Chapter 7

Open Problems for further research

From the literature it is apparent that although the spectral properties of graphs have been investigated quite extensively, only the surface of this subject has been scratched so far. Many interesting problems are still open and following are some of them.

7.1 Adjacency Spectrum

1. The characterization of singular graphs by their graph theoretic properties is not solved. Although the singular bipartite graphs have been studied to some extent, the singularity of non-bipartite graphs has not been studied that much.

2. The problem of determining the graphs with maximal adjacency spectral radius has received much attention and has been studied extensively. But the similar problem regarding the minimal adjacency spectral radius has not been investigated as much. For example, following are some open problems in this direction.

   (i) Determining the graphs with minimal adjacency spectral radius among
all graphs with a given number of cut-vertices.

(ii) Determining the graphs with minimal adjacency spectral radius among all graphs with a given number of cut-edges.

(iii) Determining the graphs with minimal adjacency spectral radius among all graphs with a given vertex connectivity.

(iv) Determining the graphs with minimal adjacency spectral radius among all graphs with a given edge connectivity.

7.2 Distance Spectrum

The study of the spectral properties of the distance matrix of a graph is relatively new and is much more difficult than that of the adjacency matrix.

1. Some open extremal problems regarding the distance matrix are as follows.

   (i) Determining the graphs with maximal distance spectral radius among all graphs with a given number of cut-vertices.

   (ii) Determining the graphs with maximal distance spectral radius among all graphs with a given vertex connectivity.

   (iii) Determining the graphs with maximal distance spectral radius among all graphs with a given number of cut-vertices and given matching.

2. Even much less is known about the spectrum of the distance matrix of a graph as a whole. In other words, the characterization of graphs by their distance spectrum is an open problem, which is much more difficult than the corresponding problem regarding the adjacency matrix of a graph.