CHAPTER 3
MODEL DESCRIPTION OF HYPersonic wind tunnel

3.1. INTRODUCTION

Hypersonic wind tunnels can operate in hypersonic speeds with Mach number ranging from 5 to 12. The wind tunnel under consideration in this work is a hypersonic intermittent blowdown type. In the directory of wind tunnel facilities in India, released on the occasion of the national conference on wind tunnel testing [2009], different types of wind tunnel available in our country are listed along with their essential features useful for the national aerospace agencies, professionals and the academia.

Figure 3.1 Arrangement of the components of the Hypersonic Wind Tunnel
(Courtesy : Directory of Wind tunnel facilities in India)

Hypersonic Intermittent blowdown type horizontal wind tunnel is being developed at the Vikram Sarabhai Space Centre in Trivandrum. It can
be used in the aerospace and industrial applications for performing steady and unsteady force, moment and pressure measurements, aero-elastic tests, flow visualization tests etc. The typical test duration of the tunnel is 20sec to 50 sec and the speed range is from 6 mach to 12 Mach. The usual pressure that has to be maintained in the settling chamber for the above range of operation is 70 bar to 100 bar. The wind tunnel is used for testing Reusable Launch Vehicle (RLV) and the Air Breathing Propulsion (ABP) projects of VSSC.

3.2 SYSTEM MODEL

Hypersonic intermittent blow down-type wind tunnel is a test facility in the ground to simulate flight conditions of space vehicles by blowing air at hypersonic speeds, for short periods at regular intervals, instead of blowing air continuously. Block schematic of the tunnel system is shown in Figure 3.2. The different subsystems are high pressure system, wind tunnel system which consists of Pressure Regulating valve (PRV), Heater, Settling chamber, Nozzle, Test section, the Diffuser (DIFF), After Cooler (AFCL) and Vacuum Chamber (Vac). Air is compressed and stored in the high pressure system. It is released through a pressure regulating valve to create the desired pressure in the settling chamber. Heater is used to heat the air while passing through the heater bed to avoid liquefaction when it is expanded through the nozzle to get high Mach numbers. The pressure in the settling chamber is controlled by the proper operation of the control valve so that flow through test section meets the Mach number and mass flow rate specified for the test conditions.

![Figure 3.2 Block schematic of Hypersonic wind tunnel](image)
General Specifications of Hypersonic Wind Tunnel are as follows:

Max. Storage Pressure : 300bar
Max. Temperature     : 1700K
Max. mass flow rate  : 232Kg/s
Size of test section : 1X1m
Mach No:             : 6,8,10and12
Run Time             : 40-60s

For studying the feasibility and performance of various controllers for regulating pressure in the settling chamber of the wind tunnel, a mathematical model is required. The pressure in the settling chamber is varied by adjusting the Pressure Regulating Valve (PRV). So the model should represent the dynamics existing between the PRV connected at the outlet of the high pressure system and the settling chamber pressure. Once the settling chamber pressure is controlled, air flow with the desired Mach number will be available through the nozzle to the test section. Thus, in the present work, modeling of the test section, diffuser, after cooler and vacuum units are not needed. So the mathematical model required for this work reduces to the one as shown in Figure 3.3.

Wind tunnel process is highly nonlinear and so obtaining its mathematical model is very complicated. The mathematical model is developed considering the total system as three pressure vessels namely the high pressure system, the heater and the settling chamber as shown in Figure 3.3.
Figure 3.3 Block diagram of Hypersonic Wind Tunnel to be modeled.

The physical parameters for modeling the system are as follows:

Volume of the compressed air storage tank (vessel 1) : 132m³
Throttle Control valve (Mach 6 operation) : 12 inch
Volume of Heater for Mach 6 operation (vessel 2) : 18.24m³
Volume of Settling chamber and associated pipelines (vessel 3) : 2.7m³
Area of Mach 6 Nozzle : 0.0130394m²

For nominal test condition, P₁=300bar, T₁=300 K; T₂=700 K; T₃=539 K
Flow rate of compressible fluid F₁ is given by

\[ F₁ = mCᵥN₈FₚP₁Y \sqrt{\frac{XM}{T₁Z}} \]  

(3.1)

where m is the stem position of the valve,
Cᵥ is the valve coefficient = 0.13055,
N₈ is the constant for engineering units = 0.00948,
Fₚ is the constant for pipeline geometry = 1,
M is molecular weight of air = 29,
Z is the compressibility factor = 1.077,
X_i is critical pressure drop ratio factor = 0.562 and
F_k is the ratio of specific heats factor = 1

\[ Y = 1 - \frac{X}{3F_kX}, \]
where, Y is the Expansion factor, X_i is critical pressure drop ratio factor, F_k is the ratio of specific heats factor and \( X = \frac{P_1 - P_2}{P_1} \), where X is the Pressure difference ratio, P_2 is the downstream pressure of PRV.

Heater chamber is considered as one pressure vessel and connected pipelines and settling chamber together considered another pressure vessel. The outflow from heater F_2 is given by

\[ F_2 = C_vN_8F_pP_2Y \sqrt{X/M \over T_2Z} \]  (3.2)

where, the Expansion factor \( Y = 1 - \frac{X}{3F_kX} \) and \( X = \frac{P_2 - P_3}{P_2} \)
and P_3 is the settling chamber pressure.

For different Mach number simulations fixed nozzles with different cross section area are used. It always maintains a choked flow through nozzle. The mass flow rate through nozzle F_3 is given by,

\[ F_3 = \frac{K_n P_3}{\sqrt{T_3}} \]  (3.3)

where k_n is the nozzle constant and P_3 is the settling chamber pressure and T_3 is the settling chamber temperature.

The continuity equations for three pressure vessels may be written as,

\[ C_1 \frac{dP_1}{dt} = -F_1 \quad C_2 \frac{dP_2}{dt} = F_1 - F_2 \quad C_3 \frac{dP_3}{dt} = F_2 - F_3 \]  (3.4)
where, \( C_1 = \frac{V_1}{nRT_1} \), \( C_2 = \frac{V_2}{nRT_2} \) and \( C_3 = \frac{V_3}{nRT_3} \)

Using the three flow equations (3.1), (3.2) and (3.3), and the continuity equations (3.4), the nonlinear model of the hypersonic wind tunnel is made in SIMULINK.

3.3. REALIZATION OF SIMULINK MODEL OF HYPersonic WIND TUNNEL

SIMULINK tool in MATLAB is used for realizing the nonlinear mathematical model of the hypersonic wind tunnel.

Figure 3.4 SIMULINK implementation of \( F_1 \) equation.
The implementation of equation (3.1), which is the output flow from the high pressure system, ie, the first chamber in simulink is shown in Figure 3.4.

Figure 3.5 shows the SIMULINK realization of equation (3.2), which is the flow from the heater ie, second chamber.

Figure 3.5. SIMULINK implementation of $F_2$ equation

The implementation of equation (3.3), which is the output flow from the settling chamber, ie, the third vessel is shown in Figure 3.6.
Figure 3.6. SIMULINK implementation of $F_3$ equation

Figure 3.7 SIMULINK model of the hypersonic wind tunnel using subsystems
The mathematical model of the wind tunnel is developed by combining the subsystems using the continuity equations in equation (3.4) as shown in Figure 3.7. It is formed into a single subsystem, which represents the hypersonic wind tunnel system as given in Figure 3.8, where \( m \) is the position of the valve and \( P_3 \) represents the pressure in the settling chamber.

![Figure 3.8 Final SIMULINK model of the hypersonic wind tunnel](image)

3.4. PERFORMANCE ANALYSIS AND VALIDATION OF THE MODEL

During the wind tunnel testing, the first vessel is charged to a pressure of 300 bar and the output valve is opened. So pressure builds up in the second and third vessel. In the second and third vessel the pressure increases and then reduces to zero as the air is escaping through the nozzle fitted in the third vessel outlet at a specific mass flow rate and Mach number. The pressure variation of the three vessels at 100% valve opening is obtained.

Figure 3.9 shows the pressure drop in the first chamber when valve is opened to 100%. The pressure reduces to zero from 300 bar in about 800 sec. Figure 3.10 shows the pressure drop in the second chamber when valve is opened to 100%. The pressure increases from zero to about 250 bar and then reduces to zero in about 800 sec.
Figure 3.9: Pressure drop in the first chamber when valve is opened to 100%

Figure 3.10 Pressure drop in the second chamber when valve is opened to 100%

Figure 3.11 shows the pressure drop in the third chamber when valve is opened to 100%. The pressure increases from zero to about 200 bar and then reduces to zero in about 800 sec.
Figure 3.11 Pressure drop in the third chamber when valve is opened to 100%

From the simulation results, it is observed that the pressure in the second and third vessel is increased, and then drops to zero, while that of in first vessel is decreased from 300 bar and becomes zero, which is in well conformance with the practical behavior of a wind tunnel. Hence we can infer that the model developed is well representing the actual system behavior.

3.5. ANALYSIS OF NONLINEARITY

In order to analyze the system nonlinearity, the following test conditions are given. Initially, the pressure in the high pressure system is kept at 350 bar, and the valve is opened to 20% and the pressure in the settling chamber is noted. Next, the valve is opened to 40% from 20% opening to note the pressure in the settling chamber. The procedure is continued for a step increase of 20% valve opening. The same procedure is repeated for different constant pressure in the high pressure system. The values are tabulated and given in Table 3.1 and the graph showing the variation of settling chamber pressure for different valve openings keeping pressure constant in the high pressure system is given in Figure 3.12.
Figure 3.12 Variation of pressure for different valve openings.

The sensitivity of each of the region is calculated from the values in the Table 3.1.

Table 3.1 Variation of settling chamber pressure for different valve openings keeping constant pressure in the High Pressure System

<table>
<thead>
<tr>
<th>% Valve opening</th>
<th>Pressure in the Settling Chamber (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1 = 350$ bar</td>
</tr>
<tr>
<td>20</td>
<td>122</td>
</tr>
<tr>
<td>40</td>
<td>210</td>
</tr>
<tr>
<td>60</td>
<td>250</td>
</tr>
<tr>
<td>80</td>
<td>268</td>
</tr>
<tr>
<td>100</td>
<td>276</td>
</tr>
</tbody>
</table>

In the case of ideal pressure of 350 bar, sensitivity of first region is 4.4 bar / % change in valve displacement, and sensitivity of second region is 2 bar / % change in valve displacement, sensitivity of third region is 0.9 bar / % change in valve displacement whereas sensitivity of fourth region is
0.4 bar / % change in valve displacement. Similarly, for an ideal pressure of 150 bar, sensitivity of first region is 1.95 bar / % change in valve displacement and sensitivity of second region is 0.8 bar / % change in valve displacement, sensitivity of third region is 0.4 bar / % change in valve displacement, whereas the sensitivity of third region is 0.15 bar / % change in valve displacement.

Graph in Figure 3.12 show only five cases. Infinite number of similar graphs can be drawn, since the pressure is continuously varying in actual case. Static sensitivity (gain of the system) attains infinite values during the process, which shows the non linear behavior.

In the case of Hypersonic Wind tunnel, the flow rate from the high pressure system is very large (12 inch pipe). So the pressure will be rapidly decreasing in the compressor which causes severe transients in the pressure inlet to the next stage which results in the non linear behavior.

Thus the non linearity of the system is obvious from the plot showing the variation of settling chamber pressure for different valve openings keeping pressure constant in the high pressure system.

3.6. LINEARIZED MODEL OF HYPERSONIC WIND TUNNEL

Linearization is the process of approximating the nonlinear system with linear one. It is used in the study of process dynamics and design of control systems.

Substituting the numerical values and simplifying using binomial expansion, we can reduce equations (3.1), (3.2) and (3.3) as
\[ F_1 = 2.39 \times 10^{-5} m(P_1 - 0.5P_2) \]  
\[ F_2 = 1.569 \times 10^{-5} (P_2 - 0.5P_3) \]  
\[ F_3 = 2.26 \times 10^{-4} P_3 \]

The continuity equations for three pressure vessels may be written as,

\[ C_1 \frac{dP_1}{dt} = -F_1 \]

\[ C_2 \frac{dP_2}{dt} = F_1 - F_2 \quad \text{and} \]

\[ C_3 \frac{dP_3}{dt} = F_2 - F_3 \]

where, \[ C_1 = \frac{V_1}{nRT_1} \quad ; \quad C_2 = \frac{V_2}{nRT_2} \quad ; \quad C_3 = \frac{V_3}{nRT_3} \]

For obtaining the linearized approximate model of the hypersonic wind tunnel, the parameters at the point of linearization is considered as follows:

\[ V_1 = 132 \text{m}^3, \quad V_2 = 18.24 \text{m}^3, \quad V_3 = 2.7 \text{m}^3, \quad T_1 = 300K, \quad T_2 = 700K, \quad T_3 = 539K \]

\[ P_1 = 250 \text{ bar}, \quad P_2 = 150 \text{ bar}, \quad P_3 = 70 \text{ bar}, \quad n = 1.73, \quad R = 287 \text{ and } m = 0.3. \]

Figure 3.13 shows the schematic of the linear approximation technique of the nonlinear model of hypersonic wind tunnel.
For chocked flow, \( F_1 = f(P_1, m) \). To linearize the equation, the total differential for a function of two variables can be written as,

\[
dF_1 = \frac{\partial F_1}{\partial P_1} dP_1 + \frac{\partial F_1}{\partial m} dm
\]

where,

\[
F_1 = 2.39 \times 10^{-5} m(P_1 - 0.5P_2)
\]

\[
dF_1 = 2.39 \times 10^{-5} m dP_1 + (2.39 \times 10^{-5} P_1 - 2.39 \times 10^{-5} \times 0.5 P_2) dm
\]

Substitute \( m = 0.3, P_1 = 250\text{bar} \) and \( P_2 = 70\text{ba} \)

\[
F_1 = 0.717 \times 10^{-5} P_1 + 418.25 m
\]

Similarly \( F_2 \) can be written as \( F_2 = f(P_2, P_3) \)

\[
dF_2 = \frac{\partial F_2}{\partial P_2} dP_2 + \frac{\partial F_2}{\partial P_3} dP_3
\]
where,

\[ dF_2 = 1.569 \times 10^{-5} dP_2 - 0.7845 \times 10^{-5} dP_3 \]

\[ F_2 = 1.569 \times 10^{-5} P_2 - 0.7845 \times 10^{-5} P_3 \]  \hspace{1cm} (3.9)

Similarly, \[ F_3 = 2.26 \times 10^{-4} P_3 \]  \hspace{1cm} (3.10)

Equations 3.8, 3.9 and 3.10 is of the general form,

where, \( k_1 = 0.717 \times 10^{-5}, k_2 = 418.25 \)

\[ k_3 = 1.569 \times 10^{-5}, k_4 = -0.7845 \times 10^{-5} \]

\[ k_5 = 2.26 \times 10^{-4} \]

The state space model of hypersonic wind tunnel, is given by:

\[
\begin{bmatrix}
\dot{P}_1 \\
\dot{P}_2 \\
\dot{P}_3
\end{bmatrix} = \begin{bmatrix}
-k_i/C_i & 0 & 0 \\
 k_i/C_i & -k_j/C_j & -k_j/C_j \\
 0 & k_i/C_i & (k_i-k_j)/C_j
\end{bmatrix} \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} + \begin{bmatrix}
k_i/C_i \\
k_j/C_j \\
0
\end{bmatrix} m \\
Y = [0 \hspace{0.5cm} 0 \hspace{0.5cm} 1] \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} \hspace{1cm} (3.11)
\]

Substituting the numerical values the state space model of the hypersonic wind tunnel is obtained as below:

\[
\begin{bmatrix}
\dot{P}_1 \\
\dot{P}_2 \\
\dot{P}_3
\end{bmatrix} = \begin{bmatrix}
-8.092 \times 10^{-3} & 0 & 0 \\
 8.092 \times 10^{-3} & -0.2994 & -0.1494 \\
 0 & 1.5565 & -23.199
\end{bmatrix} \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} + \begin{bmatrix}
-50.5790 \times 10^4 \\
85.390625 \times 10^4 \\
0
\end{bmatrix} m \\
Y = [0 \hspace{0.5cm} 0 \hspace{0.5cm} 1] \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} \hspace{1cm} (3.12)
\]
The transfer function of the system is obtained as

$$\frac{P_i(s)}{m(s)} = \frac{1.3291 \times 10^7 s + 0.0101 \times 10^7}{s^3 + 23.5065 s^2 + 7.3685 s + 0.0581}$$  \hspace{1cm} (3.13)

3.6.1 Analysis of the Linearised Model

The analysis of linearized mathematical model developed, can be done in the time domain and frequency domain. The time response of a system is the output of the system as a function of time, when subjected to a known input. Time response analysis investigates the time-domain transient behavior of linear models for particular classes of inputs and disturbances. Figure 3.14 shows the step response and impulse response of the system in time domain and also the response of the system for a sine wave input.

From the step response we can say that the time constant of the system is about 4 sec. It indicates how fast the system reaches the final value. The system characteristics like rise time, settling time, overshoot, and steady-state error can be determined from the time response.
Root locus is a graphical method of determining the stability of a system in time domain, and it is obtained by sketching the locus of roots in the s- plane as the parameter of the system is varied. Figure 3.15 shows the root locus of the system. Since the root locus is totally on the left side of the s plane, the system is stable for all values of gain K, ranging from 0 to infinity.

![Root locus plot of the system](image)

Figure 3.15: Root locus plot of the system

Frequency response of a system is the response of the system for sinusoidal input signal of various frequencies. The important frequency domain specifications are resonant frequency, gain margin and phase margin. The frequency-domain analysis can be done using the standard plots like Bode plot and Nyquist plot. The magnitude and phase plot of the Bode plot is shown in Figure 3.16, with gain margin and phase margin calculated and noted. Since the gain margin of the system is infinity and phase margin is positive, the closed loop system is always stable for all the values of gain.
Figure 3.16: Frequency Response analysis using Bode plot.

Figure 3.17: Frequency Response Analysis using Nyquist plot.

Nyquist plot is a semi graphical method that determines the stability of a closed loop transfer function from the open loop transfer function, using the
Nyquist stability criterion. Figure 3.17 shows the Nyquist plot of the system. In the plot, since there is no encirclements around -1+j0 point, the closed loop system will always be stable.

3.7. SIGNIFICANCE OF OPTIMUM CONTROLLER FOR WIND TUNNEL OPERATION

Maintaining constant pressure in the settling chamber of the hypersonic intermittent blow down type wind tunnel is an important task for its effective performance. The system is highly uncertain and non linear. The set pressure has to be reached within few seconds. So it is a challenging task to develop a controller that should adapt for different set pressure values, inlet pressure values mass flow and temperature. The set pressure values changes as per the requirement. The inlet pressure value varies as the test begins. These stringent requirements and the need for high reliability systems make the role of controller design important. The design of controller becomes very much challenging as the process exhibits wide variation in static sensitivity.