CHAPTER 5

ORDERED EXTREMALLY AND ORDERED BASICALLY DISCONNECTED SPACES VIA INTUITIONISTIC FUZZY PRESEMI OPEN SETS

Ordered fuzzy topology was introduced by KATSARAS [48]. The concept of pairwise fuzzy extremally disconnected spaces was introduced by CHANDRASEKAR and BALASUBRAMANIAN [29]. Pairwise ordered fuzzy extremally disconnected spaces was introduced by UMA, ROJA and BALASUBRAMANIAN [78]. In this chapter, ordered intuitionistic fuzzy presemi extremally disconnected spaces and ordered intuitionistic fuzzy presemi basically disconnected spaces are introduced as in [47, 48, 70, 77, 78]. Also, Tietze extension theorem for ordered intuitionistic fuzzy presemi extremally disconnected spaces, ordered intuitionistic fuzzy presemi basically disconnected spaces and pairwise ordered intuitionistic fuzzy presemi extremally disconnected spaces has been discussed as in [47, 48, 70, 77, 78].
5.1 ORDERED INTUITIONISTIC FUZZY PRESEMI EXTREMALLY DISCONNECTED SPACES

In this section, the concept of ordered intuitionistic fuzzy presemi extremally disconnected space is introduced and studied.

**Definition 5.1.1**

An ordered set \((X, \leq)\) on which there is given an intuitionistic fuzzy topology \(T\) is called an ordered intuitionistic fuzzy topological space and it is denoted by \((X, T, \leq)\).

**Definition 5.1.2**

An intuitionistic fuzzy set \(A\) in \((X, T, \leq)\) is said to be

(a) an increasing intuitionistic fuzzy set if \(x \leq y\) implies \(A(x) \subseteq A(y)\)

That is, \(\mu_x \leq \mu_y\) and \(\gamma_x \geq \gamma_y\)

(b) a decreasing intuitionistic fuzzy set if \(x \leq y\) implies \(A(x) \supseteq A(y)\)

That is, \(\mu_x \geq \mu_y\) and \(\gamma_x \leq \gamma_y\)

**Definition 5.1.3**

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space.

For any intuitionistic fuzzy set \(A\) in \((X, T, \leq)\)

\[ I^{IFPS}(A) = \text{increasing intuitionistic fuzzy presemi closure of } A \]

\[ = \bigcap B/B \text{ is an increasing intuitionistic fuzzy presemi closed set and } B \supseteq \]

\[ D^{IFPS}(A) = \text{decreasing intuitionistic fuzzy presemi closure of } A \]
\[ = \bigcap B/B \text{ is a decreasing intuitionistic fuzzy presemi closed set and } B \supseteq \]

\[ I^{0IFPS}(A) = \text{increasing intuitionistic fuzzy presemi interior of } A \]

\[ = \bigcup B/B \text{ is an increasing intuitionistic fuzzy presemi open set and } B \subseteq \]

\[ D^{0IFPS}(A) = \text{decreasing intuitionistic fuzzy presemi interior of } A \]

\[ = \bigcup B/B \text{ is a decreasing intuitionistic fuzzy presemi open set and } B \subseteq \]

Clearly, \( I^{IFPS}(A) \) (resp., \( D^{IFPS}(A) \)) is the smallest increasing (resp., decreasing) intuitionistic fuzzy presemi closed set containing \( A \) and \( I^{0IFPS}(A) \) (resp., \( D^{0IFPS}(A) \)) is the largest increasing (resp., decreasing) intuitionistic fuzzy presemi open set contained in \( A \).

**Proposition 5.1.1**

For any intuitionistic fuzzy set \( A \) of an ordered intuitionistic fuzzy topological space \( (X,T,\leq) \) the following statements hold:

(a) \( I^{IFPS}(A) = \bar{A} \).

(b) \( D^{IFPS}(A) = \bar{\bar{A}} \).

(c) \( I^{0IFPS}(A) = \bar{\bar{\bar{A}}} \).

(d) \( D^{0IFPS}(A) = \bar{A} \).

**Proof**

(a) Let \( (X,T,\leq) \) be an ordered intuitionistic fuzzy topological space and \( A \) be any intuitionistic fuzzy set in \( (X,T,\leq) \)
is an increasing intuitionistic fuzzy presemi closed set in \((X,T,\leq)\) and \(B \supseteq\)

Taking complement on both sides,

\[
\overline{I^{\text{IFPS}}(A)} = \bigcup \overline{B}/\overline{B} \quad \text{is an increasing intuitionistic fuzzy presemi open set in } (X,T,\leq) \quad \text{and } \overline{B} \subseteq
\]

\[= D^{\text{IFPS}}(A).\]

Also, (b), (c) and (d) can be proved in a similar manner.

**Definition 5.1.4**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy topological space. Let \(A\) be any increasing (resp., decreasing) intuitionistic fuzzy presemi open set in \((X,T,\leq)\). If \(I^{\text{IFPS}}(A)\) (resp., \(D^{\text{IFPS}}(A)\)) is increasing (resp., decreasing) intuitionistic fuzzy presemi open in \((X,T,\leq)\) then \((X,T,\leq)\) is said to be upper (resp., lower) intuitionistic fuzzy presemi extremally disconnected. An intuitionistic fuzzy topological space \((X,T,\leq)\) is said to be an ordered intuitionistic fuzzy presemi extremally disconnected if it is both upper and lower intuitionistic fuzzy presemi extremally disconnected.

**Proposition 5.1.2**

For an ordered intuitionistic fuzzy topological space \((X,T,\leq)\) the following statements are equivalent:

(a) \((X,T,\leq)\) is upper intuitionistic fuzzy presemi extremally disconnected.
(b) For each decreasing intuitionistic fuzzy presemi closed set $A$,

$$D^{0\text{IFPS}}(A)$$ is decreasing intuitionistic fuzzy presemi closed.

(c) For each increasing intuitionistic fuzzy presemi open set $A$,

$$D^{\text{IFPS}}(\overline{I^{\text{IFPS}}(A)}) =$$

(d) For each pair of increasing intuitionistic fuzzy presemi open set $A$

and decreasing intuitionistic fuzzy presemi open set $B$ in $(X,T,\leq)$

with

$$\overline{I^{\text{IFPS}}(A)} = D^{\text{IFPS}}(B) =$$

Proof

(a) ⇒ (b): Let $A$ be any decreasing intuitionistic fuzzy presemi closed set

$(X,T,\leq)$ We claim that $D^{0\text{IFPS}}(A)$ is decreasing intuitionistic fuzzy

presemi closed. Now, $\overline{A}$ is increasing intuitionistic fuzzy presemi open.

By assumption (a) and Proposition 5.1.1, it follows that

$$I^{\text{IFPS}}(\overline{A}) =$$

is an increasing intuitionistic fuzzy presemi open set. That

is, $D^{0\text{IFPS}}(A)$ is decreasing intuitionistic fuzzy presemi closed.

(b) ⇒ (c): Let $A$ be an increasing intuitionistic fuzzy presemi open set in

$(X,T,\leq)$ Then, $\overline{A}$ is a decreasing intuitionistic fuzzy presemi closed set

and by (b), $D^{0\text{IFPS}}(\overline{A})$ is a decreasing intuitionistic fuzzy presemi closed

set. Consider

$$D^{\text{IFPS}}(\overline{I^{\text{IFPS}}(A)}) = D^{\text{IFPS}}(D^{0\text{IFPS}}(\overline{A})) =$$

Hence,

$$D^{\text{IFPS}}(\overline{I^{\text{IFPS}}(A)}) = I^{\text{IFPS}}(A).$$ Therefore, (c) holds.
(c) \Rightarrow (d): Let \( A \) be any increasing intuitionistic fuzzy presemi open set and \( B \) be any decreasing intuitionistic fuzzy presemi open set such that \( I^{IFPS}(A) = A \). By Proposition 5.1.1, \( B = \overline{\overline{A}} \).

By (c), \( D^{IFPS}(I^{IFPS}(A)) = D^{IFPS}(\overline{\overline{A}}) = A \).

Therefore, \( D^{IFPS}(I^{IFPS}(A)) = D^{IFPS}(B) \).

By (c), \( D^{IFPS}(D^{0IFPS}(A)) = B \). But, \( B = \overline{\overline{A}} \).

Therefore, \( D^{IFPS} B = \overline{\overline{A}} \).

(d) \Rightarrow (a): Let \( A \) be any increasing intuitionistic fuzzy presemi open set. Clearly, \( B \) is an decreasing intuitionistic fuzzy presemi open set. From (d), it follows that \( D^{IFPS}(B) = \overline{\overline{A}} \). That is, \( I^{IFPS}(A) \) is a decreasing intuitionistic fuzzy presemi closed set which implies \( I^{IFPS}(A) \) is a increasing intuitionistic fuzzy presemi open set. Hence, \((X,T,\leq)\) is intuitionistic fuzzy presemi extremally disconnected space.

**Proposition 5.1.3**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy topological space. Then \((X,T,\leq)\) is an intuitionistic fuzzy presemi extremally disconnected space if and only if for any decreasing intuitionistic fuzzy presemi open set \( A \) and decreasing intuitionistic fuzzy presemi closed set \( B \) such that \( A \subseteq D^{IFPS}(A) \subseteq B \).

**Proof**

Suppose that \((X,T,\leq)\) is an intuitionistic fuzzy presemi extremally disconnected space. Let \( A \) be any decreasing intuitionistic
fuzzy presemi open set, $B$ be any decreasing intuitionistic fuzzy presemi closed set such that $A \subseteq B$. Then by (b) of Proposition 5.1.2, $D_{\text{IFPS}}^0(B)$ is a decreasing intuitionistic fuzzy presemi closed set. Also, since $A$ is decreasing intuitionistic fuzzy presemi open and $A \subseteq B$, it follows that $A \subseteq B$. Again, since $D_{\text{IFPS}}^0(B)$ is decreasing intuitionistic fuzzy presemi closed, it follows that $D_{\text{IFPS}}^0(A) \subseteq B$.

To prove the converse, let $B$ be any decreasing intuitionistic fuzzy presemi closed set. Then $D_{\text{IFPS}}^0(B)$ is decreasing intuitionistic fuzzy presemi open and it is clear that $D_{\text{IFPS}}^0(B) \subseteq B$. Therefore by assumption,

$$D_{\text{IFPS}}^0(D_{\text{IFPS}}^0(B)) \subseteq B$$

It is known that,

$$D_{\text{IFPS}}^0(D_{\text{IFPS}}^0(B)) \supseteq B$$

Thus,

$$D_{\text{IFPS}}^0(D_{\text{IFPS}}^0(B)) = B$$

This implies that $D_{\text{IFPS}}^0(B)$ is a decreasing intuitionistic fuzzy presemi closed set. Hence by (b) of Proposition 5.1.2, it follows that $(X, T, \leq)$ is intuitionistic fuzzy presemi extremally disconnected.

**Notation 5.1.1**

An ordered intuitionistic fuzzy set which is both decreasing (resp., increasing) intuitionistic fuzzy presemi open and intuitionistic fuzzy presemi closed is called a decreasing (resp., increasing) intuitionistic fuzzy presemi clopen set.
**Remark 5.1.1**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy presemi extremally disconnected space. Let \(A_i, \bar{B}_i \ (i \in I)\) be a collection such that \(A_i\)'s are decreasing intuitionistic fuzzy presemi open sets, \(B_i\)'s are decreasing intuitionistic fuzzy presemi closed sets and let \(A, \bar{B}\) be decreasing intuitionistic fuzzy presemi open and increasing intuitionistic fuzzy presemi open sets respectively. If \(A_i \subseteq \subseteq \) and \(A_j \subseteq \subseteq \) for all \(i,j \in I\) then there exists a decreasing intuitionistic fuzzy presemi clopen set \(C\) such that \(D_{\text{IFPS}}(A_i) \subseteq \subseteq (B_j)\) for all \(i,j \in I\).

**Proof**

By Proposition 5.1.3,

\[
D_{\text{IFPS}}(A_i) \subseteq \cap \subseteq (i,j \in I).
\]

Put \(C = \cap D_{\text{IFPS}}^0(B)\). Now, \(C\) satisfies our required condition.

**Proposition 5.1.4**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy presemi extremally disconnected space. Let \(A_q\) and \(B_q\) be the monotone increasing collections of decreasing intuitionistic fuzzy presemi open sets and decreasing intuitionistic fuzzy presemi closed sets of \((X,T,\leq)\) respectively and suppose that \(A_q \subseteq \subseteq \) whenever \(q_1 <_\infty \) (\(\mathbb{Q}\) is the set of rational numbers). Then there exists a
monotone increasing collection \( C_q \) of decreasing intuitionistic fuzzy presemi clopen sets of \((X,T,\leq)\) such that \(D^{IFPS}(A_{q_1}) \subseteq B_{q_2} \) and 
\( C_{q_1} \subseteq (B_{q_2}) \) whenever \( q_1 < q_2 \).

**Proof**

Let us arrange into a sequence \( q_n \) of rational numbers without repetitions. For every \( n \geq 1 \) define inductively a collection
\[
\begin{align*}
C_{q_i} / 1 \leq i &\leq n \subseteq \zeta_i \quad \text{such that} \\
D^{IFPS}(A_q) &\subseteq (, , )< , > \quad (S_n)
\end{align*}
\]
for all \( i < n \).

By Proposition 5.1.3, the countable collections \( D^{IFPS}(A_q) \) and \( D^{0IFPS}(B_q) \) satisfying \( D^{IFPS}(A_{q_1}) \subseteq (, , )B_{q_2} \) if \( q_1 < q_2 \) By Remark 5.1.1, there exists a decreasing intuitionistic fuzzy presemi clopen set \( D_1 \) such that \( D^{IFPS}(A_{q_1}) \subseteq (, , )B_{q_2} \). Setting \( C_{q_1} = \) we get \( (S_2) \).

Assume that the intuitionistic fuzzy sets \( C_{q_i} \) are already defined for \( i < n \) and satisfy \( (S_n) \).

Define
\[
\begin{align*}
\Sigma & = \bigcup_{i < n, q_i < q_n} A_{q_n} \quad \text{and} \\
\Phi & = \bigcap_{j < n, q_j > q_n} B_{q_n}
\end{align*}
\]
Then, 
\[
D_{\text{IFPS}}(C_{q_i}) \subseteq \sum_{i < q} \quad \text{and} \quad D_{\text{IFPS}}(C_{q_j}) \subseteq \Phi \subseteq \sum_{j > q}
\]
whenever \( q_i < q_j \) (i, j < n) as well as \( A_q \subseteq \sum_{i < q} \) and \( A_q \subseteq \Phi \subseteq \sum_{j > q} \) whenever \( q < q_n \).

This shows that the countable collections
\[
C_{q_i} / i < n, q_i < q_n \cup A_q / q < q_n \quad \text{and} \quad C_{q_j} / j < n, q_j > q_n \cup B_q / q > q_n
\]
together with \( \sum \) and \( \Phi \) fulfill all the conditions of Remark 5.1.1.

Hence, there exists a decreasing intuitionistic fuzzy presemi clopen set \( D_n \) such that \( D_{\text{IFPS}}(D_n) \subseteq \) if \( q_n < q \), \( A_q \subseteq (D_n) \) if \( q < q_n \),

\[
D_{\text{IFPS}}(C_{q_i}) \subseteq D_n \quad \text{if} \quad q_i < q_n,
\]

\[
D_{\text{IFPS}}(D_n) \subseteq C_{q_j} \quad \text{if} \quad q_n < q_j \quad \text{where} \quad 1 \leq j \leq n
\]

Now, setting \( C_{q_n} = \) we obtain the intuitionistic fuzzy sets \( C_{q_1}, C_{q_2}, \ldots, C_{q_n} \) that satisfy \((S_{n+})\). Therefore the collection

\[
C_{q_i} / i = \ldots \quad \text{has the required property.}
\]
5.2 PROPERTIES AND CHARACTERIZATIONS OF ORDERED INTUITIONISTIC FUZZY PRESEMI EXTREMALLY DISCONNECTED SPACES

In this section, various properties and characterizations of ordered intuitionistic fuzzy presemi extremely disconnected spaces are discussed.

Definition 5.2.1

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space. A mapping \(f : X \rightarrow R_I\) is called ordered lower (resp., ordered upper) intuitionistic fuzzy presemi continuous, if \(f^{-1}(R_{I}^{t})\) (resp., \(f^{-1}(\nu_{I}^{t})\)) is an increasing or decreasing intuitionistic fuzzy presemi open (resp., increasing or decreasing intuitionistic fuzzy presemi clopen) set for each \(t \in I\).

Lemma 5.2.1

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space. Let \(A \in \zeta\) and let \(f : X \rightarrow R_I\) be such that

\[
f(x)(t) = \begin{cases} 
    t < 0, \\
    \leq \\
    t > 1,
\end{cases}
\]

for all \(x \in X\) and \(t \in I\). Then \(f\) is ordered lower (resp., ordered upper) intuitionistic fuzzy presemi continuous iff \(A\) is an increasing or decreasing intuitionistic fuzzy presemi open (resp., increasing or decreasing intuitionistic fuzzy presemi clopen) set.
Proof

\[ f^{-1}(R^I_t) = \begin{cases} t < & \text{if } t < \end{cases} \]

implies that \( f \) is ordered lower intuitionistic fuzzy presemi continuous iff \( A \) is increasing or decreasing intuitionistic fuzzy presemi open set.

\[ f^{-1}(\nu^I_t') = \begin{cases} t \leq & \text{if } t \leq \end{cases} \]

implies that \( f \) is an ordered upper intuitionistic fuzzy presemi continuous functions iff \( A \) is an increasing or decreasing intuitionistic fuzzy presemi clopen set.

**Proposition 5.2.1**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy topological space and let \( A \in \zeta \). Then, \( \psi \) is ordered lower (resp., ordered upper) intuitionistic fuzzy presemi continuous iff \( A \) is increasing or decreasing intuitionistic fuzzy presemi open (resp., increasing or decreasing intuitionistic fuzzy presemi clopen) set.

**Proof** The proof follows from Lemma 5.2.1.

**Definition 5.2.2**

Let \((X,T,\leq)\) and \((Y,S,\leq)\) be ordered intuitionistic fuzzy topological spaces. A mapping \( f : (X,T,\leq) \rightarrow (Y,S,\leq) \) is called increasing (resp., decreasing) intuitionistic fuzzy strongly presemi continuous iff \( f^{-1}(\nu^I_t) \) is increasing (resp., decreasing) intuitionistic fuzzy presemi clopen in \((X,T,\leq)\) for every increasing (resp., decreasing) intuitionistic
fuzzy presemi open set in \((Y,S,\leq)\). If \(f\) is both increasing and decreasing intuitionistic fuzzy strongly presemi continuous, then it is called ordered intuitionistic fuzzy strongly presemi continuous.

**Proposition 5.2.2**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy topological space. Then the following are equivalent:

(a) \((X,T,\leq)\) is ordered intuitionistic fuzzy presemi extremally disconnected,

(b) If \(g,h:X \rightarrow I\) \(g\) is ordered lower intuitionistic fuzzy presemi continuous, \(h\) is ordered upper intuitionistic fuzzy presemi continuous and \(g \subseteq \) then there exists an increasing intuitionistic fuzzy strongly presemi continuous function, \(f:(X,T,\leq) \rightarrow R_{I}\) such that \(g \subseteq \subseteq \)

(c) If \(A\) is increasing intuitionistic fuzzy presemi open and \(B\) is decreasing intuitionistic fuzzy presemi open such that \(B \subseteq \) then there exists an increasing intuitionistic fuzzy strongly presemi continuous function \(f:(X,T,\leq) \rightarrow I_{I}\) such that \(B \subseteq \subseteq \substack{\uparrow\ 1} \subseteq \)

**Proof**

(a) \(\Rightarrow\) (b): Define \(H_r = \) and \(G_r = \) \(r \in \) Thus we have two monotone increasing families respectively decreasing intuitionistic fuzzy presemi open and increasing intuitionistic fuzzy presemi closed sets of \((X,T,\leq)\). Moreover \(H_r \subseteq \) if \(r < \) By Proposition 5.1.4, there
exists a monotone increasing family \( F_r \) of decreasing intuitionistic fuzzy pre semi clopen sets of \( (X,T,\leq) \) such that \( D^{IFPS}(H_r) \subseteq \) and \( F_r \subseteq (G_{s}) \) whenever \( r < s \in \) Let \( V_t = \cap_{r \in T} I^{IFPS}(V_t) \) for all \( t \in T \) we define a monotone decreasing family \( V_t/t \in \zeta \). Moreover we have \( I^{IFPS}(V_t) \subseteq V_s \) whenever \( s < t \). We have

\[
\bigcup_{t \in R} V_t = \bigcup_{t \in R} \cap_{r \in T} F_r \\
\supseteq \bigcup_{t \in R} \cap_{r \in T} G_r \\
= \bigcup_{-} \cap_{r \in T} g_{-} \cup_{r \in T} \\
= \bigcup_{-} L_t \\
= g \bigcup_{t \in R} L_t^t.
\]

Similarly, \( \cap_{t \in T} V_t = \) Now define a function \( f : X \to I \) satisfying the required properties. Let \( f(x)(t) = \) for all \( x \in X \) and \( t \in T \).

By the above discussion, it follows that \( f \) is well defined.

To prove, \( f \) is increasing intuitionistic fuzzy strongly presemi continuous, we observe that

\[
\bigcup_{s > t} V_s = \bigcup_{s > t} I^{IFPS}(V_s) \quad \text{and} \quad \bigcap_{s < t} V_s = \bigcap_{s < t} I^{IFPS}(V_s).
\]

Then, \( f^{-1} \cap_{t \in T} l_t = \bigcup_{s > t} = \bigcup_{s > t} I^{IFPS}(V_s) \) is increasing intuitionistic fuzzy presemi open. And \( f^{-1} \cap_{t \in T} l_t = \bigcap_{s < t} = \bigcap_{s < t} I^{IFPS}(V_s) \) is increasing intuitionistic fuzzy presemi clopen. Therefore \( f \) is increasing.

117
intuitionistic fuzzy strongly presemi continuous. It remains to show that \( g \subseteq \subseteq \) that is

\[
g^ {-} \subseteq \subseteq \subseteq \quad \text{and}
\]

\[
g^ {-} \subseteq \subseteq \subseteq \,
\]

for each \( t \in \)

We have, \( g^ {-} \left( L^ l_t \right) = \bigcap_s \ g^ {-} \left( R^ l_r \right) \)

\[
= \bigcap_s \ \bigcap_{r < s} g^ {-} \left( R^ l_r \right)
\]

\[
= \bigcap_s \ \bigcap_{r < s} G_r
\]

\[
\subseteq \bigcap_s \ \bigcap_{r < s} F_r
\]

\[
= \bigcap_s \ V_s = f \ L^ l_t
\]

And, \( f^ {-} \left( L^ l_t \right) = \bigcap_s \ V_s \)

\[
= \bigcap_s \ \bigcap_{r < s} F_r
\]

\[
\subseteq \bigcap_s \ \bigcap_{r < s} H_r
\]

\[
= \bigcap_s \ \bigcap_{r < s} h^ {-} \left( L^ l_s \right)
\]

\[
= \bigcap_s \ h \ L^ l_s
\]

Similarly, \( g^ {-} \left( R^ l_t \right) = \bigcup_s \ g^ {-} \left( R^ l_s \right) \)

\[
= \bigcup_s \ \bigcup_{r > s} g^ {-} \left( R^ l_r \right)
\]

\[
= \bigcup_s \ \bigcup_{r > s} G_r
\]

\[
\subseteq \bigcup_s \ \bigcap_{r < s} F_r
\]
\[ V_s = f^t R_i^t \]

And,
\[ f^{-1}(R_i^t) = \bigcup_s V_s \]
\[ = \bigcup_s \bigcap_{r < s} \overline{F}_r \]
\[ \subseteq \bigcup_s \bigcup_{r > s} \overline{H}_r \]
\[ = \bigcup_s \bigcup_{r > s} h^{-1}_r L_r^t \]
\[ = \bigcup_{s > t} h^t R_s^t \]

Thus (b) is proved.

**Proof of (b)⇒(c):** Suppose \( \overline{A} \) is an increasing intuitionistic fuzzy presemi open set and \( B \) is a decreasing intuitionistic fuzzy presemi open set, such that \( B \subseteq A \). Then, \( \psi_\alpha \subseteq \psi_\beta \) and \( \psi_\beta \psi_\alpha \) are ordered lower and ordered upper intuitionistic fuzzy presemi continuous functions respectively.

Hence by (b), there exists an intuitionistic fuzzy strongly presemi continuous function \( f : (X, T, \leq) \rightarrow R_1 \) such that
\[ \psi_\alpha \subseteq \psi_\beta \]
Clearly, \( f(x) \in I_1 \) for all \( x \in \) and
\[ B = \overline{T_1} \psi_\alpha \subseteq \overline{T_1} \subseteq \overline{I_1} \subseteq \overline{I_1} \psi_\alpha = \]

Therefore,
\[ B = \overline{T_1} \subseteq \overline{T_1} \subseteq \overline{T_1} \]

**Proof of (c)⇒(a):** This follows from Proposition 5.1.3 and the fact that \( (L_1^t)f \) and \( R_0^t f \) are decreasing intuitionistic fuzzy presemi closed and decreasing intuitionistic fuzzy presemi open sets respectively.
Therefore, \((X,T,\leq)\) is an ordered intuitionistic fuzzy presemi extremally disconnected space.

**Note 5.2.1**

The Propositions 5.1.1 to 5.1.4, 5.2.2 and Remark 5.1.1 can be discussed for other cases also.

### 5.3 Tietze Extension Theorem for Ordered Intuitionistic Fuzzy Presemi Extremally Disconnected Spaces.

In this section, Tietze extension theorem for ordered intuitionistic fuzzy presemi extremally disconnected space is studied.

**Proposition 5.3.1**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy presemi extremally disconnected space and let \(A \subseteq X\) be such that \(\chi_A\) is an increasing intuitionistic fuzzy presemi open set in \((X,T,\leq)\). Let \(f:(A, T/A) \to I\) be an increasing intuitionistic fuzzy strongly presemi continuous function. Then, \(f\) has an increasing intuitionistic fuzzy strongly presemi continuous extension over \((X,T,\leq)\).

**Proof**

Let \(g, h : X \to I\) be such that \(g = = \) on \(A\), and \(g(x) = \)

\[
h(x) = \begin{cases} \chi_A \geq & \text{if } x \not\in A \\ \chi_A < 0, & \text{if } x \in A \end{cases}
\]

We now have,

\[
R^t g = \left\{ \bigcap_{t \geq 0} g \right\} \geq \begin{cases} \chi_A \geq & \text{if } t < 0, \\ \chi_A < 0, & \text{if } t \geq 0, \end{cases}
\]
where $B_t$ is increasing intuitionistic fuzzy presemi open set such that

$$B_t/A = I_t$$

and

$$L^t_I h = \bigcap \leq \begin{cases} \text{if } t > 1, \end{cases}$$

where $C_t$ is increasing intuitionistic fuzzy presemi open such that

$$C_t/A = I_t$$

Thus, $g$ is ordered lower intuitionistic fuzzy presemi continuous, $h$ is ordered upper intuitionistic fuzzy presemi continuous and $g \subseteq$  By Proposition 5.2.2, there is an increasing intuitionistic fuzzy strongly presemi continuous function $F : (X,T,\leq \rightarrow I_t)$ such that $g \subseteq \subseteq$ hence $F =$ on $A$.

**Note 5.3.1**

The above proposition can be discussed for other case also.

**5.4. ORDERED INTUITIONISTIC FUZZY PRESEMI BASICALLY DISCONNECTED SPACES**

In this section, the concept of ordered intuitionistic fuzzy presemi basically disconnected spaces is introduced. Some of its characterizations and properties are studied.

**Definition 5.4.1**

Let $(X,T,\leq$ be an intuitionistic fuzzy topological space. For any intuitionistic fuzzy set $A$ in $(X,T,\leq$

$I^{IFPSG\delta}(A) =$ increasing intuitionistic fuzzy presemi $G_\delta$ closure of $A$
= \bigcap B/B is an increasing intuitionistic fuzzy presemi closed

G_δ set and B \supseteq

D^{IFPSG_δ \ldots} = decreasing intuitionistic fuzzy presemi G_δ closure of A

= \bigcap B/B is a decreasing intuitionistic fuzzy presemi closed

G_δ set and B \supseteq

I^{0IFPSF_\sigma \ldots} = increasing intuitionistic fuzzy presemi F_\sigma interior of A

= \bigcup B/B is an increasing intuitionistic fuzzy presemi open

F_\sigma set and B \subseteq

D^{0IFPSF_\sigma \ldots} = decreasing intuitionistic fuzzy presemi F_\sigma interior of A

= \bigcup B/B is a decreasing intuitionistic fuzzy presemi open

F_\sigma set and B \subseteq

Clearly, I^{IFPSG_δ \ldots} (resp., D^{IFPSG_δ \ldots}, is the smallest increasing (resp., decreasing) intuitionistic fuzzy presemi closed G_δ set containing A and I^{0IFPSF_\sigma \ldots} (resp., D^{0IFPSF_\sigma \ldots}, is the largest increasing (resp., decreasing) intuitionistic fuzzy presemi open F_\sigma set contained in A.

**Proposition 5.4.1**

For any intuitionistic fuzzy set A of an ordered intuitionistic fuzzy topological space \((X,T,\leq)\) the following statements are hold:

(a) \(I^{IFPSG_δ \ldots} = \ldots\)

(b) \(D^{IFPSG_δ \ldots} = \ldots\)
Proof

(a) Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space.

Let \(A\) be any intuitionistic fuzzy set in \((X, T, \leq)\)

\[
I_{\text{IFPSG}_{\delta \leq}} = \bigcap B / B
\]

is an increasing intuitionistic fuzzy presemi closed set set in \((X, T, \leq)\) and \(B \supseteq\)

Taking complement on both sides,

\[
\overline{I_{\text{IFPSG}_{\delta \leq}}} = \bigcup \overline{B} / \overline{B}
\]

is an increasing intuitionistic fuzzy presemi open set in \((X, T, \leq)\) and \(\overline{B} \subseteq\)

\[
= D_{\text{IFPS}}(A).
\]

The proofs of (b), (c) and (d) can be proved in a similar manner.

Definition 5.4.2

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space.

Let \(A\) be any increasing (resp., decreasing) intuitionistic fuzzy presemi open \(F_{\sigma}\) set in \((X, T, \leq)\) If \(I_{\text{IFPSG}_{\delta \leq}}, \) (resp., \(D_{\text{IFPSG}_{\delta \leq}}\)) is increasing (resp., decreasing) intuitionistic fuzzy presemi open in \((X, T, \leq)\) then \((X, T, \leq)\) is said to be upper (resp., lower) intuitionistic fuzzy presemi basically disconnected. An intuitionistic fuzzy topological space \((X, T, \leq)\) is said to be ordered intuitionistic fuzzy presemi basically disconnected if it is both upper and lower intuitionistic fuzzy presemi basically disconnected.
Proposition 5.4.2

For an intuitionistic fuzzy topological space \((X,T,\leq)\) the following statements are equivalent:

(a) \((X,T,\leq)\) is upper intuitionistic fuzzy presemi basically disconnected.

(b) For each decreasing intuitionistic fuzzy presemi closed \(G_\delta\) set \(A\),

\[
D^{0IFPS}_{\sigma} \rightarrow \text{is decreasing intuitionistic fuzzy presemi closed.}
\]

(c) For each increasing intuitionistic fuzzy presemi open \(F_\sigma\) set \(A\),

\[
D^{IFPS} (I^{IFPSG}_\delta \rightarrow \text{is increasing intuitionistic fuzzy presemi open}) = \ldots.
\]

(d) For each pair of increasing intuitionistic fuzzy presemi open \(F_\sigma\) set \(A\) and any decreasing intuitionistic fuzzy presemi open \(F_\sigma\) set \(B\) in \((X,T,\leq)\) with \(I^{IFPSG}_\delta \rightarrow \text{is increasing intuitionistic fuzzy presemi open}) = D^{IFPS} (B) = \ldots.

Proof. (a) \Rightarrow (b): Let \(A\) be any decreasing intuitionistic fuzzy presemi closed \(G_\delta\) set in \((X,T,\leq)\). We claim that \(D^{0IFPS}_{\sigma} \rightarrow \), is decreasing intuitionistic fuzzy presemi closed. Now, \(\overline{A}\) is increasing intuitionistic fuzzy presemi open \(F_\sigma\).

Therefore by assumption (a) and Proposition 5.4.1, \(I^{IFPSG}_\delta \rightarrow \text{is increasing intuitionistic fuzzy presemi open}) = \overline{A} \rightarrow \), is increasing intuitionistic fuzzy presemi open. That is, \(D^{0IFPS}_{\sigma} \rightarrow \), is decreasing intuitionistic fuzzy presemi closed.

(b) \Rightarrow (c): Let \(A\) be an increasing intuitionistic fuzzy presemi open \(F_\sigma\) set in \((X,T,\leq)\). Then, \(\overline{A}\) is a decreasing intuitionistic fuzzy presemi
closed $G_\delta$ set and by (b), $D_{IIFS}\sigma_\ldots$, is decreasing intuitionistic fuzzy presemi closed set.

Therefore, $D_{IFPS}(D_{IIFS}\sigma_\ldots) = \ldots$

Hence, $D_{IFPS}(D_{IIFS}\sigma_\ldots) = I_{IFPSG}\delta_\ldots$

By Proposition 5.4.1, (c) holds.

(c) $\Rightarrow$ (d): Let $A$ be any increasing intuitionistic fuzzy presemi open $F_{\sigma}$ set and $B$ be any decreasing intuitionistic fuzzy presemi open $F_{\sigma}$ set such that $I_{IFPSG}\delta_\ldots = \ldots$

By (c), $D_{IFPS}(D_{IIFS}\sigma_\ldots) = I_{IFPSG}\delta_\ldots$

But, $B = \ldots$

Therefore, $D_{IFPS}(B) = \ldots$

(d) $\Rightarrow$ (a): Let $A$ be any increasing intuitionistic fuzzy presemi open $F_{\sigma}$ set. Then, $I_{IFPSG}\delta_\ldots = \ldots$

From (d), it follows that $D_{IFPS}(B) = \ldots$

Therefore, $I_{IFPSG}\delta_\ldots$ is increasing intuitionistic fuzzy presemi closed.

Thus, $I_{IFPSG}\delta_\ldots$ is increasing intuitionistic fuzzy presemiopen.

Hence, $(X,T,\leq)$ is upper intuitionistic fuzzy presemi basically disconnected space.
**Proposition 5.4.3**

Let \((X,T,\leq)\) be an ordered intuitionistic fuzzy topological space. Then, \((X,T,\leq)\) is an upper intuitionistic fuzzy presemi basically disconnected space if and only if for any decreasing intuitionistic fuzzy presemi open \(F_\sigma\) set \(A\) and decreasing intuitionistic fuzzy presemi closed \(G_\delta\) set \(B\) such that \(A \subseteq D^{IFPS}(A) \subseteq \sigma_{\sim}\).

**Proof**

Suppose that \((X,T,\leq)\) is an upper intuitionistic fuzzy pre semi basically disconnected space. Let \(A\) be any decreasing intuitionistic fuzzy pre semi open \(F_\sigma\) set, \(B\) be any decreasing intuitionistic fuzzy pre semi closed \(G_\delta\) set such that \(A \subseteq \). Since, \(A \subseteq \), it follows that \(A \subseteq \sigma_{\sim}\). Then by (b) of Proposition 5.4.2, \(D^{\sigma_{\sim}}\), is a decreasing intuitionistic fuzzy pre semi closed set. Now, \(D^{IFPS}(A) \subseteq \sigma_{\sim}\). To prove the converse, let \(B\) be any decreasing intuitionistic fuzzy pre semi closed \(G_\delta\) set.

Then \(D^{\sigma_{\sim}}\), is decreasing intuitionistic fuzzy pre semi open \(F_\sigma\) and it is clear that \(D^{\sigma_{\sim}}\) \(\subseteq \).

Therefore by assumption,

\[
D^{IFPS}(D^{\sigma_{\sim}}) \subseteq \sigma_{\sim},
\]

\[
\Rightarrow \quad A \subseteq \sigma_{\sim},
\]

it follows that,

\[
D^{IFPS}(D^{\sigma_{\sim}}) \subseteq \sigma_{\sim}.
\]
Conversely, let

\[ D_{F_{\sigma}}^{IFPS}(A) \subseteq T \cap \sigma \cdot \cdot \cdot \]

But always

\[ D_{F_{\sigma}}^{0IFPS} \subseteq 4 \cdot \cdot \cdot \]

Thus,

\[ D_{\sigma}^{IFPS}(A) = T \cdot \cdot \cdot \]

This implies that \( D_{F_{\sigma}}^{0IFPS} \), is a decreasing intuitionistic fuzzy pre semi closed set. Hence by (b) of Proposition 5.4.2, it follows that \((X,T,\leq)\) is upper intuitionistic fuzzy pre semi basically disconnected.

**Notation 5.4.1**

An ordered intuitionistic fuzzy which is both decreasing (resp., increasing) intuitionistic fuzzy pre semi open \( F_{\sigma} \) and decreasing (resp., increasing) intuitionistic fuzzy pre semi closed \( G_{\delta} \) is called a decreasing (resp., increasing) intuitionistic fuzzy pre semi \( COG_{\delta} \) set.

**Remark 5.4.1**

Let \((X,T,\leq)\) be an upper intuitionistic fuzzy pre semi basically disconnected space. Let \( A_i, B_i \) be a collection of all are decreasing intuitionistic fuzzy pre semi open \( F_{\sigma} \) sets and let \( A, B \) be decreasing intuitionistic fuzzy pre semi open \( F_{\sigma} \) and increasing intuitionistic fuzzy pre semi open \( F_{\sigma} \) sets respectively. If \( A_i \subseteq B_j \) and \( A_i \subseteq A_j \) for all \( i, j \in \mathbb{N} \) then there exists a decreasing intuitionistic fuzzy pre semi \( COG_{\delta} \) set \( C \) such that

\[ D_{\sigma}^{IFPS}(A_i) \subseteq T \cap \sigma \cdot \cdot \cdot \]

**Proof**

By Proposition 5.4.3,
\[ D^{\text{IFPS}}(A_i) \subseteq \bigcap_{i \neq j} \bigcap_{\sigma, \omega} A_i \cap A_j, \quad i, j \in \mathbb{N} \]

Put \( C = \bigcap_{\sigma, \omega} \sigma_{\omega} \). Now, \( C \) satisfies our required condition.

**Proposition 5.4.4**

Let \((X, T, \leq)\) be an upper intuitionistic fuzzy presemi basically disconnected space. Let \( A_{q_1} \) and \( B_{q_2} \) be the monotone increasing collections of decreasing intuitionistic fuzzy presemi open \( F_\sigma \) sets and decreasing intuitionistic fuzzy presemi closed \( G_\delta \) sets of \((X, T, \leq)\) respectively and suppose that \( A_{q_1} \subseteq \bar{\sigma}_{q_2} \) whenever \( q_1 < q_2 \) (\( Q \) is the set of rational numbers).

Then there exists a monotone increasing collection \( C_{q_3} \) of decreasing intuitionistic fuzzy presemi \( COG_{\delta} \) sets of \((X, T, \leq)\) such that \( D^{\text{IFPS}}(A_{q_1}) \subseteq \bar{\sigma}_{q_2} \) and \( C_{q_3} \subseteq \sigma_{q_2} \) whenever \( q_1 < q_2 \).

**Proof**

Let us arrange into a sequence \( q_n \) of rational numbers without repetitions. For every \( n \geq 1 \) define inductively a collection \( C_{q_i} \) such that

\[
\begin{align*}
D^{\text{IFPS}}(A_{q_i}) & \subseteq \bigcap_{i < j} \bigcap_{\sigma, \omega} A_{q_i} \cap A_{q_j}, \\
C_{q_i} & \subseteq \bigcap_{i < j} \sigma_{q_j}.
\end{align*}
\]

for all \( i < n \).
By Proposition 5.4.3, the countable collections $D^{IFPS}(A_q)$ and $D^{0IFPS_S}_{\sigma, \mathcal{q}}$, satisfying $D^{IFPS}(A_q) \subseteq \mathcal{r}_q \mathcal{s}_q$ if $q_1 < q_2$.

By Remark 5.4.1, there exists a decreasing intuitionistic fuzzy pre semi $\text{COG}_g \mathcal{r}_g \sigma$ set $D_1$ such that $D^{IFPS_S}_{\mathcal{q}_1} \subseteq \mathcal{r}_q \mathcal{s}_q$, setting, $C_{q_1} = \text{we get } (S_2)$.

Assume that intuitionistic fuzzy sets $C_{q_i}$ are already defined for $i < n$ and satisfy $(S_n)$.

Define $\Sigma = \bigcup_{i < n} / i < n, q_i < q_n \bigcup A_q$ and

$\Phi = \bigcap_{i < n} / j < n, q_j > q_n \bigcap B_q$.

Then, $D^{IFPS}(C_{q_i}) \subseteq \Sigma \subseteq \mathcal{r}_q \mathcal{s}_q \quad \text{and} \quad D^{IFPS}(C_{q_i}) \subseteq \Phi \subseteq \mathcal{r}_q$,

whenever $q_i < \mathcal{r}_q \mathcal{s}_q (i,j < n)$ as well as $A_q \subseteq \Sigma \subseteq \mathcal{r}_q \mathcal{s}_q$ and

$A_q \subseteq \Phi \subseteq \mathcal{r}_q$, whenever $q < q_n < q$.

This shows that the countable collections

$C_{q_i} / i < n, q_i < q_n \bigcup A_q / q < q_n$ and $C_{q_j} / j < n, q_j > q_n \bigcup B_q / q > q_n$ together with $\Sigma$ and $\Phi$ fulfill all conditions of Remark 5.4.1.

Hence, there exists a decreasing intuitionistic fuzzy pre semi $\text{COG}_g \mathcal{r}_g \sigma$ set $D_n$ such that $D^{IFPS}(D_n) \subseteq \mathcal{r}_q \mathcal{s}_q$ if $q < q_n$, $A_q \subseteq \mathcal{r}_q \mathcal{s}_q$, if $q < q_n$.
\[ D^{IFPS}(C_{q_i}) \subseteq \omega_{\omega}, \text{ if } q_i < q_n, \]
\[ D^{IFPS}(D_n) \subseteq \omega_{-q_j}, \text{ if } q_n < q_j \text{ where } l \leq j \leq n. \]

Now, setting \( C_{q_n} = \) we obtain the intuitionistic fuzzy sets \( C_{q_1}, C_{q_2}, \ldots, C_{q_n} \) that satisfy \((S_{p+})\). Therefore the collection \( C_{q_i}/i = \) has required property.

### 5.5 Properties and Characterizations of an Ordered Intuitionistic Fuzzy Presemi Basically Disconnected Spaces

In this section, various properties and characterizations of an ordered intuitionistic fuzzy presemi basically disconnected spaces are discussed.

**Definition 5.5.1**

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space. A function \( f : X \rightarrow R_l \) is called lower (resp., upper) intuitionistic fuzzy presemi continuous, if \( f^{-1}(R_l') \) (resp., \( f^{-1}(\omega_{l}') \)) is an increasing or decreasing intuitionistic fuzzy presemi open \( F_\sigma \) (resp., increasing or decreasing intuitionistic fuzzy presemi \( COG_{\sigma'} \)) set for each \( t \in \)

**Lemma 5.5.1**

Let \((X, T, \leq)\) be an ordered intuitionistic fuzzy topological space. Let \( A \in \zeta \) and let \( f : X \rightarrow R_l \) be such that

\[
f(x)(t) = \begin{cases}
  t < 0, & \leq \leq \\
  t > 1,
\end{cases}
\]
for all $x \in X$ and $t \in T$. Then $f$ is lower (resp., upper) intuitionistic fuzzy presemi continuous iff $A$ is an increasing or decreasing intuitionistic fuzzy presemi open $F_{\sigma}^-$ (resp., increasing or decreasing intuitionistic fuzzy presemi $COG_{\delta}^-$) set.

**Proof**

\[
\begin{align*}
    f^{-1}(R_t^1) = & \begin{cases} 
    t < \\
    \leq < \\
    t \geq 
    \end{cases} \\
    f^{-1}(L_t^1) = & \begin{cases} 
    t \leq \\
    < \leq \\
    t > 
    \end{cases}
\end{align*}
\]

implies that $f$ is lower $F_{\sigma}$ intuitionistic fuzzy presemi continuous iff $A$ is increasing or decreasing intuitionistic fuzzy presemi open $F_{\sigma}^-$ set.

implies that $f$ is an upper intuitionistic fuzzy presemi continuous iff $A$ is an increasing or decreasing intuitionistic fuzzy presemi $COG_{\delta}^-$ set.

**Proposition 5.5.1**

Let $(X, T, \leq)$ be an ordered intuitionistic fuzzy topological space and let $A \in \zeta$. Then $\psi_\omega$ is lower (resp., upper) intuitionistic fuzzy presemi continuous iff $A$ is an increasing or decreasing intuitionistic fuzzy presemi open $F_{\sigma}^-$ (resp., increasing or decreasing intuitionistic fuzzy presemi $COG_{\delta}^-$) set.

**Proof** The proof follows from Lemma 5.5.1.
Definition 5.5.2

Let $(X,T,\leq)$ and $(Y,S,\leq)$ be ordered intuitionistic fuzzy topological spaces. A function $f:(X,T,\leq) \rightarrow (Y,S,\leq)$ is called increasing (resp., decreasing) intuitionistic fuzzy strongly presemi continuous if $f^{-1}(\alpha)$ is increasing (resp., decreasing) intuitionistic fuzzy presemi $\text{COG}_\delta^{-\sigma}$ in $(X,T,\leq)$ for every increasing (resp., decreasing) intuitionistic fuzzy presemi open $F_\sigma$ set in $(Y,S,\leq)$ If $f$ is both increasing and decreasing intuitionistic fuzzy strongly presemi continuous, then it is called ordered intuitionistic fuzzy strongly presemi continuous.

Proposition 5.5.2

Let $(X,T,\leq)$ be an ordered intuitionistic fuzzy topological space. Then the following statements are equivalent:

(a) $(X,T,\leq)$ is upper intuitionistic fuzzy presemi basically disconnected.

(b) If $g,h:X \rightarrow R_I$ $g$ is lower intuitionistic fuzzy presemi continuous, $h$ is upper intuitionistic fuzzy presemi continuous and $g \subseteq$ then there exists an increasing intuitionistic fuzzy strongly presemi continuous function, $f:(X,T,\leq) \rightarrow R_I$ such that $g \subseteq$.

(c) If $\overline{A}$ is increasing intuitionistic fuzzy presemi open $F_\sigma$ set and $B$ is decreasing intuitionistic fuzzy presemi open $F_\sigma$ set such that $B \subseteq$ then there exists an increasing intuitionistic fuzzy
strongly presemi continuous function \( f : (X, T, \leq) \to I_f \) such that \( B \subseteq I \subseteq C \).

**Proof**

(a) \( \Rightarrow \) (b): Define \( H_r = \quad \) and \( G_r = \quad r \in \quad \) Thus we have two monotone increasing families respectively decreasing intuitionistic fuzzy presemi open \( F_\sigma \) and increasing intuitionistic fuzzy presemi closed \( G_\delta \) sets of \( (X, T, \leq) \). Moreover \( H_r \subseteq \quad \) if \( r < \) By Proposition 5.4.4, there exists a monotone increasing family \( F_{r \in} \) of decreasing intuitionistic fuzzy presemi \( COG_\delta \) sets of \( (X, T, \leq) \) such that \( D^{IFPS}(H_r) \subseteq \quad \) and \( F_r \subseteq \quad \sigma \), whenever \( r < \). Let \( V_t = \quad \) for all \( t \in \quad \) we define a monotone decreasing family \( V_t / t \in \quad \subseteq \zeta \). Moreover we have \( I^{IFPS}(V_t) \subseteq \quad \), whenever \( s \prec t \). We have

\[
\bigcup_{t \in R} V_t = \bigcup_{t \in R} \bigcap_r F_r \\
\supseteq \bigcup_{t \in R} \bigcap_r G_r \\
= \bigcup_{t \in R} \bigcap_r \sigma, \quad \text{if} \quad r < \\
= g \quad \bigcup_{t \in R} L^{I}_t, \\
= g \quad \bigcup_{t \in R} L^{I}_t = \\
= \]

Similarly, \( \bigcap_{t \in R} V' = \)

Now, define a function \( f : X \to \quad \) satisfying the required properties. Let \( f(x)(t) = \quad \) for all \( x \in \quad \) and \( t \in \quad \) By the above
discussion, it follows that \( f \) is well defined. To prove \( f \) is increasing intuitionistic fuzzy strongly presemi continuous, we observe that

\[
\bigcup_{s > t} V_s = \bigcup \overline{\text{IFPSF}}_{\sigma} \cap \overline{V}_s \quad \text{and} \quad \bigcap_{s < t} V_s = \bigcap \overline{\text{IFPSG}}_{\delta} \cap \overline{V}_s.
\]

Then, \( f^{-} (\omega_i) = \bigcup \overline{\text{IFPSF}}_{\sigma} \cap \overline{V}_s \), is increasing intuitionistic fuzzy presemi open \( F_{\sigma} \), and \( f^{-} (\omega_i) = \bigcap \overline{\text{IFPSG}}_{\delta} \cap \overline{V}_s \), is increasing intuitionistic fuzzy presemi closed \( G_{\delta} \). Therefore, \( f \) is increasing intuitionistic fuzzy strongly presemi continuous.

It remains to show that \( g \subseteq \subseteq \) that is,

\[
g^{-} (\omega_i) \subseteq \subseteq , \quad \text{and} \quad g^{-} (\omega_i) \subseteq \subseteq \quad \text{for each } t \in \]

We have,

\[
g^{-} (L_i^I) = \bigcap_s g^{-} (L_i^I),
\]

\[
= \bigcap_s \bigcap_{s < r} g^{-} (R_i^I),
\]

\[
= \bigcap_s \bigcap_{s < r} \overline{G}_r,
\]

\[
\subseteq \bigcap_s \bigcap_{s < r} \overline{F}_r
\]

\[
= \bigcap_s \bigcap_{s < r} V_s.
\]

And,

\[
f^{-} (L_i^I) = \bigcap_s V_s
\]

\[
= \bigcap_s \bigcap_{s < r} \overline{F}_r
\]

\[
\subseteq \bigcap_s \bigcap_{s < r} \overline{F}_r
\]

\[
= \bigcap_s \bigcap_{s < r} \overline{H}_r
\]

\[
= \bigcap_s \bigcap_{s < r} h^{-} (L_i^I),
\]

\[
= \bigcap_s \bigcap_{s < r} h^{-} (L_i^I).
\]
Similarly, \( g^-(R^l_s) = \bigcup_s g^-(R^l_s) \)
\[ = \bigcup_s \bigcup_{r > s} g^-(R^l_r) \]
\[ = \bigcup_s \bigcup_{r > s} \overline{G_r} \]
\[ \subseteq \bigcup_s \bigcap_{r < s} F_r \]
\[ = \bigcup_s V_s = f \cdot R^l_t \]

And, \( f^-(R^l_t) = \bigcup_s V_s \)
\[ = \bigcup_s \bigcap_{r < s} F_r \]
\[ \subseteq \bigcup_s \bigcup_{r > s} \overline{H_r} \]
\[ = \bigcup_s \bigcup_{r > s} h^-L^l_r \]
\[ = \bigcup_{s > t} R^l_s, = \]

Thus (b) is proved.

\textbf{(b) \Rightarrow (c):} Suppose \( \overline{A} \) is an increasing intuitionistic fuzzy presemi open \( F_\delta \) set and \( B \) is a decreasing intuitionistic fuzzy presemi closed \( G_\delta \) set, such that \( B \subseteq A \). Then, \( \psi^-_\alpha \subseteq \psi^-_\alpha \) and \( \psi^-_\alpha \psi^-_\alpha \) are lower and upper intuitionistic fuzzy presemi continuous functions respectively.

Hence by (b), there exists an increasing intuitionistic fuzzy strongly presemi continuous function \( f:(X,T,\leq) \to R_I \) such that \( \psi^-_\alpha \subseteq \psi^-_\alpha \). Clearly, \( f(x) \in I_I(I) \) for all \( x \in A \) and \( B = \overline{T'} \psi^-_\alpha \subseteq \overline{T'} \).

\[ \subseteq \bigcup_{\nu} \bigcup \nu \psi^-_\alpha \]

Therefore, \( B = \overline{T'} \subseteq \overline{\overline{T'}} \subseteq \]
(c) \implies (a): This follows from Proposition 5.4.3 and the fact that $(\overline{L^I}_k)f$ and $R^I_0(f)$ are decreasing intuitionistic fuzzy presemi $COG_{\delta}^{-\sigma}$ sets respectively. Therefore, $(X,T,\leq)$ is upper intuitionistic fuzzy presemi basically disconnected.

**Note 5.5.1**

The Propositions 5.4.1 to 5.4.4, 5.5.2 and Remark 5.4.1 can be discussed for other cases also.

**5.6 Tietze Extension Theorem for Ordered Intuitionistic Fuzzy Presemi Basically Disconnected Spaces.**

In this section, Tietze extension theorem for ordered intuitionistic fuzzy presemi basically disconnected spaces is studied.

**Proposition 5.6.1**

Let $(X,T,\leq)$ be an upper intuitionistic fuzzy presemi basically disconnected space and let $A \subseteq X$ be such that $\gamma$ is increasing intuitionistic fuzzy presemi open $F_\sigma$ set in $(X,T,\leq)$. Let $f:(A, T/A) \rightarrow$ be an increasing intuitionistic fuzzy strongly presemi continuous function. Then $f$ has an increasing intuitionistic fuzzy strongly presemi continuous extension over $(X,T,\leq)$.

**Proof**

Let $g, h : X \rightarrow$ be such that $g = \gamma$ on $A$, and $g(x) =$

$h(x) = \text{if } x \notin A$

We now have,
where $B_t$ is increasing intuitionistic fuzzy presemi open $F_\sigma$ set such that $B_t/A = \chi$ and

$$L_t^{'}h = \begin{cases} 
\bigcap & \leq \\
& \text{if } t>1,
\end{cases}$$

where $C_t$ is increasing intuitionistic fuzzy presemi open $F_\sigma$ such that $C_t/A = \chi$. Thus, $g$ is lower intuitionistic fuzzy presemi continuous, $h$ is upper intuitionistic fuzzy presemi continuous and $g \subseteq$ By Proposition 5.5.2, there is an increasing intuitionistic fuzzy strongly presemi continuous function $F: (X,T_\leq) \to I_I$ such that $g \subseteq \subseteq$ hence $F \equiv$ on $A$.

**Note 5.6.1**

The above proposition can be discussed for other case also.

### 5.7 PAIRWISE ORDERED INTUITIONISTIC FUZZY PRESEMI EXTREMALLY DISCONNECTED SPACES

In this section, the concept of pairwise ordered intuitionistic fuzzy presemi extremally disconnected spaces is introduced. Characterizations and properties are studied.

**Definition 5.7.1**

An intuitionistic fuzzy bitopological space is a triple $(X,T_1,T_2)$ where $X$ is a set and $T_1$, $T_2$ are any two intuitionistic fuzzy topologies on $X$. 

\[
R_t^{'}g = \begin{cases} 
\bigcap & \geq \\
& \text{if } t<0,
\end{cases}
\]
Definition 5.7.2

An ordered set $(X, \leq)$ on which there is given intuitionistic fuzzy topologies $T_1$, $T_2$ is called an ordered intuitionistic fuzzy bitopological space and it is denoted by $(X, T_1, T_2, \leq)$.

Definition 5.7.3

Let $(X, T_1, T_2, \leq)$ be an ordered intuitionistic fuzzy bitopological space. Let $A$ be any $T_1$-increasing (resp., $T_1$-decreasing) intuitionistic fuzzy presemi open set in $(X, T_1, T_2, \leq)$. If $I_{T_2}^{IFS}(A)$ (resp., $D_{T_2}^{IFS}(A)$) is $T_2$-increasing (resp., $T_2$-decreasing) intuitionistic fuzzy presemi open in $(X, T_1, T_2, \leq)$, then $(X, T_1, T_2, \leq)$ is said to be $T_1$-upper (resp., $T_1$-lower) intuitionistic fuzzy presemi extremally disconnected. Similarly we can define $T_2$-upper (resp., $T_2$-lower) intuitionistic fuzzy presemi extremally disconnected. An ordered intuitionistic fuzzy bitopological space $(X, T_1, T_2, \leq)$ is said to be pairwise upper intuitionistic fuzzy presemi extremally disconnected if it is both $T_1$-upper intuitionistic fuzzy presemi extremally disconnected and $T_2$-upper intuitionistic fuzzy presemi extremally disconnected. Similarly we can define pairwise lower intuitionistic fuzzy presemi extremally disconnected. An ordered intuitionistic fuzzy bitopological space $(X, T_1, T_2, \leq)$ is said to be pairwise ordered intuitionistic fuzzy presemi extremally disconnected if it is both pairwise upper intuitionistic fuzzy presemi extremally disconnected and pairwise lower intuitionistic fuzzy presemi extremally disconnected.
Proposition 5.7.1

For an ordered intuitionistic fuzzy bitopological space 
\((X, T_1, T_2, \leq)\), the following statements are equivalent:

(a) \((X, T_1, T_2, \leq)\) is pairwise upper intuitionistic fuzzy presemi extremally disconnected.

(b) For each \(T_1\)-decreasing intuitionistic fuzzy presemi closed set \(A\), 
\(D_{T_2}^{0IFPS}(A)\) is \(T_2\)-decreasing intuitionistic fuzzy presemi closed.
Similarly, for each \(T_2\)-decreasing intuitionistic fuzzy presemi closed set \(A\), 
\(D_{T_1}^{0IFPS}(A)\) is \(T_1\)-decreasing intuitionistic fuzzy presemi closed.

(c) For each \(T_1\)-increasing intuitionistic fuzzy presemi open set \(A\), 
\(D_{T_2}^{IFPS}(I_{T_2}^{IFPS}(A)) = \_\). Similarly, for each \(T_2\)-increasing intuitionistic fuzzy presemi open set \(A\), 
\(D_{T_1}^{IFPS}(I_{T_1}^{IFPS}(A)) = \_\).

(d) For each pair of a \(T_1\)-increasing intuitionistic fuzzy presemi open set \(A\) and a \(T_1\)-decreasing intuitionistic fuzzy presemi open set \(B\) in \((X, T_1, T_2, \leq)\) with 
\(I_{T_2}^{IFPS}(A) = \_\) and \(D_{T_2}^{IFPS}(B) = \_\). Similarly, for each pair of a \(T_2\)-increasing intuitionistic fuzzy presemi open set \(A\) and \(T_2\)-decreasing intuitionistic fuzzy presemi open set \(B\) in 
\((X, T_1, T_2, \leq)\) with 
\(I_{T_1}^{IFPS}(A) = \_\) and \(D_{T_1}^{IFPS}(B) = \_\).
**Proof**

(a) \(\Rightarrow\) (b): Let \(A\) be any \(T_1\)-decreasing intuitionistic fuzzy presemi closed set in \((X,T_1,T_2,\leq)\). We claim that \(D^{\text{IFPS}}_{T_2}(A)\) is \(T_2\)-decreasing intuitionistic fuzzy presemi closed. Now, \(\overline{A}\) is \(T_1\)-increasing intuitionistic fuzzy presemi open. So by assumption (a), \(I^{\text{IFPS}}_{T_2}(\overline{A}) = \overline{I^{\text{IFPS}}_{T_2}(A)}\) is \(T_2\)-increasing intuitionistic fuzzy presemi open. That is, \(D^{\text{IFPS}}_{T_2}(A)\) is \(T_2\)-decreasing intuitionistic fuzzy presemi closed.

(b) \(\Rightarrow\) (c): Let \(A\) be an \(T_1\)-increasing intuitionistic fuzzy presemi open set in \((X,T_1,T_2,\leq)\). Then, \(\overline{A}\) is a \(T_1\)-decreasing intuitionistic fuzzy presemi closed set and by (b), \(D^{\text{IFPS}}_{T_2}(\overline{A}) = \overline{D^{\text{IFPS}}_{T_2}(A)}\) is a \(T_2\)-decreasing intuitionistic fuzzy presemi closed set. Now, consider

\[
D^{\text{IFPS}}_{T_2}(I^{\text{IFPS}}_{T_2}(A)) = D^{\text{IFPS}}_{T_2}(D^{\text{IFPS}}_{T_2}(A)) = \overline{D^{\text{IFPS}}_{T_2}(A)} = \overline{\overline{A}}.
\]

Hence, \(D^{\text{IFPS}}_{T_2}(I^{\text{IFPS}}_{T_2}(A)) = I^{\text{IFPS}}_{T_2}(A)\). By Proposition 5.1.1, (c) holds.

(c) \(\Rightarrow\) (d): Let \(A\) be any \(T_1\)-increasing intuitionistic fuzzy presemi open set and \(B\) be any \(T_1\)-decreasing intuitionistic fuzzy presemi open set such that \(I^{\text{IFPS}}_{T_2}(A) = \overline{\overline{A}}\). By Proposition 5.1.1, \(B = \overline{\overline{A}}\).

By (c), \(D^{\text{IFPS}}_{T_2}(I^{\text{IFPS}}_{T_2}(A)) = D^{\text{IFPS}}_{T_2}(D^{\text{IFPS}}_{T_2}(A)) = \overline{D^{\text{IFPS}}_{T_2}(A)} = \overline{\overline{A}}\).

Therefore, \(D^{\text{IFPS}}_{T_2}(I^{\text{IFPS}}_{T_2}(A)) = D^{\text{IFPS}}_{T_2}(B)\).

By (c), \(D^{\text{IFPS}}_{T_2}(D^{\text{IFPS}}_{T_2}(A)) = I^{\text{IFPS}}_{T_2}(A)\).
But, \( B = \overline{\overline{A}} \). Thus, \( I_{T_2}^{IFPS}(A) = \overline{\overline{A}} \).

\((d) \Rightarrow (a):\) Let \( A \) be any \( T_1 \)-increasing intuitionistic fuzzy presemi open set. Clearly, \( B \) is an \( T_2 \)-decreasing intuitionistic fuzzy presemi open set.

Let \( I_{T_2}^{IFPS}(A) = \overline{\overline{A}} \). From \((d)\), it follows that \( D_{T_2}^{IFPS}(B) = \overline{\overline{B}} \). That is, \( I_{T_2}^{IFPS}(A) \) is a \( T_1 \)-decreasing intuitionistic fuzzy presemi closed set.

This implies that \( I_{T_2}^{IFPS}(A) \) is \( T_2 \)-increasing intuitionistic fuzzy presemi open set. Hence, \((X, T_1, T_2, \leq)\) is \( T_1 \)-upper intuitionistic fuzzy presemi extremally disconnected. Similarly, we can prove that \((X, T_1, T_2, \leq)\) is \( T_2 \)-upper intuitionistic fuzzy presemi extremally disconnected space.

Hence, \((X, T_1, T_2, \leq)\) is pairwise upper intuitionistic fuzzy presemi extremally disconnected space

**Proposition 5.7.2**

Let \((X, T_1, T_2, \leq)\) be an ordered intuitionistic fuzzy bitopological space. Then \((X, T_1, T_2, \leq)\) is a pairwise ordered intuitionistic fuzzy presemi extremally disconnected space if and only if for a \( T_1 \)-decreasing intuitionistic fuzzy presemi open set \( A \) and \( T_2 \)-decreasing intuitionistic fuzzy presemi closed set \( B \) such that \( A \subseteq B \), we have \( D_{T_1}^{IFPS}(A) \subseteq B \).

**Proof**

Suppose that \((X, T_1, T_2, \leq)\) is a pairwise upper intuitionistic fuzzy presemi extremally disconnected space. Let \( A \) be any \( T_1 \)-decreasing
intuitionistic fuzzy presemi open set, \( B \) be any \( T_2 \)-decreasing intuitionistic fuzzy presemi closed set such that \( A \subseteq \). Then by (b) of Proposition 5.7.1, \( D_{T_1}^{0\text{IFPS}}(B) \) is a \( T_1 \)-decreasing intuitionistic fuzzy presemi closed set. Also, since \( A \) is \( T_1 \)-decreasing intuitionistic fuzzy presemi open and \( A \subseteq \), it follows that \( A \subseteq \).\( B \). Again, since \( D_{T_1}^{0\text{IFPS}}(B) \) is \( T_1 \)-decreasing intuitionistic fuzzy presemi closed, it follows that \( D_{T_1}^{IFPS}(A) \subseteq \). To prove the converse, let \( B \) be any \( T_2 \)-decreasing intuitionistic fuzzy presemi closed set. Then, \( D_{T_1}^{0\text{IFPS}}(B) \) is \( T_1 \)-decreasing intuitionistic fuzzy presemi open and it is clear that \( D_{T_1}^{0\text{IFPS}}(B) \subseteq \). Therefore by assumption, \( D_{T_1}^{IFPS}(D_{T_1}^{0\text{IFPS}}(B)) \subseteq \).\( B \). But always \( D_{T_1}^{0\text{IFPS}}(B) \subseteq \).\( D_{T_1}^{0\text{IFPS}}(B) \)).

Thus, \( D_{T_1}^{IFPS}(D_{T_1}^{0\text{IFPS}}(B))= \).\( B \). This implies that \( D_{T_1}^{0\text{IFPS}}(B) \) is a \( T_1 \)-decreasing intuitionistic fuzzy presemi closed set. Hence by (b) of Proposition 5.7.1, it follows that \((X,T_1,T_2,\leq )\) is \( T_1 \)-upper intuitionistic fuzzy presemi extremally disconnected. Similarly we can prove also the other cases.

**Notation 5.7.1**

An ordered intuitionistic fuzzy set which is both \( T_1 \) or \( T_2 \)-decreasing (resp., \( T_1 \) or \( T_2 \)-increasing) intuitionistic fuzzy presemi open set and \( T_1 \) or \( T_2 \)-decreasing (resp., \( T_1 \) or \( T_2 \)-increasing) intuitionistic
fuzzy presemi closed is called a $T_1$ or $T_2$-decreasing (resp., $T_1$ or $T_2$-increasing) intuitionistic fuzzy presemi clopen set.

**Remark 5.7.1**

Let $(X, T_1, T_2, \leq)$ be a pairwise ordered intuitionistic fuzzy presemi extremally disconnected space. Let $A_i, B_i/i \in \mathbb{I}$ be a collection such that $A_i$’s are $T_1$-decreasing intuitionistic fuzzy presemi open sets, $B_i$’s are $T_2$-decreasing intuitionistic fuzzy presemi closed sets and let $A, B$ be $T_1$-decreasing intuitionistic fuzzy presemi open and $T_2$-increasing intuitionistic fuzzy presemi open sets respectively. If $A_i \subseteq \subseteq$ and $A_i \subseteq \subseteq$ for all $i, j \in \mathbb{I}$ then there exists a $T_1$ and $T_2$-decreasing intuitionistic fuzzy presemi clopen set $C$ such that

$$D_{T_1}^{IFPS}(A_i) \subseteq \subseteq \cup (B_j) \text{ for all } i, j \in \mathbb{I}.$$

**Proof**

By Proposition 5.7.2,

$$D_{T_1}^{IFPS}(A_i) \subseteq \subseteq \cap \subseteq \subseteq (i, j \in \mathbb{I}).$$

Put $C = \cap \cap \cap \cap D_{T_1}^{0IFPS}(B)$. Now, $C$ satisfies our required

**Proposition 5.7.3**

Let $(X, T_1, T_2, \leq)$ be a pairwise ordered intuitionistic fuzzy presemi extremally disconnected space. Let $A_q, B_q/q \in \mathbb{Q}$ be the monotone increasing collections of $T_1$-decreasing intuitionistic fuzzy presemi open sets and $T_2$-decreasing intuitionistic fuzzy presemi
closed sets of \((X, T_1, T_2, \leq)\) respectively and suppose that \(A_{q_1} \subseteq \ldots\) whenever \(q_1 < \ldots\) (\(Q\) is the set of rational numbers). Then there exists a monotone increasing collection \(C_q, q \in T_1\) and \(T_2\)-decreasing intuitionistic fuzzy presemi clopen sets of \((X, T_1, T_2, \leq)\) such that \(D_{n_1}^{IFPS}(A_{q_1}) \subseteq \ldots\) and \(C_{q_1} \subseteq \ldots\) \((B_{q_2})\) whenever \(q_1 < \ldots\)

**Proof**

Let us arrange into a sequence \(q_n\) of rational numbers without repetitions. For every \(n \geq\), define inductively a collection \(C_{q_i}, 1 \leq i \leq \ldots\) such that

\[
\begin{align*}
D_{n_1}^{IFPS}(A_{q_1}) & \subseteq \ldots < \ldots < \ldots \quad (S_n) \\
C_{q_i} & \subseteq \ldots < \ldots \quad (S_n)
\end{align*}
\]

for all \(i < n\).

By Proposition 5.7.2, the countable collections \(D_{n_1}^{IFPS}(A_{q_1})\) and satisfy \(D_{n_1}^{IFPS}(A_{q_1}) \subseteq \ldots \subseteq (B_{q_2})\) if \(q_1 < \ldots\)

By Remark 5.7.1, there exists a \(T_1\) and \(T_2\)-decreasing intuitionistic fuzzy presemi clopen set \(D_{n_1}\) such that \(D_{n_1}^{IFPS}(A_{q_1}) \subseteq \ldots \subseteq B_{q_2}\). Setting, \(C_{q_1} = \ldots\) we get \((S_2)\).

Assume that \(T_1\)-intuitionistic fuzzy sets \(C_{q_i}\) are already defined for \(i <\) and satisfy \((S_n)\).

Define
\[
\sum = \bigcup_{i} / i < n, q_i < q_n \bigcup A_{q_n} \quad \text{and} \quad \Phi = \bigcap_{j} / j < n, q_j > q_n \bigcap B_{q_n}.
\]

Then,
\[
D_{T_1}^{IFPS}(C_{q_i}) \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup \sum \subseteq \bigcap_{j} / j < n, q_j > q_n \bigcap \Phi
\]
whenever \( q_i < \ldots < q_j \) and as well as \( A_{q} \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup \sum \subseteq \bigcap_{j} / j < n, q_j > q_n \bigcap \Phi \),
and \( A_{q} \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup \sum \subseteq \bigcap_{j} / j < n, q_j > q_n \bigcap \Phi \), whenever \( q < q_n < \)

This shows that the countable collections
\[
C_{q_i} / i < n, q_i < q_n \bigcup A_{q} / q < q_n \quad \text{and} \quad C_{q_j} / j < n, q_j > q_n \bigcup B_{q} / q > q_n
\]
together with \( \sum \) and \( \Phi \) fulfill all conditions of Remark 5.7.1.

Hence, there exists a \( T_1 \) and \( T_2 \)-decreasing intuitionistic fuzzy presemi clopen set \( D_n \) such that
\[
D_{T_1}^{IFPS}(D_n) \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup A_{q} \bigcap \Phi \text{ if } q_n < q, A_{q} \bigcap \Phi \text{ if } q < q_n,
\]
\[
D_{T_1}^{IFPS}(C_{q_i}) \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup D_n \text{ if } q_i < q_n,
\]
\[
D_{T_1}^{IFPS}(D_n) \subseteq \bigcup_{i} / i < n, q_i < q_n \bigcup C_{q_j} \text{ if } q_n < q_j \text{ where } 1 \leq j \leq n - \]

Now, setting \( C_{q_n} = \) we obtain the \( T_1 \)-intuitionistic fuzzy sets
\[
C_{q_1}, C_{q_2}, \ldots, C_{q_n} \text{ that satisfy } (S_{n+}) \cdot \text{ Therefore the collection}
\]
\[
C_{q_i} / i = \text{ has the required property.}
\]

**Definition 5.7.4**

Let \( (X, T_1, T_2, \leq) \) be an ordered intuitionistic fuzzy bitopological space. A function \( f : X \rightarrow R_{IFPS} \) is called \( T_1 \)-lower (resp., \( T_1 \)-upper)
intuitionistic fuzzy presemi continuous, if \( f^{-1}(R_{t}^L) \) (resp., \( f^{-1}(\cup_{t}^L) \)) is a \( T_{1} \)-increasing or \( T_{1} \)-decreasing intuitionistic fuzzy presemi open set (resp., \( T_{1} \)-increasing or \( T_{1} \)-decreasing intuitionistic fuzzy presemi clopen) for each \( t \in \). Similarly we can define \( T_{2} \)-lower (resp., \( T_{2} \)-upper) intuitionistic fuzzy presemi continuous.

**Lemma 5.7.1**

Let \((X,T_{1},T_{2},\leq)\) be an ordered intuitionistic fuzzy bitopological space. Let \( A \in \zeta \) be a \( T_{1} \)-intuitionistic fuzzy set and let \( f : X \to R_{I} \) be such that

\[
f(x)(t) = \begin{cases} t < 0, \\ \leq, \\ t > 1, 
\end{cases}
\]

for all \( x \in X \) and \( t \in I \). Then \( f \) is \( T_{1} \)-lower (resp., \( T_{1} \)-upper) intuitionistic fuzzy presemi continuous iff \( A \) is an \( T_{1} \)-increasing or \( T_{1} \)-decreasing intuitionistic fuzzy presemi open (resp., \( T_{1} \)-increasing or \( T_{1} \)-decreasing intuitionistic fuzzy presemi clopen) set.

**Proof**

\[
f^{-1}(R_{t}^L) = \begin{cases} t < \\ \leq, \\ t \geq \end{cases}
\]

implies that \( f \) is \( T_{1} \)-lower intuitionistic fuzzy presemi continuous iff \( A \) is \( T_{1} \)-increasing or \( T_{1} \)-decreasing intuitionistic fuzzy presemi open set.
implies that \( f \) is \( T_i \)-upper intuitionistic fuzzy presemi continuous iff \( A \) is an \( T_i \)-increasing or \( T_i \)-decreasing intuitionistic fuzzy presemi clopen set.

**Proposition 5.7.4**

Let \( (X, T_1, T_2, \leq) \) be an ordered intuitionistic fuzzy bitopological space, and let \( A \in \zeta \) be an \( T_i \)-intuitionistic fuzzy set. Then, \( \psi \) is \( T_i \)-lower (resp., \( T_i \)-upper) intuitionistic fuzzy presemi continuous iff \( A \) is an \( T_i \)-increasing or \( T_i \)-decreasing intuitionistic fuzzy presemi open set (resp., \( T_i \)-increasing or \( T_i \)-decreasing intuitionistic fuzzy presemi clopen) set.

**Proof.** The proof follows from Lemma 5.7.1.

**Definition 5.7.5**

Let \( (X, T_1, T_2, \leq) \) and \( (Y, S_1, S_2, \leq) \) be ordered intuitionistic fuzzy bitopological spaces. A function \( f: (X, T_1, T_2, \leq) \to (Y, S_1, S_2, \leq) \leq \) is called \( T_i \)-increasing (resp., \( T_i \)-decreasing) intuitionistic fuzzy strongly presemi continuous if \( f^{-1}(\zeta) \) is \( T_i \)-increasing (resp., \( T_i \)-decreasing) intuitionistic fuzzy presemi clopen in \( (X, T_1, T_2, \leq) \) for every \( S_1 \) or \( S_2 \)-intuitionistic fuzzy presemi open set in \( (Y, S_1, S_2, \leq) \). If \( f \) is both \( T_i \)-increasing and \( T_i \)-decreasing intuitionistic fuzzy strongly presemi continuous, then it is called ordered \( T_i \)-intuitionistic fuzzy strongly
presemi continuous. Similarly we can define for ordered $T_2$-intuitionistic fuzzy strongly presemi continuous functions.

**Proposition 5.7.5**

Let $(X, T_1, T_2, \leq)$ be an ordered intuitionistic fuzzy bitopological space. Then the following statements are equivalent:

(a) $(X, T_1, T_2, \leq)$ is pairwise ordered intuitionistic fuzzy presemi extremally disconnected,

(b) If $g, h : X \to R_I$, $g$ is $T_1$-lower intuitionistic fuzzy presemi continuous, $h$ is $T_2$-upper intuitionistic fuzzy presemi continuous and $g \subseteq I$, then there exists a $T_1$ and $T_2$-increasing intuitionistic fuzzy strongly presemi continuous function, $f : (X, T_1, T_2, \leq) \to I_I$ such that $g \subseteq I$

(c) If $A$ is $T_2$-increasing intuitionistic fuzzy presemi open set and $B$ is $T_1$-decreasing intuitionistic fuzzy presemi open set such that $B \subseteq I$, then there exists an $T_1$ and $T_2$-increasing intuitionistic fuzzy strongly presemi continuous function $f : (X, T_1, T_2, \leq) \to I_I$ such that $B \subseteq I_I$

**Proof**

(a) $\implies$ (b): Define $H_r = \ldots$ and $G_r = \ldots$, $r \in \ldots$ Thus we have two monotone increasing families respectively $T_1$-decreasing intuitionistic fuzzy presemi open and $T_2$-decreasing intuitionistic fuzzy presemi closed sets of $(X, T_1, T_2, \leq)$. Moreover $H_r \subseteq I$ if $r < \ldots$ By Proposition
5.7.3, there exists a monotone increasing family $F_r$ of $T_1$ and $T_2$-

decreasing intuitionistic fuzzy presemi clopen sets of $(X,T_1,T_2,\leq)$ such
that $D_{I_{T_1}}^{IFPS}(H_r) \subseteq$ and $F_r \subseteq_1 G_2$ whenever $r <$. Let
$V_t=\bigcap_{t \in \zeta}$ for all $t \in \zeta$ we define a monotone decreasing family
$V_t/t \in \zeta$. Moreover, we have $I_{T_1}^{IFPS}(V_t) \subseteq_1 V_s$ whenever $s < t$.

We have

\[ \bigcup_{t \in R} V_t = \bigcup_{t \in R} \bigcap_{r} \overline{F_r} \]

\[ \supseteq \bigcup_{t \in R} \bigcap_{r} \overline{G_r} \]

\[ = \bigcap_{r} \bigcup_{t \in R} L^T_t \]

\[ = \bigcap_{r} \bigcup_{t \in R} L_t \]

Similarly, $\bigcap_{t \in \zeta}$.

We now define a function $f : X \rightarrow$ satisfying the required
properties. Let $f(x(t)) = $ for all $x \in X$ and $t \in T$. By the above
discussion, it follows that $f$ is well defined.

To prove $f$ is $T_1$ and $T_2$-increasing intuitionistic fuzzy strongly
presemi continuous, we observe that, $\bigcup_{s > t} V_s = \bigcup_{s > t}^{IFPS}(V_s)$ and
$\bigcap_{s < t} V_s = \bigcap_{s < t}^{PS}(V_s)$.
Then $f^{-1}(\mathcal{U}_t) = \bigcup \mathcal{T} \mathcal{I} = \bigcup \mathcal{T} \mathcal{P} (V_s)$ is $T_1$ and $T_2$-increasing intuitionistic fuzzy presemi clopen. And $f^{-1}(\mathcal{U}_t) = \bigcap \mathcal{T} \mathcal{I} = \bigcap \mathcal{T} \mathcal{P} (V_s)$ is $T_1$ and $T_2$-increasing intuitionistic fuzzy presemi closed. Therefore, $f$ is $T_1$ and $T_2$-increasing intuitionistic fuzzy strongly presemi continuous. To conclude the proof, it remains to show that $g \subseteq \subseteq$ that is $g^{-1}(\mathcal{U}_t) \subseteq \subseteq$, and $g^{-1}(\mathcal{U}_t) \subseteq \subseteq$, for each $t \in$

We have, $g^{-1}(L^t_i) = \bigcap_s g \cdot (L^t_s)$

$$= \bigcap_s \bigcap_{r<s} g^{-1}(R^t_r),$$

$$= \bigcap_s \bigcap_{r<s} \overline{G_r},$$

$$\subseteq \bigcap_s \bigcap_{r<s} \overline{F_r},$$

$$= \bigcap_s V_s = f^{-1}(L^t_i).$$

And, $f^{-1}(L^t_i) = \bigcap_s \bigcap_s \bigcup \mathcal{T} \mathcal{I} = \bigcup \mathcal{T} \mathcal{P} (V_s)$

$$= \bigcap_s \bigcap_{r<s} \overline{F_r},$$

$$\subseteq \bigcap_s \bigcap_{r<s} \overline{H_r},$$

$$= \bigcap_s \bigcap_{r<s} h^{-1}(L^t_r),$$

$$= \bigcap_s h \cdot (L^t_s),$$

Similarly, $g^{-1}(R^t_i) = \bigcup_s g \cdot (R^t_s)$

$$= \bigcup_s \bigcup_{r>s} g^{-1}(R^t_r).$$
\[= \bigcup_s \bigcup_{r>s} \overline{G_r}\]
\[\subseteq \bigcup_s \bigcap_{r<s} \overline{F_r}\]
\[= \bigcup_s V_s = f^{-1}(R^l_i)\]
And, \[f^{-1}(R^l_i) = \bigcup_s V_s\]
\[= \bigcup_s \bigcap_{r<s} \overline{F_r}\]
\[\subseteq \bigcup_s \bigcup_{r>s} \overline{H_r}\]
\[= \bigcup_s \bigcup_{r>s} h^- \cap L^l_i,\]
\[= \bigcup_{s>t} h^- \cap R^l_s,\]

Thus (b) is proved.

(b) \(\Rightarrow\) (c): Suppose \(A\) is an \(T_2\)-increasing intuitionistic fuzzy presemi open set and \(B\) is a \(T_1\)-decreasing intuitionistic fuzzy presemi closed set, such that \(B \subseteq \bigcup_{s \geq t} h^- \cap L^l_{s+t}\). Then \(\psi_{\alpha} \subseteq \psi_{\alpha'}\) and \(\psi_{\alpha'} \psi_{\alpha}\) are \(T_1\)-lower and \(T_2\)-upper intuitionistic fuzzy presemi continuous functions respectively. Hence by (b), there exists a \(T_1\) and \(T_2\)-increasing intuitionistic fuzzy strongly presemi continuous function \(f:(X,T_1,T_2, \leq) \rightarrow \bigcup_{s \geq t} h^- \cap L^l_{s+t}\) such that \(\psi_{\alpha} \subseteq \psi_{\alpha} \subseteq \psi_{\alpha}\). Clearly, \(f(x) \in I_f(I)\) for all \(x \in \bigcup_{s \geq t} h^- \cap L^l_{s+t}\) and \(B = \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} = \bigcup_{s \geq t} h^- \cap L^l_{s+t}\). Therefore, \(B = \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} \subseteq \overline{T_1} \psi_{\alpha} = \bigcup_{s \geq t} h^- \cap L^l_{s+t}\).

(c) \(\Rightarrow\) (a): This follows from Proposition 5.7.2 and the fact that \((L_1)f\) and \(R_0f\) are \(T_1\)-decreasing intuitionistic fuzzy presemi closed and \(T_2\)-decreasing intuitionistic fuzzy presemi open sets respectively.
Note 5.2.2

The Propositions 5.7.1 to 5.7.3, 5.7.5 and Remark 5.7.1 can be discussed for other cases also.

5.8 TIETZE EXTENSION THEOREM FOR PAIRWISE ORDERED INTUITIONISTIC FUZZY PRESEMI EXTREMALLY DISCONNECTED SPACES.

In this section, Tietze extension theorem for pairwise ordered intuitionistic fuzzy presemi extremally disconnected spaces is studied.

Proposition 5.8.1

Let \((X, T_1, T_2, \leq)\) be an pairwise ordered intuitionistic fuzzy presemi extremally disconnected space and let \(A \subseteq X\) be such that \(\chi_A\) is \(T_1\) and \(T_2\)-increasing intuitionistic fuzzy presemi open set in \((X, T_1, T_2, \leq)\). Let \(f : (A, T_1/A, T_2/A, \leq) \to (X, T_1, T_2, \leq)\) be \(T_1\) and \(T_2\)-increasing intuitionistic fuzzy strongly presemi continuous and isotone function. Then \(f\) has a \(T_1\) and \(T_2\)-increasing intuitionistic fuzzy strongly presemi continuous extension over \((X, T_1, T_2, \leq)\).

Proof

Let \(g, h : X \to \mathbb{R}\) be such that \(g = 0\) on \(A\) and \(g(x) = 1\) if \(x \notin A\). We now have,

\[ R^I f g = \left\{ \begin{array}{ll} \bigcap & \geq \text{ if } t < 0, \\
& \end{array} \right. \]

Where \(B_t\) is \(T_1\) and \(T_2\)-increasing intuitionistic fuzzy presemi open set such that \(B_t/A = \) and
where $C_t$ is $T_1$ and $T_2$-increasing intuitionistic fuzzy presemi open set such that $C_t / A = I^t$. Thus, $g$ is $T_1$-lower intuitionistic fuzzy presemi continuous, $h$ is $T_2$-upper intuitionistic fuzzy presemi continuous and $g \subseteq h$. By Proposition 5.7.5, there is a $T_1$ and $T_2$-increasing intuitionistic fuzzy strongly presemi continuous function $F : (X, T_1, T_2, \leq \rightarrow$ such that $g \subseteq h$ hence $F \equiv$ on $A$.

**Note 5.8.1**

The above proposition can be discussed for other case also.