CHAPTER - 1

INTRODUCTION

Topology is a branch of Mathematics through we elucidate and investigate the ideas of continuity, within the framework of Mathematics. The study of topological spaces, their continuous mappings and general properties make up one branch of topology known as “General Topology”. The pivotal problems of general topology are concerned with the theories of convergence and limits, separation axioms, covering properties, connectedness and continuous or like functions. Continuous explorations of these and many other neighbouring problems shaped and enriched the subject topology through ages enabling the researchers, specially real and functional analysis, to fruitfully carry on their investigations. The role of continuous function has a number of deep rooted and widespread applications in developing topology. The main purpose of studying general topology is to study the invariance of topological properties. The applications of topology in various fields are such as Physics [51], Remote Sensing [42] and Internet Banking [48].
1.1 REVIEW OF LITERATURE

Topology began its development with the first attempt of classifying spaces by Riemann [44] followed by Frechet [12] through the study of metric spaces, the work of Riesz [45], where the primitive version of the notion of condensation point was first used to describe abstract spaces. The next step towards a unified topological structure was taken by Weyl [52] who introduced the notion of neighbourhood system which culminated in 1914 to an elegant form, with the epic paper of Hausdorff [17], who found the right axiom system for neighbourhoods made them propitious for a suitable abstraction and thus founded modern topology. In fact, “General topology” as it is understood today, began with the publication of “Grundzuge der Mengenlehre” by Hausdorff.

The field of mathematical sciences which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Semiopen sets were first introduced by Levine [29], where the author studied the notion of semi continuity.

After that semi convergence and semi compactness were investigated and characterized by Dorsett [6, 7], Latif [27] introduced the concept of semi-convergence of filters and investigated functions. Min [31] used the idea of semi-convergence of filters to introduce a new class of sets called $\gamma$-open sets and notions of $\gamma$-closure, $\gamma$-interior and $\gamma$-continuity. The topological aspect of function spaces was first considered by Fox [11], Lambrinos [26], Porter [36, 37] and other
topologists introduced different set-open topologies on function spaces like bounded open topology, open-open topology, regular – open topology etc.

The method of centred system in the theory of topological spaces was introduced by Illiadis and Fomin[18]. Fletcher and Lindgren [9] studied the concept of compactification for quasi-uniform spaces. Further contributions in this direction, were given in [10] and [43]. The structure of $T_0^*$-compactifications of a quasi-uniform space was carried out in [46]. As a generalization of closed sets, $\tilde{g}$-closed sets were introduced by Jafari et al [20]. This notion was further studied by Rajesh and Ekici [38-41].

Andre weil [1] formulated the concept of uniform space which is a generalization of a metric space. The concept of hyperspace topology was first initiated and extensively studied by Hausdorff [17]. Michael[30] considered different collections of subsets of a topological spaces as hyperspaces. Fell [8] introduced a new topology known as Fell topology on hyperspace. Caldas and Jafari [3] investigated some applications of b-open sets in topological spaces. The concept of $\theta$-compact spaces interms of nets and filterbases was introduced by Jafari [19] and $\theta$-complete accumulation points by using $\theta$-compact spaces was introduced by Caldas, Jafari, Navalagi and Shankrikopp [4].

1.2 OUTLINE OF THE THESIS

This section presents a chapterwise summary of results obtained on $\gamma^*$-open set, $\gamma^*$-continuous mapping, almost strongly $\theta$-$\gamma^*$-continuous
mapping, super $\gamma^*$-continuous mapping, $\gamma^*$-splitting, $\gamma^*$-jointly continuous, maximal $G_\delta$-centred system, $G_\delta$-extremally disconnected space, the absolute $\omega^*(R)$ of space $R$, $G_\delta$-perfect mapping, centred quasi-uniform structure spaces, centred quasi-uniform bicomplete space, centered $T_\omega^*$-compactification, b-open symmetric in uniform space, uniform-bregular space, uniform b-normal space, $\theta^*$-closed set, $H^*$-closed set, b-nets, b-accumulation point, b-complete accumulation point and b-compact spaces.

In Chapter 2, the concept of $\gamma$-open open topology for function spaces was introduced in [13]. Based on this concept, $\gamma^*$-open set, $\gamma^*$-open open topology, $\gamma^*$-continuous mapping, almost $\gamma^*$-continuous mapping, weakly $\gamma^*$-continuous mapping, weakly $\theta^*$-continuous mapping, super $\gamma^*$-continuous mapping and almost strongly $\theta^*$-continuous mapping are introduced.

The method of centred system in the theory of topological spaces was introduced in [18]. Motivated by these concepts, the concepts of maximal $G_\delta$-maximal structure, $G_\delta$-extremally disconnected space, $G_\delta$-$\theta$-continuous mapping, Alexandrov Urysohn $G_\delta$-compactness criterion, the fundamental theorem on $G_\delta$-irreducible and $G_\delta$-perfect mapping are introduced in Chapter 3.

Kunzi, Romaguera and Sanchez – Granero [25] studied the concept of $T_\omega^*$-compactification in the hyperspace. In chapter 4, the concepts of centred quasi-uniform structure space, centred quasi-
uniform hereditary precompact space and centred quasi-uniform structure bicomplete space are introduced. Also many interesting characterizations are established.

Neelamegarjan, Rajesh and Erdal Ekici [32] studied the concept of $\tilde{g}$-regular and $\tilde{g}$-normal spaces. Based on these concepts, the concepts of b-open symmetric in uniform topological space, uniform b-regular space, uniform b-normal space and uniform b-$T_{\frac{1}{2}}$ space are introduced in chapter 5. Also some interesting properties are studied.

In Chapter 6, S. Ganguly, Sandip Jana and Ritusen [14] studied the hyperspace topology. Based on this concept, the $\theta^*$-closed set is introduced. Further, $T_i$ spaces ($i=0,1,2,\frac{1}{2}$) and Urysohn*space are introduced and their interrelations are discussed.

In chapter 7, the concepts of b-net, b-accumulation point, b-complete accumulation point and b-compact spaces are introduced as in [4]. Interesting properties and some characterizations are obtained.

1.3 BASIC CONCEPTS IN TOPOLOGICAL SPACES

In this section, some basic definitions like continuous functions, interior, closure have been recalled. Also, related results, important theorems and propositions are collected from various research papers.

**Definition 1.3.1. [33]**

A topology on a set $X$ is a collection $T$ of subsets of $X$ having the following properties:

(i) $\emptyset$ and $X$ are in $T$. 

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(ii). The union of elements of any sub collection of $T$ is in $T$.

(iii). The intersection of the elements of finite subcollection of $T$ is $T$.

The ordered pair $(X, T)$ is called the topological space.

**Definition 1.3.2.** [29]

A subset $S$ of $X$ is called a semi-open set if $S \subseteq \text{cl}(\text{int}(S))$. The complement of a semi-open set is called a semi-closed set. The family of all semi-open sets in a topological space $(X, \tau)$ will be denoted by $\text{SO}(X)$.

**Definition 1.3.3.** [29]

A subset $M(x)$ of a space $X$ is called a semi-neighbourhood of a point $x \in X$ if there exists a semi-open set $S$ such that $x \in S \subseteq M(x)$.

**Definition 1.3.4.** [2]

A subset $A$ of $X$ is called a $b$-open set if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$.

The family of all $b$-open (resp. $b$-closed) sets in $(X, T)$ will be denoted by $\text{BO}(X, T)$ (resp. $\text{BC}(X, T)$).

**Definition 1.3.5.** [29]

Let $S(x) = \{ A \in \text{SO}(X) : x \in A \}$ and let $S_x = \{ A \subseteq X : \text{there exists } \mu \subseteq S(x) \text{ such that } \mu \text{ is finite and } \bigcap \mu \subseteq A \}$. Then $S_x$ is called the semi-neighbourhood filter at $x$.

**Definition 1.3.6.** [29]

For any filter $\Gamma$ on $X$, we say that $\Gamma$ semi-converges to $x$ if and only if $\Gamma$ is finer than the semi-neighbourhood filter at $x$. 
Definition 1.3.7. [28]

A subset $U$ of $X$ is called a $\gamma$-open set if whenever a filter $\Gamma$ semi-converges to $x$ where $x \in U$, and $U \in \Gamma$.

The complement of a $\gamma$-open set is called a $\gamma$-closed set.

Definition 1.3.8. [28]

The intersection of all $\gamma$-closed sets containing $A$ is called the $\gamma$-closure of $A$ denoted by $\text{cl}_\gamma(A)$. A subset $A$ is $\gamma$-closed if $A = \text{cl}_\gamma(A)$. The family of all $\gamma$-open sets of $(X, \tau)$ is denoted by $\tau^\gamma$. It is shown in [10] that $\tau^\gamma$ is a topology on $X$. In a topological space $(X, \tau)$, it is always true that $\tau \subseteq \text{SO}(X) \subseteq \tau^\gamma$.

Definition 1.3.9. [28]

A point $x \in X$ is said to be a $\gamma$-interior point of $A$ if there exists a $\gamma$-open set $U$ containing $x$ such that $U \subseteq A$. The set of all $\gamma$-interior points of $A$ is said to be $\gamma$-interior of $A$ and is denoted by $\text{int}_\gamma(A)$.

Proposition 1.3.1. [28]

For a subset $A$ of a space $X$, $\text{int}_\gamma(X \setminus A) = X \setminus \text{cl}_\gamma(A)$.

Definition 1.3.10. [22]

A net is a pair $(S, \succeq)$ such that $S$ is a function and $\succeq$ directs the domain of $S$. (A net is sometimes called a directed set).

Definition 1.3.11. [28]

A net $\{x_\lambda : \lambda \in \Lambda\}$ in $X$ is said to $\gamma$-converge to $x \in X$ if $\{x_\lambda : \lambda \in \Lambda\}$ is eventually in each $\gamma$-open set containing $x$, denoted by $x_\lambda \rightarrow$.
Definition 1.3.12. [15]

A net \( \{ x_\lambda : \lambda \in \Lambda \} \) in \( X \) is weakly \( \theta \)-converge to \( x \in X \) if every neighbourhood \( U \) of \( x \), there exists a \( \lambda_0 \in \Lambda \) such that \( x_\lambda \in \text{int}(\text{cl}(U)) \) for all \( \lambda \geq \lambda_0 \).

Definition 1.3.13. [31]

A function \( f : X \to Y \) is \( \gamma \)-continuous if the inverse image of every open set of \( Y \) is \( \gamma \)-open in \( X \). The set of all \( \gamma \)-continuous function from \( X \) into \( Y \) is denoted by \( \gamma C(X, Y) \).

Definition 1.3.14. [49]

Let \( X \) be a topological space. A point \( x \in X \) is in the \( \delta \)-closure (respectively, \( \theta \)-closure) of a subset \( A \) of \( X \) denoted by \( x \in \text{cl}_\delta(A) \) (respectively, \( x \in \text{cl}_\theta(A) \)), if for each open neighbourhood \( V \) of \( x \), \( A \cap \text{int}(\text{cl}(V)) \neq \emptyset \) (respectively, \( A \cap \text{cl}(V) \neq \emptyset \)). \( A \) is said to be \( \delta \)-closed (respectively, \( \theta \)-closed) if \( \text{cl}_\delta(A) = A \) (respectively, \( \text{cl}_\theta(A) = A \)).

Definition 1.3.15. [47]

A set \( A \subset X \) in a topological space \( (X, T) \) is called a \( G_\delta \)-set if \( A = \bigcap_{n=1}^\infty A_n \) where each \( A_n \in T \).

The complement of \( G_\delta \)-set is called a \( F_\sigma \) set.

Definition 1.3.16. [21]

A topological space is a Hausdorff space iff whenever \( x \) and \( y \) are distinct points of the space there exists disjoint neighbourhoods of \( x \) and \( y \).
**Definition 1.3.17. [18]**

Let $R$ be a Hausdorff space. A system $p = \{U_\alpha\}$ of open sets of $R$ is called centred if any finite collection of sets of the system has a nonempty intersection. The system $p$ is called a maximal centred system or briefly an end if it cannot be included in any larger centred system of open sets.

**Definition 1.3.18. [18]**

Let $f$ be a mapping of a space $X$ into a space $Y$ with $f(x) = y$. Then $f$ is called $\theta$-continuous at $x$ if for every neighbourhood $O_y$ of $y$ there exists neighbourhood $O_x$ of $x$ such that $f(O_x) \subseteq O_y$. The mapping is called $\theta$-continuous if it is $\theta$-continuous at every point of $X$. A mapping that is one-to-one and $\theta$-continuous in both directions is called a $\theta$-homeomorphism.

It is clear that a continuous mapping is $\theta$-continuous. An example of a $\theta$-continuous mapping that is not continuous. Let $I$ be the interval $[0,1]$ with the usual topology, and $I'$ the same interval with the following topology: the neighbourhoods of every point $x \neq 0$ are the same as those in the half-open interval $(0,1]$, but the neighbourhoods of $x = 0$ are the sets of the form $[0, \varepsilon) \setminus D$, where $D$ is the set of all points $1/n$ ($n = 1, 2, \ldots; 0 < \varepsilon < 1$). It is easy to see that the space obtained is not regular at 0. Let $f$ be the identity mapping of $[0,1]$ onto itself. It is easy to verify that this mapping of $I$ onto $I'$ is $\theta$-continuous, we have also obtained a $\theta$-homeomorphism that is not a homeomorphism. It is
essential here that the space $I'$ is not regular, since it is easy to show that if the image is regular, then a $\theta$-continuous mapping is automatically continuous.

**Remark 1.3.1. [18]**

The canonical open sets (sets of the form $I (\bar{U})$ where $U$ is open) form a base.

**Definition 1.3.19. [33]**

A filter on a set $X$ is a set $F$ of subsets of $X$ which has the following properties

(i). Every subset of $X$ which contains a set $F$ belongs to $F$

(ii). Every finite intersection of sets of $F$ belongs to $F$

(iii). The empty set is not in $F$

**Definition 1.3.20. [33]**

A uniformity for a set $X$ is a non-void family $U$ of subsets of $X \times X$ such that

(i). Each member of $U$ contains the diagonal $\Delta$

(ii). If $U \in U$ the $U^{-1} \in U$

(iii). If $U \in U$, then $V \circ V = U$ for some $V$ in $U$

(iv). If $U$ and $V$ are members of $U$, then $U \cap V \in U$

(v). If $U \in U$ and $V \in X \times X$, then $V \in U$.

The pair $(X, U)$ is a uniform space. The sets of $U$ called entourages of the uniformity defined on $X$ by $U$. 
Definition 1.3.21. [34]

A quasi-uniformity on a set $X$ is a filter $\mathcal{U}$ on $X \times X$ such that each member of $\mathcal{U}$ contains the diagonal of $X \times X$ and if $U \in \mathcal{U}$, then $V \circ V \subseteq U$ for some $V \in \mathcal{U}$. The pair $(X, \mathcal{U})$ is called a quasi-uniform space. $\mathcal{U}$ generates a topology $\tau(\mathcal{U})$ containing all subsets $G$ of $X$ such that for $x \in G$, there exists $U \in \mathcal{U}$ with $U(x) \subseteq G$, where $U(x) = \{ y \in X : (x,y) \in U \}$.

Definition 1.3.22. [22]

A topological space is a $T_0$-space iff for each pair $x$ and $y$ of distinct points, there is a neighbourhood of one point to which the other does not belong.

Definition 1.3.23. [22]

A filter $\mathcal{F}$ on a uniform space $X$ is called a Cauchy filter if for each entourage $V$ of $X$ there is a subset of $X$ which is $V$-small and belongs to $\mathcal{F}$. In otherwords a Cauchy filter is one containing arbitrarily small sets.

Definition 1.3.24. [25]

The minimal elements of the set of Cauchy filters on a uniform space $X$ are called minimal Cauchy filters on $X$.

Definition 1.3.25. [25]

A quasi uniform space $(X, \mathcal{U})$ is called point symmetric if $\mathcal{T}(\mathcal{U}) = \mathcal{T}(\mathcal{U}^{-1})$.

Definition 1.3.26. [25]

A compactification of a $T_1$ quasi-uniform space $(X, \mathcal{U})$ is a compact $T_1$ quasi-uniform space $(Y, \mathcal{V})$ that has a $\mathcal{T}(\mathcal{V})$ dense subspace quasi-isomorphic to $(X, \mathcal{U})$. 
**Definition 1.3.27. [25]**

A quasi-uniform space \((X, \mathcal{U})\) is said to be bicomplete if each Cauchy filter on \((X, \mathcal{U}')\) converges with respect to the topology \(\mathcal{T}(\mathcal{U}')\) i.e., if the uniform space \((X, \mathcal{U}')\) is complete.

**Lemma 1.3.1. [23]**

A \(T_1\) quasi-uniform space \((X, \mathcal{U})\) is \(*\)-compactifiable iff it is point symmetric and its bicompletion is compact.

**Lemma 1.3.2. [23]**

Let \((X, \mathcal{U})\) be a \(T_1\) quasi-uniform space such that \(\mathcal{U}^{-1}\) is hereditarily precompact. Then \((X, \mathcal{U})\) is \(*\)-compactifiable iff it is point symmetric and precompact.

**Definition 1.3.28. [31]**

A space \((X, \mathcal{T})\) is said to be \(\tilde{g}\)-regular if for every \(\tilde{g}\)-closed set \(F\) and each point \(x \notin F\), there exists disjoint open sets \(U\) and \(V\) such that \(F \subseteq U\) and \(x \in V\).

**Definition 1.3.29. [31]**

A topological space \((X, \mathcal{T})\) will be termed symmetric iff for \(x\) and \(y\) in \((X, \mathcal{T})\), \(x \in \text{cl}(y)\) implies that \(y \in \text{cl}(x)\).

**Definition 1.3.30. [31]**

A topological space \((X, \mathcal{T})\) is said to be \(\tilde{g}\)-normal if for any pair of disjoint \(\tilde{g}\)-closed sets \(A\) and \(B\) there exists disjoint open sets \(U\) and \(V\) such that \(A \subseteq U\) and \(B \subseteq V\).
**Definition 1.3.31. [49]**

A point $x \in X$ is said to be a $\theta$-contact point of a set $A \subseteq X$ if for every neighbourhood $U$ of $x$, $\text{cl}_X U \cap A \neq \emptyset$.

The set of all $\theta$-contact points of a set $A$ is called the $\theta$-closure of $A$ and denote this set by $\theta\text{-cl}_X A$. A set $A$ is called $\theta$-closed if $A = \theta\text{-cl}_X A$. A set $A$ is called $\theta$-open if $X \setminus A$ is $\theta$-closed.

**Remark 1.3.2. [49]**

The collection of all $\theta$-open sets in a space $X$ forms a topology on $X$. In this connection $\theta(X)$ a new topology $T$ is defined as $\theta(X) = \{A \subseteq X ; A \neq \emptyset \text{ and } A \text{ is } \theta\text{-closed in } X\}$ where $X$ is a topological space.

**Definition 1.3.32. [49]**

A $T_2$-space $X$ is called H-closed if any open cover $u$ of $X$ has a finite proximate subcover, i.e., a finite subcollection $u_o$ of $u$ whose union is dense in $X$.

A set $A \subseteq X$ is called an H-set if any cover $\{U_\alpha : \alpha \in \Lambda\}$ of $A$ by open sets in $X$ has a finite subfamily $\{U_{\alpha_i} : i = 1, 2, \ldots, n\}$ such that $A \subseteq \bigcup_{i=1}^{n} \text{cl}_X U_{\alpha_i}$.

**Definition 1.3.33. [22]**

A topological space is a $T_0$-space iff for each pair $x$ and $y$ of distinct points, there is a neighbourhood of one point to which the other does not belong.
Definition 1.3.34. [22]
A topological space is a $T_1$-space iff each set which consists of a single point is closed.

Definition 1.3.35. [22]
A topological space is a $T_2$-space iff whenever $x$ and $y$ are distinct points of the space there exists disjoint neighbourhoods of $x$ and $y$.

Definition 1.3.36. [22]
A set $A$ is dense in a topological space $X$ iff the closure of $A$ is $X$.

Definition 1.3.37. [5]
Let $R$ be a usual metric (i) Let $A \subseteq R$ be a nonempty set and $x \in R$. $x \in \text{cl}(A)$ iff there is a sequence in $A$ converging to $x$. (ii) A function $f : R \to R$ is continuous at $x_0$ iff $(x_n) \to x_0 \Rightarrow (f(x_n)) \to f(x_0)$ for every sequence $(x_n)$.

Definition 1.3.38. [5]
Let $(X, \tau)$ and $(Y, \nu)$ be any two topological spaces. Then the following are two well known results : (i) Let $A \subseteq X$ be nonempty and $x \in X$. Then $x \in \text{cl}(A)$ iff there is a net in $A$ converging to $x$. (ii) A function $f : X \to Y$ is $(\tau, \nu)$ continuous at $x_0$ iff whenever the net $(x_i) \to (x_0)$ in $X$, the net $(f(x_i)) \to f(x_0)$ in $Y$.

Remark 1.3.3 [5]
Let $(X, \mu)$ be a topological space and $C_\mu(A)$ is the $\mu$-closure of $A$. It is clear that, in a topological space $(X, \mu)$ $x \in C_\mu(A)$ iff $G \cap x \neq \emptyset$ for every $\mu$-open set $G$ containing $x$. Every $\mu$-open set containing $x \in X$ is
called a neighbourhood of $x$ and the family of neighbourhoods of $x$ is denoted by $\mu(x)$. If $\mu(x) \neq \emptyset$, then the following statements hold.

(i). $x \in U$ for every $U \in \mu(x)$

(ii). If $U, V \in \mu(x)$, then $U \cup V \in \mu(x)$

(iii). If $y \in U \in \mu(x)$, then $U \in \mu(y)$.

**Definition 1.3.39. [5]**

Let $(X, \mu)$ and $(Y, \lambda)$ be any two topological spaces. A function $f : (X, \mu) \to (Y, \lambda)$ is said to be $(\mu, \lambda)$-continuous if the inverse image of every $\lambda$-open subset of $Y$ is a $\mu$-open subset of $X$.

**Definition 1.3.40 [49]**

Let $(X, \tau)$ and $(Y, \sigma)$ be any two topological spaces. Let the interior and closure of a set $A$ is denoted by $\text{int}(A)$ and $\text{cl}(A)$. A point $x \in X$ is called a $\theta$-adherent point of $A$ if $A \cap \text{cl}(U) \neq \emptyset$ for every open set $U$ of $X$ containing $x$.

**Definition 1.3.41. [49]**

The set of all $\theta$-adherent points of $A$ is called the $\theta$-closure of $A$ denoted by $\theta\text{cl}(A)$.

**Definition 1.3.42. [49]**

The subset $A$ is called $\theta$-closed if $A = \theta\text{cl}(A)$. The complement of a $\theta$-closed set is called $\theta$-open. The collection of all $\theta$-open (respectively, $\theta$-closed) sets is denoted by $\theta\text{O}(X, \tau)$ (resp. $\theta\text{cl}(X, \tau)$).

**Definition 1.3.43. [50]**

Let $A$ be a directed set. A net $\xi = \{X_\alpha / \alpha \in \Lambda\}$ $\theta$-accumulates at a point $x \in X$ if the net is frequently in every $U \in \theta\text{O}(X, x)$. The net $\xi$ $\theta$-converges to a point $x$ of $X$ if it is eventually in every $U \in \theta\text{O}(X, x)$.
Definition 1.3.44. [50]

A filterbase \( \Theta = \{ F_\alpha / \alpha \in \Gamma \} \) \( \theta \) - accumulates at a point \( x \in X \) if \( x \in \bigcap \alpha \text{cl}(F_\alpha). \) For a given set \( S \) with \( S \subseteq X \), a \( \theta \)-cover of \( S \) is a family of \( \theta \)-open subsets \( U_\alpha \) of \( X \) for each \( \alpha \in I \) of \( X \) such that \( S \subseteq \bigcup \alpha U_\alpha. \)

Definition 1.3.45. [50]

A filterbase \( \Theta = \{ F_\alpha / \alpha \in \Gamma \} \) \( \theta \)-converges to a point \( x \) in \( X \) for each \( U \in \Theta \) \( O \) \( (X, x) \), there exists an \( F_\alpha \) in \( \Theta \) such that \( F_\alpha \subseteq U. \)

Definition 1.3.46. [4]

A point \( x \) in a space \( X \) is said to be a \( \theta \)-complete accumulation point of a subset \( S \) of \( X \) if \( \text{card} (S \cap U) = \text{card}(S) \) for each \( U \in \Theta(X, x) \) where \( \text{card} (S) \) denotes the cardinality of \( S. \)

Lemma 1.3.3. (Zorn’s Lemma) [22]

If each chain in a partially ordered set has an upper bound, then there is a maximal element of the set.

Definition 1.3.47. [50]

A space \( X \) is said to be \( \theta \)-compact if every \( \theta \)-open cover of \( X \) has a finite subcover which covers \( X. \)