CHAPTER 6

DESIGN OF BACKSTEPPING CONTROLLER FOR
HYPERSONIC WIND TUNNEL

6.1 INTRODUCTION

Back stepping is a nonlinear control design technique, developed by V. Kokotovic in 1990’s. This approach is suitable for the design of a large class of feedback linearizable systems in strict feedback form. They are built from subsystems that radiate out from irreducible subsystem, which are then stabilized using Lyapunov synthesis. The Backstepping can also accommodate large nonlinearities and uncertainties in the system’s model. The complex nonlinear system is broken down into smaller subsystems and the Lyapunov function is applied for each of the subsystems.

The back stepping approach presents a systematic method for designing a control system to stabilize a reference signal, by selecting an appropriate Lyapunov function. The designer can start the design process at the known stable system and ‘back out’ new controllers that progressively stabilize each other subsystem. The process terminates when the final external control is reached. Hence, the process is known as Backstepping. Thus the main concept of the Backstepping design is to treat the system variable as an independent input for subsystems and each step results in a new virtual controller for the next step. For each step, the virtual control law is adopted with satisfaction of selected Lyapunov functions such that the stability of each subsystem can be guaranteed. Owing to its systematic design concept, design
of Backstepping controller has been explored to wide class of nonlinear systems. So if a Backstepping controller is designed for the pressure regulation in hypersonic wind tunnel, which is a highly non linear process, there can be an improvement in the performance parameters.

For applying the Backstepping technique, the system should be in pure feedback form, which many physical systems lack. There are two approaches for designing a Backstepping controller, which are the Stabilizing design and Tracking design.

### 6.2 STABILIZING DESIGN OF BACKSTEPPING CONTROLLER

The design of the back stepping controller using the stabilizing technique for a hypersonic wind tunnel is as follows.

From the modeling equations in chapter 3 (section 3.2) the state equations of the system can be written as follows

\[
\begin{align*}
    \dot{x}_1 &= \frac{F_2 - F_3}{C_3}, \\
    \dot{x}_2 &= \frac{F_1 - F_2}{C_2}, \\
    \dot{x}_3 &= \frac{-F_3}{C_1}
\end{align*}
\]

(6.1)

Then, the three flow equations is obtained as below

\[
\begin{align*}
    F_1 &= 2.39 \times 10^{-5} m \sqrt{P_1^2 - P_2 P_3} \\
    F_2 &= 1.57 \times 10^{-5} \sqrt{P_2^2 - P_2 P_3} \\
    F_3 &= 2.24 \times 10^{-4} P_3
\end{align*}
\]

(6.2) \hspace{1cm} (6.3) \hspace{1cm} (6.4)

Also,

\[
\begin{align*}
    x_1 &= P_3; x_2 &= P_2; x_3 &= P_1
\end{align*}
\]
Now using binomial expansion, the state equations can be approximated as

\[
\dot{x}_1 = 1.557x_2 - 22.97x_1 \\
\dot{x}_2 = m(0.455x_3 - 0.227x_2) - 0.29x_2 + 0.14x_1
\]

(6.5)

(6.6)

Control Lyapunov function is selected as \( V_1(x) = \frac{1}{2}x_1^2 \)

Derivative of \( V_1(x) \) becomes,

\[
\dot{V}_1(x) = x_1 \dot{x}_1 \\
\dot{V}_1(x) = x_1(1.557x_2 - 22.97x_1)
\]

(6.7)

Now the desired value of \( x_2 \) to make (6.7) negative definite is given by,

\( x_2(des) = -C_1x_1 \), where, \( C_1 \) is a positive constant.

Let, \( Z = x_2 - x_2(des) = x_2 + C_1x_1 \).

\( x_2 = (Z - C_1x_1) \)  

(6.8)

(6.9)

Now substituting (6.9) in (6.5) we get,

\[
\dot{x}_1 = 1.557Z - 1.557C_1x_1 - 22.97x_1
\]

(6.10)

From equation (6.8),

\[
\dot{Z} = \dot{x}_2 + C_1 \dot{x}_1
\]

(6.11)

Substituting equations (6.6) and (6.10) in (6.11) we get,

\[
\dot{Z} = m(0.455x_3 - 0.227x_2) - 0.29x_2 + 0.14x_1 + C_1(1.557z - 1.557C_1x_1 - 22.97x_1)
\]

(6.12)

The second Lyapunov function is selected as \( V_2(x) = \frac{1}{2}x_1^2 + \frac{1}{2}Z^2 \)

\[
\dot{V}_2(x) = x_1 \dot{x}_1 + Z \dot{Z}
\]

\[
\dot{V}_2(x) = x_1(1.557z - 1.557C_1x_1 - 22.97x_1) + z[m(0.455x_3 - 0.227x_2) - 0.29x_2 + 0.14x_1 + C_1(1.557z - 1.557C_1x_1 - 22.97x_1)]
\]

(6.13)
Now the desired value of \( m \), \( m_{(des)} \) to make (6.13) negative definite is given by

\[
m_{(des)} = -C_z z - \left( \frac{1.557 x_1 + 1.557 C_1 z}{0.455 x_3 - 0.227 x_2} \right)
\]

(6.14)

Equation 6.14 represents the controller equation based on the stabilizing design of Backstepping controller for regulating pressure in the settling chamber of a HWT. The controller is implemented in simulink and is used with the developed model of HWT as shown in the block diagram in Figure 6.1.

Figure 6.1 Controller implementation using the stabilizing design of Backstepping Controller.

The difference of the set point and the present value of the output \( P_3 \) from the settling chamber is given as one of the input to the backstepping controller developed using the stabilizing technique. The intermediate
pressures $P_1$ and $P_2$ are also given to the controller. The controller output is used to regulate the pressure regulating valve, so that the desired pressure is obtained at the settling chamber.

6.2.1 Implementation of Stabilizing Design

The step-by-step procedure for designing and simulating the Backstepping Stabilizing Controller for the HWT system is given in this section.

1. The modeling equations in equation 1.4 are simplified and binomial expansion is applied to get the approximated state equations.

2. Select suitable control Lyapunov functions, and make necessary substitutions and simplifications.

3. Obtain the controller transfer function $m_{des}$, implement it in the Simulink and make it as a subsystem.

4. The controller subsystem is connected with the Simulink model of the HWT system developed in Chapter 3.

5. The given set point is compared with the present output from the settling chamber, and the error is given to the Backstepping stabilizing controller.

6. The Backstepping stabilizing controller, produces a control output which is given to adjust the stem position of the control valve for maintaining constant pressure in the settling chamber of the hypersonic wind tunnel

7. Simulations were done at different set points and the results obtained are tabulated and compared for the various performance parameters.
6.3 TRACKING DESIGN OF BACKSTEPPING CONTROLLER

The back stepping controller is also designed using the tracking technique. The approximated state equation for the Hypersonic Wind Tunnel system is given by equation (6.5) and (6.6). For the first system, the state $x_{\text{ref}}$ is chosen as a virtual control input. The first Backstepping variable is chosen as follows:

\[
\begin{align*}
    z &= x_{\text{ref}} - x_1 \\
    \dot{z} &= x_{\text{ref}} - \dot{x}_1 \\
    \ddot{z} &= x_{\text{ref}} - 1.557 x_2 + 22.97 x_1 \\
\end{align*}
\]  

(6.15)

The Control Lyapunov function is selected as

\[
V_1(z) = \frac{1}{2} z^2
\]

Derivative of $V_1(z)$ becomes,

\[
\begin{align*}
    \dot{V}_1(z) &= \dot{z} z \\
    \dot{V}_1(z) &= z \dot{x}_{\text{ref}} - 1.557 z x_2 + 22.97 z x_1 \\
\end{align*}
\]  

(6.16)

Now the desired value of $x_2$ to make (6.16) negative definite is given by,

\[
x_2(\text{des}) = -C_1 z + \frac{22.97}{1.557} x_1 + \frac{x_{\text{ref}}}{1.557} , \\
\]

(6.17)

Where, $C_1$ is a positive constant.

Let, 

\[
y = x_2 - x_2 (\text{des}) \\
\]

(6.18)

So, 

\[
x_2 = y - C_1 z + \frac{22.97}{1.557} x_1 + \frac{x_{\text{ref}}}{1.557} , \\
\]

(6.19)

To find the derivative of $x_2$, substitute (6.19) in (6.15),

\[
\begin{align*}
    \ddot{z} &= -1.557 y - 1.557 C_1 z \\
\end{align*}
\]  

(6.20)

To find the derivative of $y$, differentiating (6.18), we get

\[
y = x_2 - x_2 (\text{des}) \\
\]

(6.21)
Substituting (6.17) in (6.21)

\[ y = x_2 + C_1 \left( - \frac{22.97}{1.557} x_1 - \frac{x_{ref}}{1.557} \right) \]  

(6.22)

Substituting (6.20) and (6.6) in (6.22) and simplifying we get,

\[ \dot{y} = m(0.455x_2 - 0.227x_2) + 0.29x_2 + 0.14x_1 + 1.557C_1y + 1.557C_1^2z \]

(6.23)

The second Lyapunov function is selected as \( V_2(z) = \frac{1}{2}z^2 + \frac{1}{2}y^2 \)

\[ V_1(z) = zz + yy \]

\[ V_2(x) = -1.557zy - 1.557C_1z^2 + my(0.455x_2 - 0.227x_2) + 0.29y x_2 + 0.14y x_1 + 1.557C_1y^2 + 1.557C_1^2yz \]  

(6.24)

Now the desired value of m, m(des) to make (6.24) negative definite is given by

\[ m(\text{des}) = -C_2y - \left( \frac{0.29x_2 - 0.14x_1 - 1.557C_1y - 1.557C_1^2z}{0.455x_2 - 0.227x_2} \right) \]  

(6.25)

Equation 6.25 represents the controller equation for the tracking design. The controller equation is implemented in simulink and is used with the model of the hypersonic wind tunnel to regulate the pressure in the settling chamber as per the block diagram shown in Figure 6.2.

Figure 6.2 Controller implementation using the tracking design of Backstepping Controller.
The set point is given to the virtual input $x_{\text{ref}}$, and the error calculation is done inside the Backstepping controller designed using the tracking technique. The present value of the settling chamber pressure $P_3$, high pressure system $P_1$ and heater pressure $P_2$ are also given as input to the backstepping controller. The controller output is used to regulate the pressure regulating valve, so that the desired pressure is obtained at the settling chamber.

### 6.3.1 Implementation of Tracking Design

The step-by-step procedure for designing and simulating the Backstepping Tracking Controller for the Hypersonic Wind Tunnel system is given in this section.

1. The modeling equations in Chapter 3, section 3.2 are simplified and binomial expansion is applied to get the approximated state equations.
2. Select a virtual control input for the innermost system for choosing the first Backstepping variable.
3. Select suitable control Lyapunov functions, and make suitable substitutions and simplifications.
4. Get the controller transfer function, implement it in the Simulink and make it as a subsystem.
5. The controller subsystem is connected to the hypersonic wind tunnel system developed in Chapter 3 using the Simulink model.
6. Set point is given to the Backstepping tracking controller through the virtual input.
7. The present output from the settling chamber, heater and high pressure tank are given to the tracking controller.

8. The Backstepping tracking controller, produces a controller output which is given to regulate the pressure in the settling chamber.

9. Simulations were done at different set points and the results obtained are tabulated and the various performance parameters are compared.

6.4 BACKSTEPPING CONTROL WITH FUZZY INFERENCE

The use of hybrid controller based on adaptation techniques along with the Backstepping controller can improve the performance of the controller with Backstepping technique alone. The adaptation mechanism can be implemented by a Fuzzy controller. Since the fuzzy rule base is formed based on human knowledge and experience, the fuzzy controller gives a human-like performance for the hybrid controller. Thus a hybrid controller is developed involving the Fuzzy controller developed for the HWT explained in Chapter 5, along with the Backstepping controller designed in section 6.2 and section 6.3 to regulate the pressure in the settling chamber.

6.4.1 Implementation of Stabilizing with Fuzzy Inference

The fuzzy controller developed in the previous chapter is combined with the Backstepping stabilizing controller to regulate the pressure in the settling chamber. The block diagram of the system is shown in Figure 6.3.
Figure 6.3 Controller implementation using the stabilizing design of Backstepping controller with Fuzzy Controller

The output pressure $P_3$ of the settling chamber in the HWT model is compared with the given set point. The error is given simultaneously to the two the controllers; the Backstepping controller developed using the stabilizing technique and the fuzzy controller. Rate of change of error is given as the second input to the fuzzy controller. The two controllers produce the controller output. The contribution of their control outputs are obtained and is used for manipulating the stem position of the control valve for obtaining the desired set pressure in the settling chamber.

The step-by-step procedure for designing and simulating the Backstepping Stabilizing with fuzzy Controller for the HWT can be summarized as follows:
1. Use the Backstepping stabilizing controller developed in section 6.2 and the fuzzy controller developed in Chapter 4.

2. The two controllers are connected with the simulink model of the HWT system developed in Chapter 3.

3. The given set point is compared with the present output from the settling chamber, and the error is given to the Backstepping stabilizing controller and the fuzzy controller.

4. The controller output from the Backstepping stabilizing and fuzzy controller is used to adjust the position of the control valve for maintaining constant pressure in the settling chamber.

5. Simulations were done at different set points and the results obtained are tabulated and compared for the various performance parameters.

### 6.4.2 Implementation of Tracking with Fuzzy inference

Next, the fuzzy controller combined with the Backstepping controller designed using the tracking technique to regulate the pressure in the settling chamber. The block diagram of the system is shown in Figure 6.4.
Figure 6.4 Controller implementation using the tracking design of Backstepping control with Fuzzy Controller.

The set point is given to the virtual input $x_{\text{ref}}$ of the Backstepping controller designed using tracking technique. The difference of the set point and the settling chamber pressure $P_3$ is given as one of the inputs to the fuzzy controller. The other input is the rate of change of error. The contributions of both the controller outputs are given as the input to the HWT so that the position of the pressure regulating valve is adjusted for obtaining the desired Mach number or pressure in the settling chamber.

The step-by-step procedure for designing and simulating the Backstepping Tracking with fuzzy Controller for the HWT system is summarized as below:
1. Use the Backstepping tracking controller developed in section 6.3 and the fuzzy controller developed in Chapter 4.

2. The two controllers are connected with the simulink model of the HWT system developed in Chapter 3.

3. The given set point is compared with the present output from the settling chamber, and the error is given to the Backstepping stabilizing controller and the fuzzy controller.

4. The controller output from the Backstepping stabilizing and fuzzy controller is used to adjust the position of the control valve for maintaining constant pressure in the settling chamber.

5. Simulations were done at different set points and the results obtained are tabulated and compared for the various performance parameters.

6.5 RESULTS AND DISCUSSION

The Backstepping controller designed using the stabilizing and tracking techniques, with and without the fuzzy inference were simulated using Simulink in MATLAB. Servo and regulator operation of the system is performed at the set points of 60 bar to 150 bar. The different performance parameters like settling time, delay time, rise time and overshoot are noted for each of the controller from the plots obtained. Calculations are done to find the test duration, IAE and ISE.
6.5.1 Response of Backstepping Controller Using Stabilizing Design

The output of servo operation for the set points of 70 bar and 100 bar is given in Figure 6.5 and Figure 6.6 respectively.

Figure 6.5 Servo operation of stabilizing design of Backstepping controller at set point of 70 bar

Figure 6.6 Servo operation of stabilizing design of Backstepping controller at set point of 100 bar
Table 6.1 gives the values of different performance parameters for all the set points from 60 bar to 150 bar.

Table 6.1 Performance parameters for servo operation of stabilizing design of Backstepping controller at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>Delay time (sec)</th>
<th>Rise time (sec)</th>
<th>% Overshoot</th>
<th>Max. Possible Test Duration (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>4.3</td>
<td>0.495</td>
<td>0.782</td>
<td>62.5</td>
<td>300</td>
<td>9.97E+07</td>
<td>5.14E+14</td>
</tr>
<tr>
<td>70</td>
<td>4.34</td>
<td>0.504</td>
<td>0.811</td>
<td>57</td>
<td>288</td>
<td>1.17E+08</td>
<td>7.04E+14</td>
</tr>
<tr>
<td>80</td>
<td>4.301</td>
<td>0.514</td>
<td>0.808</td>
<td>51.25</td>
<td>260</td>
<td>1.46E+08</td>
<td>9.68E+14</td>
</tr>
<tr>
<td>90</td>
<td>3.44</td>
<td>0.534</td>
<td>0.934</td>
<td>38.1</td>
<td>210</td>
<td>1.75E+08</td>
<td>1.31E+15</td>
</tr>
<tr>
<td>100</td>
<td>4.5</td>
<td>0.56</td>
<td>1.019</td>
<td>36.3</td>
<td>160</td>
<td>3.61E+09</td>
<td>1.91E+15</td>
</tr>
<tr>
<td>110</td>
<td>4.41</td>
<td>0.58</td>
<td>1.58</td>
<td>43.6</td>
<td>120</td>
<td>2.75E+08</td>
<td>2.39E+15</td>
</tr>
<tr>
<td>120</td>
<td>4.46</td>
<td>0.619</td>
<td>1.9</td>
<td>24.1</td>
<td>100</td>
<td>3.63E+08</td>
<td>3.35E+15</td>
</tr>
<tr>
<td>130</td>
<td>2.96</td>
<td>0.64</td>
<td>1.3</td>
<td>22</td>
<td>85</td>
<td>4.44E+08</td>
<td>4.41E+15</td>
</tr>
<tr>
<td>140</td>
<td>3.08</td>
<td>0.68</td>
<td>1.41</td>
<td>18.4</td>
<td>65</td>
<td>5.38E+08</td>
<td>5.72E+15</td>
</tr>
<tr>
<td>150</td>
<td>3.28</td>
<td>0.724</td>
<td>1.55</td>
<td>15</td>
<td>45</td>
<td>6.53E+09</td>
<td>7.37E+15</td>
</tr>
</tbody>
</table>

From Table 6.1, we can infer that for high values of set point, even though the delay time and rise time is more, the values of settling time and overshoot is less. There is only 1sec variation in settling time, for the variation of set points from 60 bar to 150 bar. The maximum test duration of 300 sec is available for the lowest set point of 60 bar. IAE and ISE are also less for low values of the set point.

Figure 6.7 and Figure 6.8, shows the output of regulator operation for the set points of 70 bar and 100 bar respectively.
Figure 6.7 Regulator operation of stabilizing design of Backstepping controller at set point of 70 bar

Figure 6.8 Regulator operation of stabilizing design of Backstepping controller at set point of 100 bar

The settling time after applying the disturbance, IAE and ISE for different set points are noted and tabulated in Table 6.2.
Table 6.2 Performance parameters for regulator operation of stabilizing design of Backstepping controller at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>25.8</td>
<td>5.95E+06</td>
<td>1.73E+12</td>
</tr>
<tr>
<td>70</td>
<td>26</td>
<td>6.15E+06</td>
<td>1.71E+12</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>6.15E+06</td>
<td>1.57E+12</td>
</tr>
<tr>
<td>120</td>
<td>26.3</td>
<td>6.92E+06</td>
<td>1.57E+12</td>
</tr>
<tr>
<td>150</td>
<td>27</td>
<td>7.82E+06</td>
<td>1.79E+12</td>
</tr>
</tbody>
</table>

6.5.2 Response of Backstepping Controller Using Tracking Design

The output of servo operation for the set points of 70 bar and 100 bar for the Backstepping controller designed using tracking technique is given in Figure 6.9 and Figure 6.10 respectively.

Figure 6.9 Servo operation of tracking design of Backstepping controller at set point of 70 bar
Figure 6.10 Servo operation of tracking design of Backstepping controller at set point of 100 bar

Table 6.3 Performance parameters for servo operation of tracking design of Backstepping controller at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>Delay time (sec)</th>
<th>Rise time (sec)</th>
<th>% Overshoot</th>
<th>Max. Possible Test Duration (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.44</td>
<td>0.497</td>
<td>0.77</td>
<td>63.6</td>
<td>380</td>
<td>9.72E+07</td>
<td>5.14E+14</td>
</tr>
<tr>
<td>70</td>
<td>4.272</td>
<td>0.498</td>
<td>0.805</td>
<td>57</td>
<td>325</td>
<td>1.12E+08</td>
<td>7.03E+14</td>
</tr>
<tr>
<td>80</td>
<td>4.311</td>
<td>0.515</td>
<td>0.864</td>
<td>49.5</td>
<td>275</td>
<td>1.40E+08</td>
<td>9.61E+14</td>
</tr>
<tr>
<td>90</td>
<td>3.56</td>
<td>0.535</td>
<td>0.941</td>
<td>40.4</td>
<td>222</td>
<td>1.73E+08</td>
<td>1.32E+15</td>
</tr>
<tr>
<td>100</td>
<td>4.225</td>
<td>0.562</td>
<td>1.01</td>
<td>35.6</td>
<td>178</td>
<td>2.11E+08</td>
<td>1.76E+15</td>
</tr>
<tr>
<td>110</td>
<td>3.59</td>
<td>0.58</td>
<td>1.1</td>
<td>30.8</td>
<td>150</td>
<td>2.68E+08</td>
<td>2.38E+15</td>
</tr>
<tr>
<td>120</td>
<td>3.07</td>
<td>0.619</td>
<td>1.2</td>
<td>26.9</td>
<td>125</td>
<td>3.62E+08</td>
<td>3.36E+15</td>
</tr>
<tr>
<td>130</td>
<td>2.92</td>
<td>0.65</td>
<td>1.3</td>
<td>22.6</td>
<td>105</td>
<td>4.45E+08</td>
<td>4.13E+15</td>
</tr>
<tr>
<td>140</td>
<td>3.06</td>
<td>0.68</td>
<td>1.44</td>
<td>35.1</td>
<td>85</td>
<td>5.40E+08</td>
<td>5.75E+15</td>
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<tr>
<td>150</td>
<td>3.218</td>
<td>0.722</td>
<td>1.53</td>
<td>15.3</td>
<td>50</td>
<td>6.51E+08</td>
<td>7.40E+15</td>
</tr>
</tbody>
</table>

Table 6.3 gives the values of different performance parameters at different set points from 60 bar to 150 bar.
From the performance parameters in the Table 6.3, we can see that there is random variation in the settling time for increasing values of set point. However there is maximum test duration, and minimum IAE and ISE for low values of the set point.

Figure 6.11 Regulator operation of tracking design of Backstepping controller at set point of 70 bar

Figure 6.12 Regulator operation of tracking design of Backstepping controller at set point of 100 bar
Figure 6.11 and Figure 6.12, shows the output of regulator operation for the set points of 70 bar and 100 bar respectively for the tracking design of Backstepping controller. The settling time after applying the disturbance is noted from the graph. IAE and ISE for different set points are calculated.

The regulator operation of the Backstepping controller designed using the tracking technique at different set points is tabulated in table 6.4, for the performance evaluation of the controller.

Table 6.4 Performance parameters for regulator operation of tracking design of Backstepping controller at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>25.4</td>
<td>5.12E+06</td>
<td>2.12E+12</td>
</tr>
<tr>
<td>70</td>
<td>25.4</td>
<td>5.05E+06</td>
<td>2.11E+12</td>
</tr>
<tr>
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<td>26.02</td>
<td>4.80E+06</td>
<td>1.98E+12</td>
</tr>
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<td>120</td>
<td>25.5</td>
<td>4.39E+06</td>
<td>1.69E+12</td>
</tr>
<tr>
<td>150</td>
<td>25.6</td>
<td>5.27E+06</td>
<td>1.68E+12</td>
</tr>
</tbody>
</table>

6.5.3 Response of Backstepping Controller Using Stabilizing Design with Fuzzy Inference

Figure 6.13 and Figure 6.14 shows the response of the Backstepping designed using tracking technique combined with fuzzy controller for the set points of 70 bar and 100 bar.

Similar simulations are done for the set points from 60 bar to 150 bar, and the different performance parameters of the system with the Backstepping tracking design with fuzzy inference are noted and calculated. Table 6.5 shows the tabulated values.
Figure 6.13 Servo operation of stabilizing design of Backstepping control with fuzzy inference at set point of 70 bar.

Figure 6.14 Servo operation of stabilizing design of Backstepping control with fuzzy inference at set point of 100 bar.
Table 6.5 Performance parameters for servo operation of stabilizing design of Backstepping control with fuzzy inference at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>Delay time (sec)</th>
<th>Rise time (sec)</th>
<th>% Overshoot</th>
<th>Max. Possible Test Duration (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.61</td>
<td>0.71</td>
<td>1.086</td>
<td>54.16</td>
<td>150</td>
<td>9.44E+08</td>
<td>2.92E+15</td>
</tr>
<tr>
<td>70</td>
<td>3.6</td>
<td>0.7018</td>
<td>1.09</td>
<td>53.2</td>
<td>130</td>
<td>1.10E+09</td>
<td>3.94E+15</td>
</tr>
<tr>
<td>80</td>
<td>3.63</td>
<td>0.71</td>
<td>1.12</td>
<td>51.8</td>
<td>107</td>
<td>1.24E+09</td>
<td>5.10E+15</td>
</tr>
<tr>
<td>90</td>
<td>3.66</td>
<td>0.72</td>
<td>1.14</td>
<td>49.1</td>
<td>100</td>
<td>1.38E+09</td>
<td>6.34E+15</td>
</tr>
<tr>
<td>100</td>
<td>3.73</td>
<td>0.72</td>
<td>1.18</td>
<td>44.3</td>
<td>72</td>
<td>1.50E+09</td>
<td>7.61E+15</td>
</tr>
<tr>
<td>110</td>
<td>3.81</td>
<td>0.51</td>
<td>1.1</td>
<td>37.9</td>
<td>40</td>
<td>1.61E+09</td>
<td>8.91E+15</td>
</tr>
<tr>
<td>120</td>
<td>3.92</td>
<td>0.75</td>
<td>1.2</td>
<td>31.8</td>
<td>30</td>
<td>1.75E+09</td>
<td>1.07E+16</td>
</tr>
<tr>
<td>130</td>
<td>4.04</td>
<td>0.77</td>
<td>1.3</td>
<td>26.3</td>
<td>18</td>
<td>1.92E+09</td>
<td>1.28E+16</td>
</tr>
<tr>
<td>140</td>
<td>4.16</td>
<td>0.8</td>
<td>1.5</td>
<td>21.7</td>
<td>10</td>
<td>2.03E+08</td>
<td>1.43E+16</td>
</tr>
<tr>
<td>150</td>
<td>4.32</td>
<td>0.82</td>
<td>1.499</td>
<td>18</td>
<td>8</td>
<td>2.20E+09</td>
<td>1.66E+16</td>
</tr>
</tbody>
</table>

From Table 6.5, it is seen that for a set point of 70 bar, settling time, delay time and rise time for the controller is the lowest, compared to other set points. It is also clear from the table that, except % overshoot; all other performance parameters are good for the low values of set point. Thus we can infer that by adding the fuzzy inference to the Backstepping controller designed using the stabilizing technique, the controller becomes more suitable for lower values of set point than higher values.

The regulator operation of the controller at 70 bar and 100 bar is shown in Figure 6.15 and Figure 6.16.
Figure 6.15 Regulator operation of stabilizing design of
Backstepping control with fuzzy inference at set point of 70 bar.

Figure 6.16 Regulator operation of stabilizing design of
Backstepping control with fuzzy inference at set point of 100 bar.

Regulator responses are done for different set points, by giving an
external disturbance to the system, and the performance values are tabulated
in Table 6.6.
Table 6.6 Performance parameters for regulator operation of stabilizing design of Backstepping controller with fuzzy inference at different set points

<table>
<thead>
<tr>
<th>Set Pint (bar)</th>
<th>Settling time (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>27</td>
<td>6.69E+08</td>
<td>3.53E+14</td>
</tr>
<tr>
<td>70</td>
<td>27</td>
<td>7.28E+08</td>
<td>3.51E+14</td>
</tr>
<tr>
<td>100</td>
<td>27.06</td>
<td>8.50E+08</td>
<td>3.59E+14</td>
</tr>
<tr>
<td>120</td>
<td>27.2</td>
<td>2.71E+09</td>
<td>1.11E+16</td>
</tr>
<tr>
<td>150</td>
<td>Not Settling</td>
<td>1.19E+09</td>
<td>5.01E+14</td>
</tr>
</tbody>
</table>

From the analysis of the values in the Table 6.6, we can infer that the controller is not well suited for the regulator operation. The controller is not settling to high values of set point. Also the IAE and ISE for the system are high for the entire variation range of the set point.

6.5.4 Response of Backstepping Controller using Tracking design with Fuzzy Inference

Figure 6.17 and Figure 6.18 shows the response of the controller for servo operation at 70 bar and 100 bar. Table 6.7 summarizes the performance parameters for servo operation of the controller at different set points.
Figure 6.17 Servo operation of tracking design of Backstepping control with fuzzy inference at set point of 70 bar

Figure 6.18 Servo operation of tracking design of Backstepping control with fuzzy inference at set point of 100 bar
Table 6.7 Performance parameters for servo operation of tracking design of Backstepping control with fuzzy inference at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>Delay time (sec)</th>
<th>Rise time (sec)</th>
<th>% Overshoot</th>
<th>Max. Possible Test Duration(sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.51</td>
<td>0.71</td>
<td>1.084</td>
<td>55.4</td>
<td>283</td>
<td>8.29E+08</td>
<td>2.93E+15</td>
</tr>
<tr>
<td>70</td>
<td>3.44</td>
<td>0.708</td>
<td>1.09</td>
<td>54.57</td>
<td>240</td>
<td>9.66E+08</td>
<td>3.96E+15</td>
</tr>
<tr>
<td>80</td>
<td>4.03</td>
<td>0.71</td>
<td>1.11</td>
<td>53</td>
<td>227</td>
<td>1.00E+09</td>
<td>5.11E+15</td>
</tr>
<tr>
<td>90</td>
<td>3.57</td>
<td>0.71</td>
<td>1.14</td>
<td>101</td>
<td>188</td>
<td>1.22E+09</td>
<td>6.36E+15</td>
</tr>
<tr>
<td>100</td>
<td>3.62</td>
<td>0.71</td>
<td>1.18</td>
<td>45.1</td>
<td>140</td>
<td>1.36E+09</td>
<td>7.61E+15</td>
</tr>
<tr>
<td>110</td>
<td>3.72</td>
<td>0.72</td>
<td>1.1</td>
<td>38.2</td>
<td>100</td>
<td>1.45E+09</td>
<td>8.90E+15</td>
</tr>
<tr>
<td>120</td>
<td>3.81</td>
<td>0.75</td>
<td>1.2</td>
<td>32.5</td>
<td>75</td>
<td>1.58E+09</td>
<td>1.07E+15</td>
</tr>
<tr>
<td>130</td>
<td>3.92</td>
<td>0.77</td>
<td>1.3</td>
<td>27.2</td>
<td>60</td>
<td>1.72E+09</td>
<td>1.28E+16</td>
</tr>
<tr>
<td>140</td>
<td>4.05</td>
<td>0.79</td>
<td>1.4</td>
<td>22.7</td>
<td>45</td>
<td>1.83E+09</td>
<td>1.43E+16</td>
</tr>
<tr>
<td>150</td>
<td>4.4</td>
<td>0.82</td>
<td>1.514</td>
<td>18.6</td>
<td>30</td>
<td>1.98E+09</td>
<td>1.66E+16</td>
</tr>
</tbody>
</table>

From Table 6.7, the controller has lowest settling time of 3.5 sec at the set point of 70 bar, as similar to the Backstepping designed using the stabilizing technique combined with fuzzy controller. It is also clear from the table that, except % overshoot, all other performance parameters are good for the low values of set point and the controller has almost similar performance as that of the one designed with stabilizing technique.

Figure 6.19 and Figure 6.20 shows the response of the tracking design of Backstepping combined with fuzzy controller for the set points of 70 bar and 100 bar. Similar responses were done for different set points from 60 bar to 150 bar, and the performance parameter values are tabulated in Table 6.8.
Figure 6.19 Regulator operation of tracking design of
Backstepping control with fuzzy inference at set point of 100 bar

Figure 6.20 Regulator operation of tracking design of
Backstepping control with fuzzy inference at set point of 100 bar
Table 6.8 Performance parameters for regulator operation of tracking design of Backstepping control with fuzzy inference at different set points.

<table>
<thead>
<tr>
<th>Set Point (bar)</th>
<th>Settling time (sec)</th>
<th>IAE</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>26.8</td>
<td>5.46E+08</td>
<td>3.86E+14</td>
</tr>
<tr>
<td>70</td>
<td>26.8</td>
<td>5.64E+08</td>
<td>3.81E+14</td>
</tr>
<tr>
<td>100</td>
<td>26.8</td>
<td>6.69E+08</td>
<td>3.58E+14</td>
</tr>
<tr>
<td>120</td>
<td>26.9</td>
<td>7.33E+08</td>
<td>3.53E+14</td>
</tr>
<tr>
<td>150</td>
<td>27.3</td>
<td>9.25E+08</td>
<td>3.79E+14</td>
</tr>
</tbody>
</table>

The comparison of performance analysis of the controller from Table 6.8 tell us that the controller settles after making a disturbance for all the values of set point. Also the value of IAE and ISE is low for the controller compared to the one designed using the stabilizing technique.

6.6 SUMMARY

The stabilizing and tracking design of the Backstepping controller is designed and simulated for a HWT. The simulations were also carried out by combining these Backstepping controllers with fuzzy inference. Compared with the PI and the fuzzy assisted PI controller, the back stepping controller has improved settling time and low IAE and ISE at different set points. When considering the performance of the Backstepping controller for the test duration, the results are not good. The back stepping controller utilizes feedback of internal states so that the controller is more predictive and feed forward in nature. This is the reason for fast settling and better disturbance rejection. When a disturbance is given, the controller takes corrective action to stabilize the system at the desired set point. From the studies made by the servo and regulator operation in the tracking and stabilizing design, we can infer that the tracking design is best for the servo operation and the stabilizing design has better performance in the regulator operation.