natural convection relative to the lid-driven forced convection. The values of the Hartmann number representing the strength of the magnetic field are 0, 25 and 100. The effects of the combined convection with electromagnetic field on the flow pattern and heat transfer characteristics have been discussed in the presented results.
5.3.1 Effects of various lengths of the heater

Figures 5.2(a)-(f) illustrate the isotherms obtained for various Richardson numbers such as $Ri = 0.01$, 1 and 100 when the lengths of the heater are $L_H = 1/3$, 2/3, 1, which are varied from bottom and top on the left sidewall, with $\frac{L}{L_H} = 1/6, 2/6, 3/6, 4/6, 5/6$. The Hartmann number is set fixed at 25. When the left sidewall is fully heated, i.e. when $L_H = 1$ and $\frac{L}{L_H} = 3/6$, it seems from Figure 5.2(a) that the temperature distribution happens almost by conduction mode for $Ri = 0.01$ and 1. But, the heat transfer occurs in convection mode when the Richardson number is increased to 100. On changing the length of the heater to $L_H = 2/3$ with $\frac{L}{L_H} = 2/6$, a similar effect on temperature distribution is observed as in the previous configuration. This can be viewed in Figure 5.2(b). It is seen that the conduction mode is slightly changed into convection mode for $Ri = 0.01$ and 1 at the heating regions for the reduced length of the heater $L_H = 1/3$ with $\frac{L}{L_H} = 1/6$. While the buoyancy force dominates, a better enhancement in temperature distribution is observed than in the previous configurations through convection. These are depicted in Figure 5.2(c). An analogue result has been observed for the length of the heater $L_H = 2/3$, which is extended form the top of the cavity, with $\frac{L}{L_H} = 4/6$, see Figure 5.2(f). It is noted that heat transfer happens in the region of heating and a high heat transfer occurs at the leading edge of the heater near the adiabatic region on the left sidewall. In general, a better convection mode is observed for the higher Richardson number, $Ri = 100$. As far as the length of the heater is concerned, for smaller length of the heater $L_H = 1/3$, the temperature distribution is worse than for the greater lengths of the heater in all these three configurations.

Figures 5.3(a)-(f) exhibit the fluid flow for various Richardson numbers and for different lengths and locations of the heater. In Figures 5.3(a)-(c), a very similar flow pattern occurs in all the three configurations when $Ri = 0.01$ and 1 for the variations in the length of the heater. The flow is characterized by a primary circular eddy rotating in the clockwise direction and it appears only in the upper-part of the cavity. Since the shear force dominates due to the motion of the lid and a resistive force due to the application of the magnetic field in the horizontal direction occurs, the motion of the fluid flow is suppressed. Hence the core region of the circulating eddy inside the cavity is near to the top of the cavity. Moreover, the flow at the bottom of the cavity becomes almost stagnant. In the case of mixed convection, the circulating eddy is stretched out and there exists an irregularity in the flow speed and so is in the flow pattern. But, there appears a remarkable change in the flow behavior when buoyancy
force dominates. The core region of the circular eddy moves to the centre of the cavity and the
rotating eddy appears almost in the entire cavity. The flow speed increases regardless of the length of the heater. Even if the magnetic field is applied, it does not affect the flow pattern considerably in the case of buoyancy-driven convection. These observations confirm that the fluid flow is independent of the length of the heater and it depends only on the magnetic field applied and the Richardson numbers.

Figure 5.4(a) depicts the local Nusselt number variations for different Richardson numbers and the extensions of the length of the heater from top and bottom when $Ha = 25$. Generally, increase in Richardson number increases the local Nusselt number. The resulting total heat transfer for different lengths of the heater is shown in Figures 5.5(a)-(b). Obviously, in the absence of magnetic field, the Nusselt number is increasing according with the increase in Richardson number. Hence, it can be concluded that the total heat transfer increases according with increase in the Richardson number. But, when $Ha = 25$, only a slight raise in the heat transfer is observed. This shows that the heat transfer rate is decreased while the Hartmann number is increased. Further, if the magnetic field is either active or inactive, an enhanced heat transfer occurs when the length of the heater is $L_H = 1/3$ either it is extended from top or bottom. For other changes in the length of the heater, a less heat transfer happens.

5.3.2 Effects of the locations of the heater

The isotherms for the different locations of the heater with constant length $L_H = 1/3$ and for the Richardson numbers $Ri = 0.01$, 1 and 100 are exemplified in Figures 5.2(c)-(e). The centers of the heaters at different locations, namely, bottom, middle, top are $\theta_2 = 1/6$, $\theta_2 = 3/6$, and $\theta_2 = 5/6$, respectively. The isotherms of different positions of heater for $Ri = 0.01$ and 1 are almost vertical except near the heating segments. By these results, it can be concluded that conduction is the dominant heat transfer mechanism in all the three configurations obtained for the positions of heater. But, in the case of $Ri = 100$, it turns into convection heat distribution which occurs in the entire cavity. Further, thin boundary layers are formed near the hot wall especially at the place where the heater is located as natural convection dominates. This becomes a good indication for the occurrence of a fine heat transfer. In all these configurations, it can be noticed that the isotherms are getting filled closer to the regions wherever the heater is placed on the left sidewall of the cavity.

The resulting streamlines of fluid flow for different locations of the heater for different Richardson numbers are illustrated by the Figures 5.3(c)-(e). There exists a primary circulating cell in the upper-part of the cavity due to the movement of top wall for $Ri = 0.01$ and 1. The
introduction of magnetic field and the dominance of forced convection become the important
factors to control the flow speed and thus the core region of circulation in the flow pattern is restricted to the upper side of the cavity. When $Ri \ll 1$, the circulating cell is elongated. In the natural convection regime, the core region moves towards the centre of the cavity and the circulating cell occupies the whole cavity. When the heater is placed at the top of the cavity, there is an upward flow near the heated wall and a downward flow near the cold wall. The change in the locations of the heater does not lead to any considerable change in the flow. But dominance of natural convection and transverse magnetic field become the key factors to control the fluid flow pattern.

The effect on the local Nusselt number for different Richardson numbers and the locations of the heater is described by Figure 5.4(b) for $Ha \ll 25$. In all the three configurations of location of the heater, heat transfer rate is increased on increasing the Richardson numbers. The average heat transfer rate for various locations of the heater is exposed in Figure 5.5(c). When the magnetic field is not present, the average heat transfer is increased with the increase in Richardson number. Nevertheless, the heat transfer for the centre location becomes better than the locations at top and bottom. If the Hartmann number is increased to 25, the increase in the heat transfer rate is not remarkable. It reveals that the increase of magnetic field parameter results in the decrease in the heat transfer rate. On comparing the heat transfer rate attained for all the three locations of the heater, it can be noticed that heat transfer rate is high for center location of the heater.

5.3.3 Effects of the Hartmann number

Figures 5.2(a)-(f) and 5.3(a)-(f) illustrate the influence of the magnetic field parameter $Ha \ll 25$ for various Richardson numbers on temperature distributions and on the fluid flow, respectively. In all the three cases, the isotherms are seen more vertical and straighten out for $Ri \ll 0.01$ and 1. Hence, the heat transfer happens through conduction mode. This is because of the existence of the magnetic field. In the case of natural convection dominance, the vertical isotherms are diminished and a convection heat transfer is observed. It shows that only a little effect of the magnetic field takes place for the natural convection regime. The main effect of magnetic field is to suppress the convective heat transfer and the fluid flow. Since the core region of the primary cell is very close to top wall, it is clear that the magnetic field opposes the flow behavior by a resistive force. In other words, the flow behavior is strongly controlled by the presence of magnetic field when $Ri \ll 0.01$ and 1. When the buoyant force dominates, the core region of the eddy moves towards the centre of the cavity in case of $Ri \ll 1$. It is important to notice that the fluid flow depends on neither the length nor the location of the heater due to
the significant effect of magnetic field in either of the cases the forced convection and the mixed convection. When the extensions in the length of the heater are considered, the average heat transfer is increased regardless of the existence of the magnetic field when the length of the heater is $L_H = 1/3$ either it is extended from top or bottom. In the case of the location of the heater, the overall heat transfer rate is better when the heater is placed at the centre of the cavity.

![Diagram](image)
Figure 5.2 Isotherms for different Richardson numbers and different size and locations of heater with $Ha \neq 25$
\( \begin{align*}
R_i & \leq 0.01 \\
R_i & \leq 1 \\
R_i & \leq 100 \\
\end{align*} \)

(a) \( L_H = 1, \varepsilon_2 = \frac{3}{6} \)

(b) \( L_H = \frac{2}{3}, \varepsilon_2 = \frac{2}{6} \)

(c) \( L_H = \frac{1}{3}, \varepsilon_2 = \frac{1}{6} \)
Figure 5.3 Streamlines for different Richardson numbers and different size and locations of heater with $Ha = 25$
Figure 5.4 Local Nusselt numbers for different Richardson numbers, $L_H$ and $\varepsilon_2$.
Figure 5.5 Average Nusselt numbers for different Richardson numbers, $L_H$ and $\varepsilon_2$. 

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