

MATHAMETAICAL ANALYSIS OF THE PROPOSED WAVEFORM GENERATORS

5.1 INTRODUCTION

In this chapter, the mathematical analysis of the proposed circuits in chapter 4 is given. All the proposed circuits are designed with one or two OTRAs along with few passive components to generate the oscillations. In present days, the VLSI technology is evolving into a high level of chip integration with low voltage and low power. In the design of analogue signal processing circuits or waveform generators, the maximum number of active components and passive components should be decreased to achieve the high level of chip integration with low voltage and low power. The proposed circuits are constructed with less number of active and passive components.

5.2 GROUNDED RESISTANCE/CAPACITANCE BASED SINUSOIDAL OSCILLATORS

The generalized configuration to realize oscillator circuits is proposed in chapter 4 and the same is shown in Fig. 5.1.

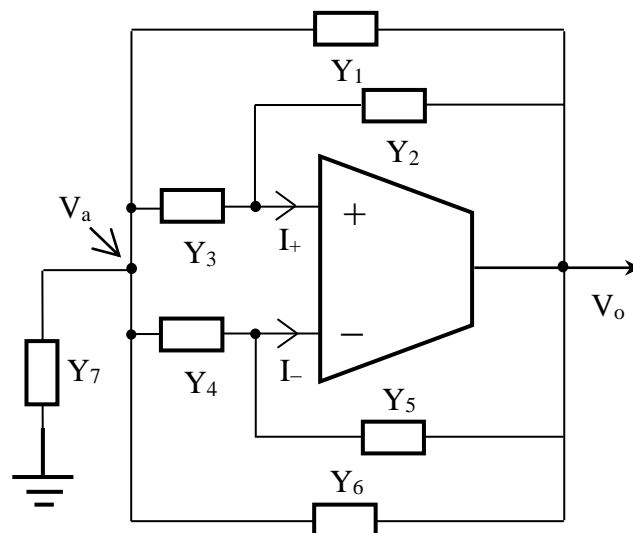


Fig. 5.1 Generalized configuration of the single OTRA based sinusoidal oscillators

The characteristic equation for the proposed generalized configuration shown in Fig. 5.1 can be derived from the ideal behavior of the OTRA. The currents at the

inverting and non-inverting terminals of the Fig. 5.1 can be written as in equations (5.1) and (5.2) are shown bellow.

$$V_0 Y_2 + V_a Y_3 = I_+ \quad (5.1)$$

$$V_0 Y_5 + V_a Y_4 = I_- \quad (5.2)$$

From the ideal terminal characteristics of the OTRA and from the equations (5.1) and (5.2), the voltage at the output terminal of the OTRA shown in Fig. 5.1 can be written as in (5.3).

$$V_0 = V_a \left(\frac{Y_4 - Y_3}{Y_2 - Y_5} \right) \quad (5.3)$$

Form the Fig. 5.1, the currents at the node V_a can be written as

$$V_0 (Y_1 + Y_6) = V_a (Y_1 + Y_3 + Y_4 + Y_6 + Y_7) \quad (5.4)$$

The generalized characteristic equation for the generalized oscillator circuit shown in Fig. 5.1 can be derived from the equations (5.3) and (5.4).

$$Y_1 Y_2 + Y_2 Y_3 + Y_2 Y_4 + Y_2 Y_6 + Y_2 Y_7 + Y_1 Y_3 + Y_3 Y_6 - Y_1 Y_5 - Y_1 Y_4 - Y_4 Y_6 - Y_3 Y_5 - Y_4 Y_5 - Y_5 Y_6 - Y_5 Y_7 = 0 \quad (5.5)$$

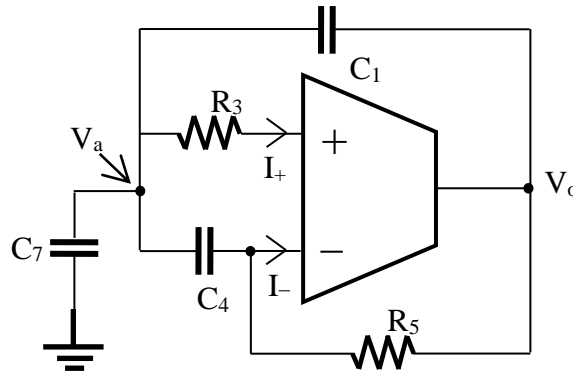


Fig. 5.2 OTRA based ooscillator circuit realized from Fig. 5.1

Where, Y_i 's are the admittances of the passive components. By proper selection of the passive components for the generalized configuration shown in Fig 5.1, many oscillator circuits can be generated. Some of the useful circuits generated from the generalized configuration are given in Fig. 4.3 and 4.4. The characteristic equations for the circuits shown in Fig. 4.3 and 4.4 can be derived by substituting admittances of the respective passive components. One of the proposed circuits in Fig. 4.4 is shown

in Fig. 5.2. The characteristic equation for the proposed circuit in Fig. 5.2 is derived from the generalized equation (5.5) by substituting the passive component $Y_1 = sC_1$, $Y_3 = R_3$, $Y_4 = sC_4$, $Y_5 = R_5$, $Y_7 = sC_7$ in place of Y_i 's. The characteristic equation for the circuit in Fig. 5.2 can be written as

$$s^2(C_1C_4) + s(G_5(C_1 + C_4 + C_7) - C_1G_3) + G_5G_3 = 0 \quad (5.6)$$

The condition of oscillation and frequency of oscillation for the circuit shown in Fig. 5.2 can be derived from the equation (5.6) as

Table 5.1. Condition of oscillations and frequency of oscillations for the proposed circuits in Fig. 4.3 and 4.4.

Oscillator circuits	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	C.O	F.O (ω_o^2)
Fig. 4.3	G_1	0	sC_2	G_3	sC_4	0	0	$C_4(R_1+R_3)=C_2R_3$	$\frac{1}{C_2C_4R_1R_3}$
Fig. 4.4 (a)	0	sC_2	G_3	sC_4	G_5	0	G_7	$C_2R_5(R_3+R_7) = C_4R_3R_7$	$\frac{(R_3 + R_7)}{C_2C_4R_3R_5R_7}$
Fig. 4.4 (b)	sC_1	0	G_3	sC_4	G_5	0	sC_7	$R_3(C_1+C_4+C_7) = C_1R_5$	$\frac{1}{C_1C_4R_3R_5}$
Fig. 4.4 (c)	G_1	0	G_3	0	sC_5	sC_6	G_7	$C_6R_1R_7 = C_5(R_3R_7 + R_1R_7 + R_1R_3)$	$\frac{1}{C_5C_6R_3R_1}$
Fig. 4.4 (d)	0	G_2	G_3	sC_4	G_5	sC_6	G_7	$R_3R_5(C_4+C_6) + R_2R_5C_6 = R_2R_3(C_6+C_4)$	$\frac{(R_3 + R_7)(R_2 - R_5)}{R_2R_3R_5R_7C_4C_6}$
Fig. 4.4 (e)	0	G_2	sC_3	G_4	0	sC_6	G_7	$R_4(C_3+C_6) = C_6R_2$	$\frac{R_4 + R_7}{R_2R_4R_7C_3C_6}$
Fig. 4.4 (f)	0	sC_2	G_3	sC_4	G_5	G_6	G_7	Equation (5.9)	Equation (5.10)
Fig. 4.4 (g)	0	sC_2	G_3	sC_4	0	G_6	G_7	$C_2(R_3+R_6+R_7) = C_4R_6R_3R_7$	$\frac{1}{C_2C_4R_3R_6}$
Fig. 4.4 (h)	sC_1	G_2	sC_3	G_4	G_5	0	0	$C_1R_4(R_2+R_5) = R_2C_3(R_4+R_5)$	$\frac{(R_2 + R_5) - R_4}{R_2R_4R_5C_1C_3}$
Fig. 4.4 (i)	sC_1	sC_2	G_3	G_4	G_5	0	0	$C_1R_4(R_3+R_5) + C_2R_3R_4 = C_2R_3R_5$	$\frac{R_3 + R_4 + R_5}{C_1C_2R_3R_4R_5}$
C.O: Condition of oscillation, F.O: Frequency of oscillation, Y's are admittance of passive components									

$$\text{Condition of oscillation (C.O): } R_3(C_1 + C_4 + C_7) = C_1R_5 \quad (5.7)$$

$$\text{Frequency of Oscillation (F.O): } f = \frac{1}{2\pi} \sqrt{\frac{1}{C_1 C_4 R_5 R_3}} \quad (5.8)$$

The condition of oscillation and frequency of oscillation for the circuit shown in Fig. 4.4(f) is given in equations (5.9) and (5.10).

$$R_5 C_2 (R_3 R_6 + R_6 R_7 + R_7 R_3) = C_4 R_7 R_3 (R_5 + R_6) \quad (5.9)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{R_3 R_6 + R_7 (R_3 + R_6 - R_5)}{C_2 C_4 R_3 R_5 R_6 R_7}} \quad (5.10)$$

Similarly, the condition of oscillation and frequency of oscillation for all the proposed circuits in Fig. 4.3 and 4.4 can be derived from the generalized characteristic equation (5.7). The condition of oscillation and the frequency of oscillation for the proposed circuits in Fig. 4.3 and 4.4 are shown in Table.1. The characteristic equations to derive the condition of oscillation and frequency of oscillation for the circuits in Fig. 4.3 and 4.4 can also be derived individually without using the generalized characteristic equation. The currents at the non-inverting and inverting terminals of the OTRA based circuits shown in Fig. 5.2 is given in below equations.

$$I_+ = \frac{V_a}{R_3} \quad (5.11)$$

$$I_- = \frac{V_o}{R_5} + V_a s C_4 \quad (5.12)$$

From the ideal terminal characteristics of the OTRA, as stated in equation (2.1), the output terminal current of the OTRA shown in Fig. 5.2 is given in equation (5.13).

$$V_o = V_a R_5 \left(\frac{1 - s C_4 R_3}{R_3} \right) \quad (5.13)$$

The equation given in (5.14) is derived by writing Kirchhoff's Current Law (KCL) at the node V_a in Fig. 5.2.

$$V_o s C_1 R_3 = V_a (1 + s R_3 (C_4 + C_7 + C_1)) \quad (5.14)$$

The characteristic equation for the circuit shown in Fig. 5.2 can be derived from the equations (5.13) and (5.14). The characteristic equation for the circuit in Fig. 5.2 is given in equation (5.15).

$$s^2 C_1 C_4 R_5 R_3 + s(R_3(C_1 + C_4 + C_7) - C_1 R_5) + 1 = 0 \quad (5.15)$$

The condition of oscillation and frequency of oscillation derived from the equation (5.15) is given in equations (5.16) and (5.17).

$$C.O = R_3(C_1 + C_4 + C_7) = C_1 R_5 \quad (5.16)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{C_1 C_4 R_5 R_3}} \quad (5.17)$$

The equations (5.7) and (5.8) for the condition of oscillation and frequency of oscillation is derived from the generalized characteristic equation and the equations (5.16) and (5.17) are the same. Likewise, the condition of oscillation and frequency of oscillation for all the proposed circuits in Fig. 4.2 and 4.3 can be derived by using generalized characteristic equation (5.5) or by applying general network laws to the respective circuits.

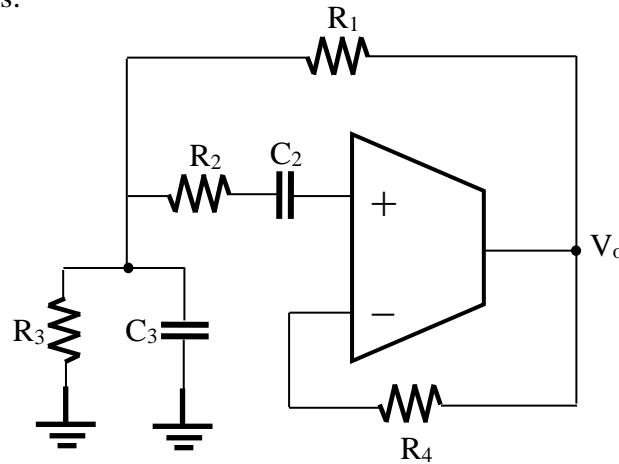


Fig. 5.3 Grounded resistance and capacitance sinusoidal oscillator circuit-I

The following passive components are chosen for the oscillator circuit shown in Fig. 5.3. $Y_1 = G_1$, $Y_2 = 0$, $Y_3 = sC_2 G_2 / (sC_2 + G_2)$, $Y_4 = 0$, $Y_5 = G_4$, $Y_6 = 0$ and $Y_7 = G_3 + sC_3$. The condition of oscillation and frequency of oscillation for the circuit shown in Fig. 5.3 are given in below equations.

$$C_2 \left(\frac{R_2 R_3 + R_1 R_3 + R_2 R_1}{R_1 R_3 R_4} \right) + \frac{C_3}{R_4} = \frac{C_2}{R_1} \quad (5.18)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3 C_2 C_3}} \quad (5.19)$$

Similarly for the circuit shown in Fig. 5.4, the passive components are chosen as $Y_1 = sC_1 G_1 / (sC_1 + G_1)$, $Y_2 = 0$, $Y_3 = G_2$, $Y_4 = 0$, $Y_5 = G_4$, $Y_6 = 0$ and $Y_7 = G_3 + sC_3$. The condition of oscillation and frequency of oscillation for the circuit shown in Fig. 5.4 is given in equations (5.20) and (5.21).

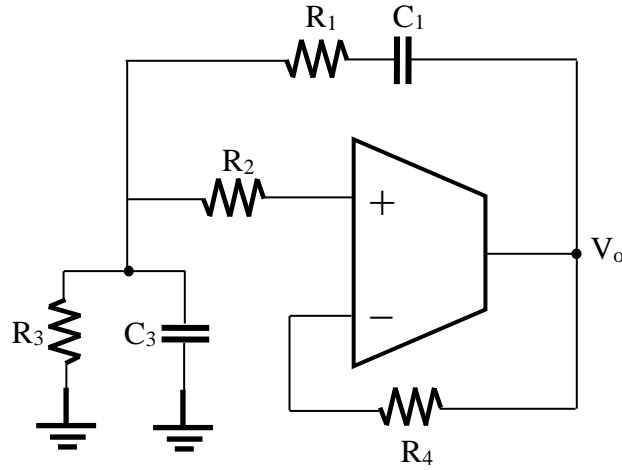


Fig. 5.4 Grounded resistance and capacitance sinusoidal oscillator circuit-II

$$C_1 \left(\frac{R_2 R_3 + R_1 R_3 + R_2 R_1}{R_2 R_3 R_4} \right) + \frac{C_3}{R_4} = \frac{C_1}{R_2} \quad (5.20)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{R_2 + R_3}{R_1 R_2 R_3 C_1 C_3}} \quad (5.21)$$

5.3 QUADRATURE SINUSOIDAL OSCILLATORS

The quadrature oscillator circuits proposed in Fig 4.6 and 4.7 are constructed by using two OTRAs and a few passive components to generate the sinusoidal oscillations with 90° phase shift. The proposed quadrature sinusoidal oscillator circuits in chapter 4 are shown in Fig. 5.5 and 5.6 along with the current directions at the input and output terminals of the OTRAs are given for deriving the condition of

oscillation and frequency of oscillation. The inverting and non-inverting terminal currents of the OTRA-1 shown in Fig. 5.5 are given in equations (5.22) and (5.23).

$$I_{1+} = \frac{V_{01}}{R_1} \quad (5.22)$$

$$I_{1-} = V_{01}sC_2 + \frac{V_{02}}{R_5} \quad (5.23)$$

From the ideal terminal characteristics of the OTRA stated in (2.1), the output terminal voltage V_{01} can be written as

$$V_{01} = V_{02} \left(\frac{R_1}{R_5(1 - sC_2R_1)} \right) \quad (5.24)$$

Similarly, the current flowing into the inverting and non-inverting terminals of the OTRA-2 is given in equations (5.25) and (5.26).

$$I_{2+} = \frac{V_{01}}{R_3} \quad (5.25)$$

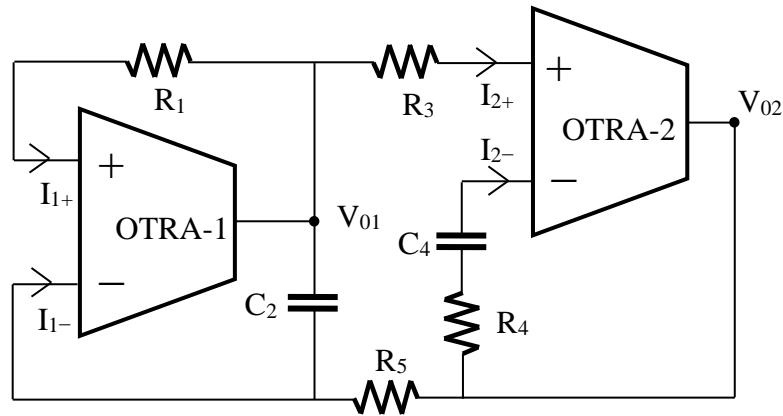


Fig. 5.5 Proposed quadrature sinusoidal oscillator circuit-I

$$I_{2-} = V_{02} \left(\frac{sC_4}{sC_4R_4 + 1} \right) \quad (5.26)$$

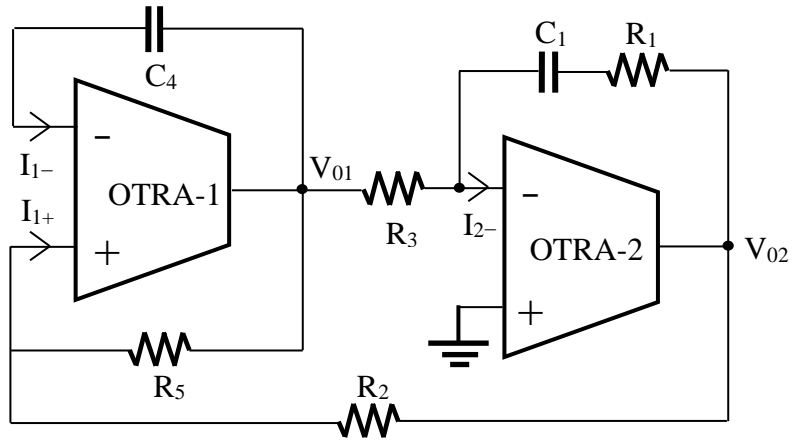


Fig. 5.6 Proposed quadrature sinusoidal oscillator circuit-II

The following equation can be written from the ideal terminal relations of the OTRA given in equation (2.1).

$$V_{01} = \frac{sC_4R_3}{sC_4R_4 + 1} V_{02} \quad (5.27)$$

The characteristic equation for the circuit shown in Fig. 5.5 can be derived from the equations (5.24) and (5.27) as

$$s^2(C_2C_4R_1R_3R_5) + sC_4(R_1R_4 - R_3R_5) + R_1 = 0 \quad (5.28)$$

From the above equation (5.28), the condition of oscillation and frequency of oscillation for the proposed quadrature sinusoidal oscillator circuit-I shown in Fig. 5.5 is given in equations (5.29) and (5.30).

$$\text{C.O: } R_1R_4 = R_3R_5 \quad (5.29)$$

$$\text{F.O: } f = \frac{1}{2\pi} \sqrt{\frac{1}{C_2C_4R_3R_5}} \quad (5.30)$$

From the equations (5.29) and (5.30), the frequency of oscillations can be controlled independently without affecting the condition of oscillation by using capacitor C_2 and C_4 . Similarly, the condition of oscillation can be controlled independently without affecting the frequency of oscillation through the resistor R_1 and R_4 .

The output voltage at the output terminal of the OTRA-1 shown in Fig. 5.6 can be derived by the ideal terminal behaviour of the OTRA. The output voltage V_{01} at the output terminal of the OTRA-1 is given in equation (5.31).

$$V_{01} = V_{02} \left(\frac{R_5}{sC_4 R_5 R_2 - R_2} \right) \quad (5.31)$$

Similarly, the output voltage V_{02} at the output terminal of the OTRA-2 shown in Fig. 5.6 is given in equation (5.32).

$$V_{02} = -V_{01} \left(\frac{sC_1 R_1 + 1}{sC_1 R_3} \right) \quad (5.32)$$

The characteristic equation for the quadrature sinusoidal oscillator circuit shown in Fig. 5.4 can be written from the equations (5.31) and (5.32).

$$s^2 C_1 C_4 R_3 R_2 R_5 + s C_1 (R_1 R_5 - R_2 R_3) + R_5 = 0 \quad (5.33)$$

The condition of oscillation and the frequency of oscillation for the circuit shown in Fig. 5.4 are given in equations (5.34) and (5.35).

$$\text{C.O: } R_1 R_5 = R_2 R_3 \quad (5.34)$$

$$\text{F.O: } f = \frac{1}{2\pi} \sqrt{\frac{1}{C_1 C_4 R_3 R_2}} \quad (5.35)$$

From the equations (5.34) and (5.35), the frequency of oscillation can be controlled independently without affecting the condition of oscillation by using capacitor C_1 and C_4 . Similarly, the condition of oscillation can be controlled independently without affecting the frequency of oscillation through the resistor R_1 and R_5 .

5.4 SQUARE WAVEFORM GENERATORS

The proposed square waveform generator using single OTRA and three passive components is shown in Fig. 5.7. From the proposed square waveform generator shown in Fig. 5.7 and from the ideal behavior of the OTRA given in equation (2.1),

the voltage V_C of the capacitor C at the non-inverting terminal of the OTRA can be written as

$$\frac{V_C}{R_1} = \frac{V_o - V_C}{R_2} - \frac{V_C}{R_1} \quad (5.36)$$

The voltage V_C of the capacitor from the above equation is given in equation (5.37).

$$V_C = \left(\frac{R_1}{2R_2 + R_1} \right) V_o \quad (5.37)$$

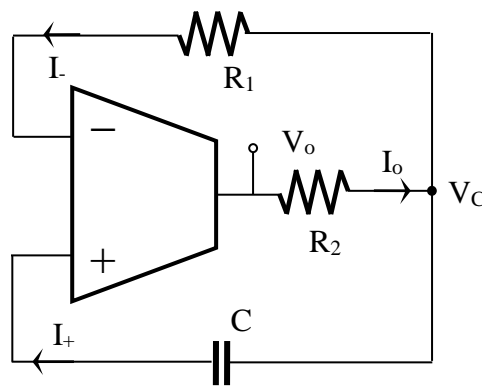


Fig. 5.7 Proposed square waveform generator using OTRA

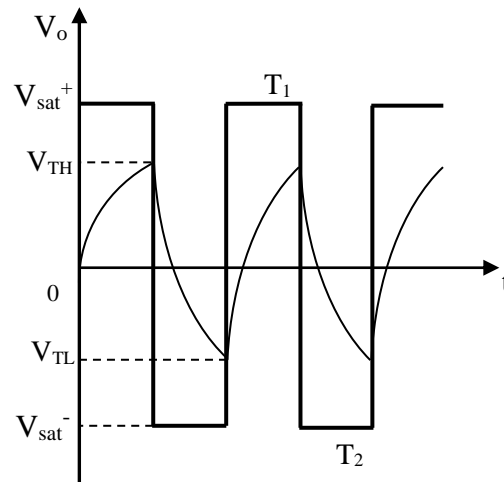


Fig. 5.8 Output waveform of the proposed circuit shown in Fig. 5.7

The output voltage V_o or the saturation levels V_{sat}^- and V_{sat}^+ changes its state when the non-inverting terminal current is equal to the inverting terminal current. Meanwhile

the capacitor voltage changes between V_{TH} and V_{TL} . Then the capacitor voltages can be derived from the equation (5.37).

$$V_{TH} = \left(\frac{R_1}{2R_2 + R_1} \right) V_{sat}^+ \quad (5.38)$$

$$V_{TL} = \left(\frac{R_1}{2R_2 + R_1} \right) V_{sat}^- \quad (5.39)$$

The equation for the capacitor voltage V_C , when it starts to increase from V_{TL} towards its final value V_{sat}^+ can be expressed as.

$$V_c(t) = (V_{TL} - V_{sat}^+) e^{\frac{-t}{R_2 C}} + V_{sat}^+ \quad (5.40)$$

The time period of the on-duty cycle T_1 can be derived from the equations (5.38), (5.39) and by making $V_C(t) = V_{TH}$ in equation (5.40).

$$T_1 = R_2 C \ln \left(\frac{\left(\frac{R_1}{2R_2 + R_1} \right) V_{sat}^- - V_{sat}^+}{\left(\frac{R_1}{2R_2 + R_1} \right) V_{sat}^+ - V_{sat}^+} \right) \quad (5.41)$$

From the output square waveform shown in Fig. 5.8, it can be written $V_{sat}^+ = -V_{sat}^-$.

$$\therefore T_1 = R_2 C \ln \left(1 + \frac{R_1}{R_2} \right) \quad (5.42)$$

The equation (5.42) is for the on-duty cycle (T_{ON}). At the end of on-duty cycle, the capacitor voltage V_C is charged up to the upper threshold voltage V_{TH} , instead of V_{sat}^+ . At this point of time, the current at the non-inverting terminal I_+ becomes less than the current at the inverting terminal I_- of the OTRA. Then the output changes its state from the upper saturation level V_{sat}^+ to the lower saturation level V_{sat}^- and the capacitor starts discharging. The voltage across capacitor C starts to decrease from higher threshold voltage V_{TH} , it can be expressed as

$$V_c(t) = (V_{TL} - V_{sat}^-)e^{\frac{-t}{R_2C}} + V_{sat}^- \quad (5.43)$$

Time period T_2 can be derived by setting $V_C(t) = V_{TL}$ in the above equation.

$$T_2 = R_2C \ln\left(\frac{V_{TH} - V_{sat}^-}{V_{TL} - V_{sat}^-}\right) \quad (5.44)$$

By substituting the V_{TH} and V_{TL} values in the equation (5.44)

$$T_2 = R_2C \ln\left(1 + \frac{R_1}{R_2}\right) \quad (5.45)$$

The equation (5.45) is for the off-duty cycle (T_{OFF}). The final time period (T) of the waveform shown in the Fig. 5.8 is the sum of the on-duty cycle time period T_1 and off-duty cycle time period T_2 cycles.

$$T = T_{ON} + T_{OFF} = T_1 + T_2$$

$$T = 2R_2C \ln\left(1 + \frac{R_1}{R_2}\right) \quad (5.46)$$

From the above equation the frequency of the proposed circuit shown in Fig. 5.7 can be written as

$$f = \frac{1}{2R_2C \ln\left(1 + \frac{R_1}{R_2}\right)} \quad (5.47)$$

The second proposed square wave generator circuit shown in Fig. 5.9 is designed to vary both the duty cycles independently. Taking no cognizance of the voltage drop of the diodes, the frequency or time period equation for the proposed square-wave generator can be derived with the help of upper threshold V_{TH} and lower threshold V_{TL} voltages.

$$V_{TH} = \left(\frac{R_{12}}{2R_2 + R_{12}}\right)V_{sat}^+ \quad (5.48)$$

$$V_{TL} = \left(\frac{R_{11}}{2R_2 + R_{11}} \right) V_{sat}^- \quad (5.49)$$

From the capacitor voltage equation (5.40), the time period for the on-duty cycle can be expressed as

$$T_1 = R_2 C \ln \left(\frac{V_{TL} - V_{sat}^+}{V_{TH} - V_{sat}^+} \right) \quad (5.50)$$

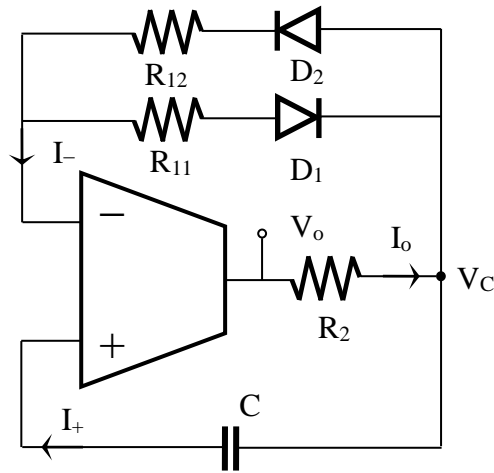


Fig. 5.9 Second proposed square-wave generator configuration

$$T_{ON} = R_2 C \ln \left(1 + \frac{R_{11}}{R_2} \right) \quad (5.51)$$

Similarly for the off-duty cycle time period, the T_{OFF} can be derived from the equation (5.37) by substituting the V_{TH} and V_{TL} from the equations (5.48) and (5.49).

$$T_{OFF} = R_2 C \ln \left(1 + \frac{R_{12}}{R_2} \right) \quad (5.52)$$

The total time period (T) for the proposed square-waveform generator shown in Fig. 5.9 can be expressed as

$$T = T_{ON} + T_{OFF}$$

$$T = R_2 C \ln \left(\left(1 + \frac{R_{11}}{R_2} \right) * \left(1 + \frac{R_{12}}{R_2} \right) \right) \quad (5.53)$$

The operating frequency of the proposed circuit shown in Fig. 5.9 can be calculated by substituting the passive component values in the below equation (5.48).

$$f = \frac{1}{R_2 C \ln \left(\left(1 + \frac{R_{11}}{R_2} \right) * \left(1 + \frac{R_{12}}{R_2} \right) \right)} \quad (5.54)$$

5.5 SUMMARY

This chapter is devoted to the mathematical analysis of the newly proposed circuits in chapter 4. All the proposed circuits are analyzed mathematically and the corresponding equations are discussed in detail. At first, the mathematical analysis of a generalized configuration is given. A generalized characteristic equation is derived from the generalized configuration. From this generalized characteristic equation, the condition of oscillation and the frequency of oscillation for all the proposed circuits are derived. The mathematical expressions of all the sinusoidal oscillator circuits are presented in a Table.

Most of the oscillator circuits realized from the generalized configuration is able to control the condition of oscillation and frequency of oscillation independently. Similarly, the same procedure is applied to derive the mathematical equations of two quadrature sinusoidal oscillators. In these quadrature sinusoidal oscillators, the condition of oscillation and frequency of oscillation can be controlled independently. Lastly, the mathematical expression for the time period of the proposed square waveform generator is given. The mathematical expressions of square waveform generator are derived by considering the charging and discharging voltages of the capacitor.

The mathematical analysis of all the proposed circuits is derived by considering the ideal terminal relation of the OTRA. The mathematical analysis based on the non-ideal model of OTRA is given in the next chapter.