Chapter 3

Forced convective heat transfer
from a circular cylinder under low
$R_m$ approximation

In this chapter, heat transfer due to the forced convection from a circular cylinder in a steady stream of viscous incompressible and electrically conducting fluid which is under the influence of an external stream-wise magnetic field is calculated at $Re = 5.40$ for $Pr = 0.065, 0.73$ and $5$. The flow velocities are obtained by solving Navier-Stokes and Maxwell’s equations using the multigrid and defect correction techniques. Higher order compact scheme (HOCS) is used to solve the governing energy equation in cylindrical polar coordinates. The behavior of local and mean Nusselt numbers are compared with recent experimental results.

3.1 Formulation of the problem

The forced convective heat transfer problem is formulated as steady, laminar flow in axis-symmetric cylindrical polar coordinates. The center of the cylinder is chosen at origin and the flow is symmetric about $\theta = \pi$ (upstream) and $\theta = 0^\circ$ (downstream). The fluid is considered to be incompressible viscous and electrically conducting. An uniform stream from infinity, $U_\infty$ is imposed from left to right at far distances from the cylinder. The magnetic Reynolds number is assumed to be small so that induced
magnetic field can be neglected and a constant magnetic field

\[ H = (-\cos\theta, \sin\theta, 0) \]  
(3.1)
is imposed opposite to the flow. The governing equations are Navier-Stokes equations and Maxwell’s equations as follows.

\[
\begin{align*}
(q \cdot \nabla) q &= -\nabla p + \frac{2}{Re} \nabla^2 q + N[(q \times H) \times H] \\
\nabla \cdot q &= 0 \\
\nabla \cdot H &= 0 \\
\nabla \times E &= 0
\end{align*}
\]  
(3.2-3.5)

where \( p \) is the pressure, \( q \) the fluid velocity, \( H \) the magnetic field, \( E \) the electric field. The kinematic Reynolds number is \( Re = \frac{2aU}{\nu} \) (here, \( a \) is the radius of the circular cylinder) and the interaction parameter \( N = \frac{\sigma H^2}{\rho U_\infty}a/\rho U_\infty \). The kinematic viscosity, density, magnetic permeability and electrical conductivity of the fluid are \( \nu, \rho, \mu \) and \( \sigma \) respectively. By taking curl of the momentum equation (3.2) we get

\[
-\nabla \times (\nabla \times \omega) = \frac{Re}{2} \nabla \times (\omega \times q) - \frac{ReN}{2} \nabla \times \{(q \times H) \times H\} \\
\omega = \nabla \times q
\]  
(3.6-3.7)
is the vorticity. Since the flow is two dimensional, \( E = 0 \). Cylindrical polar co-ordinates \((r, \theta, z)\) are used as they are most suitable in dealing with cylindrical boundaries. The co-ordinate system is set up such that the velocity and magnetic field are parallel at large distances and the flow is symmetric about \( \theta = 0^\circ \) and \( \theta = 180^\circ \). Here, \( q = (q_r, q_\theta, 0) \). These velocity components are expressed in terms of a dimensionless stream function \( \psi(\xi, \theta) \) such that the equation of continuity \( \nabla \cdot q = 0 \) is satisfied. They are

\[
q_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad q_\theta = -\frac{\partial \psi}{\partial r}
\]  
(3.8)

We use the transformation \( r = e^{\pi \xi} \) and \( \theta = \pi \eta \) and consequently, the flow equations
become

\[
\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} + \pi^2 e^{2\pi \xi} \omega = 0 \tag{3.9}
\]

\[
\frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} - \frac{Re}{2} \left[ \frac{\partial \psi}{\partial \eta} \frac{\partial \omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \omega}{\partial \eta} \right] = \frac{NRe}{2} \left[ \pi^2 e^{2\pi \xi} \omega \sin^2(\pi \eta) \\
+ \sin 2(\pi \eta) \frac{\partial^2 \psi}{\partial \xi \partial \eta} - \pi \sin 2(\pi \eta) \frac{\partial \psi}{\partial \eta} - \cos 2(\pi \eta) \frac{\partial^2 \psi}{\partial \xi^2} + \pi \cos 2(\pi \eta) \frac{\partial \psi}{\partial \xi} \right] \tag{3.10}
\]

Equations (3.9) and (3.10) must now be solved subject to the following boundary conditions. On the surface of the cylinder, no-slip condition is applied. At far off distances, uniform flow is imposed. The boundary conditions applied are (i) On the surface of the cylinder \( \psi = \partial \psi / \partial \xi = 0 \) (ii) At large distances from cylinder \( \psi \sim e^{\pi \xi} \sin(\pi \eta) \); and (iii) On the axis of symmetry \( \psi = 0, \omega = 0 \). The flow velocities are obtained by solving equations (3.1), (3.6) to (3.8) using multigrid technique as explained in chapter 1. More details of this technique and results are explained in Sekhar et. al [61]. Under the condition that the physical properties of the fluid are constant and the internal generation of heat by friction is neglected, the energy equation is given by

\[
\mathbf{q} \cdot \nabla T = \frac{2}{Re \, Pr} \nabla^2 T \tag{3.11}
\]

In Cylindrical polar coordinates, using the given transformation, the energy equation is written as

\[
\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2} = \frac{Re \Pr}{2} \left( \frac{\partial \psi}{\partial \eta} \frac{\partial T}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial T}{\partial \eta} \right) \tag{3.12}
\]

where \( T(\xi, \eta) \) is the non-dimensionalized temperature, defined by subtracting the main-flow temperature \( T_\infty \) from the temperature and dividing by \( T_s - T_\infty \). The Prandtl number \( Pr \) is given by \( Pr = \nu / \kappa \). The boundary conditions for temperature are (i) \( T = 1 \) at \( \xi = 0 \) on the surface of the cylinder, (ii) \( T \to 0 \) as \( \xi \to \infty \), at large distance from cylinder and (iii) Along the axis of symmetry (\( \eta = 0 \) and \( \eta = 1 \)); \( \frac{\partial T}{\partial \eta} = 0 \).
3.2 Fourth order Compact Scheme

Equation (3.12) is rewritten as

\[- \frac{\partial^2 T}{\partial \xi^2} - \frac{\partial^2 T}{\partial \eta^2} + c \frac{\partial T}{\partial \xi} + d \frac{\partial T}{\partial \eta} = 0\]  

(3.13)

where

\[c = \frac{RePr^2}{2 \pi e q_r}, \quad d = \frac{RePr^2}{2 \pi e q_\theta}\]  

(3.14)

The velocity components \(q_r\) and \(q_\theta\) in the equation (3.14) are obtained using usual fourth order approximations from the stream function \(\psi\). Applying standard central difference operators to equation (3.13) gives,

\[-\delta^2_{\xi} T_{i,j} - \delta^2_{\eta} T_{i,j} + c_{i,j} \delta_{\xi} T_{i,j} + d_{i,j} \delta_{\eta} T_{i,j} - \tau_{i,j} = 0\]  

(3.15)

The truncation error of (3.15) is given by

\[\tau_{i,j} = \left[ 2 \left( \frac{h^2}{12} \frac{\partial^3 T}{\partial \xi^3} + \frac{k^2}{12} \frac{\partial^3 T}{\partial \eta^3} \right) - \left( \frac{h^2}{12} \frac{\partial^4 T}{\partial \xi^4} + \frac{k^2}{12} \frac{\partial^4 T}{\partial \eta^4} \right) \right]_{i,j} + O(h^4, k^4),\]  

(3.16)

where \(h\) and \(k\) are grid spacing \((h \neq k)\) in the radial and angular directions, respectively.

From equation (3.12), we get

\[\frac{\partial^3 T}{\partial \xi^3} = - \frac{\partial^3 T}{\partial \xi \partial \eta^2} + c \frac{\partial^2 T}{\partial \xi^2} + \frac{\partial c}{\partial \xi} \frac{\partial T}{\partial \xi} + d \frac{\partial^2 T}{\partial \xi \partial \eta} + \frac{\partial d}{\partial \xi} \frac{\partial T}{\partial \eta}\]

\[\frac{\partial^4 T}{\partial \xi^4} = - \frac{\partial^4 T}{\partial \xi^2 \partial \eta^2} + d \frac{\partial^3 T}{\partial \xi^3 \partial \eta} - c \frac{\partial^3 T}{\partial \xi \partial \eta^3} + \left( \frac{2 \partial d}{\partial \xi} + cd \right) \frac{\partial^2 T}{\partial \xi \partial \eta} + \frac{2 \partial c}{\partial \xi} \frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 c}{\partial \xi^2} \frac{\partial T}{\partial \xi} + \frac{\partial d^2}{\partial \xi^2} + \frac{c \partial d}{\partial \xi} \frac{\partial T}{\partial \eta}\]

\[\frac{\partial^3 T}{\partial \eta^3} = - \frac{\partial^3 T}{\partial \xi^2 \partial \eta} + c \frac{\partial^2 T}{\partial \xi \partial \eta^2} + \frac{\partial c}{\partial \eta} \frac{\partial T}{\partial \xi} + d \frac{\partial^2 T}{\partial \eta^2} + \frac{\partial d}{\partial \eta} \frac{\partial T}{\partial \eta}\]
\[
\frac{\partial^4 T}{\partial \eta^2} = -\frac{\partial^3 T}{\partial \xi^2 \partial \eta^2} + c \frac{\partial^3 T}{\partial \xi \partial \eta^2} - d \frac{\partial^3 T}{\partial \xi^2 \partial \eta} + \left( 2 \frac{\partial c}{\partial \eta} + cd \right) \frac{\partial^2 T}{\partial \xi \partial \eta} + \left( \frac{\partial d}{\partial \eta^2} + d \frac{\partial d}{\partial \eta} \right) \frac{\partial^2 T}{\partial \xi^2} \\
+ \left( \frac{\partial^2 d}{\partial \eta^2} + d \frac{\partial d}{\partial \eta} \right) \frac{\partial T}{\partial \xi} - \frac{\partial T}{\partial \eta}.
\]

On substituting in equation (3.16) and using it in equation (3.15) gives

\[
-e_{i,j} \delta^2 \xi T_{i,j} - f_{i,j} \delta^2 \eta T_{i,j} + g_{i,j} \delta \xi T_{i,j} - o_{i,j} \delta \eta T_{i,j} - D_2 \left( \delta^2 \xi \delta^2 \eta T_{i,j} - c_{i,j} \delta \xi \delta^2 \eta T_{i,j} - d_{i,j} \delta^2 \xi \delta \eta T_{i,j} \right) + w_{i,j} \delta \eta \delta \xi T_{i,j} = 0
\]

where the coefficients \(e_{i,j}, f_{i,j}, g_{i,j}, o_{i,j}\) and \(w_{i,j}\) are given by

\[
e_{i,j} = 1 + \frac{h^2}{12} (c_{i,j} - 2 \delta \xi c_{i,j})
\]

\[
f_{i,j} = 1 + \frac{k^2}{12} (d_{i,j} - 2 \delta \eta d_{i,j})
\]

\[
g_{i,j} = c_{i,j} + \frac{h^2}{12} (\delta^2 \xi c_{i,j} - c_{i,j} \delta \xi c_{i,j}) + \frac{k^2}{12} (\delta^2 \eta c_{i,j} - d_{i,j} \delta \eta c_{i,j})
\]

\[
o_{i,j} = d_{i,j} + \frac{h^2}{12} (\delta^2 \xi d_{i,j} - c_{i,j} \delta \xi d_{i,j}) + \frac{k^2}{12} (\delta^2 \eta d_{i,j} - d_{i,j} \delta \eta d_{i,j})
\]

\[
w_{i,j} = \frac{h^2}{6} \delta \xi d_{i,j} + \frac{k^2}{6} \delta \eta c_{i,j} - D_2 c_{i,j} d_{i,j}.
\]

and

\[
D_2 = \frac{h^2 + k^2}{12}
\]

The two-dimensional cross derivative \(\delta\) operators on a uniform anisotropic mesh \((h \neq k)\) are approximated by the central difference approximation as follows:

\[
\delta \xi \delta \eta T_{i,j} = \frac{T_{i+1,j+1} + T_{i+1,j-1} - T_{i-1,j+1} - T_{i-1,j-1}}{4hk}
\]

\[
\delta^2 \xi \delta \eta T_{i,j} = \frac{T_{i+1,j+1} + T_{i-1,j+1} - T_{i+1,j-1} - T_{i-1,j-1} - 2T_{i,j+1} + 2T_{i,j-1}}{2h^2k}
\]

\[
\delta \xi \delta^2 \eta T_{i,j} = \frac{T_{i+1,j+1} + T_{i+1,j-1} - T_{i-1,j+1} - T_{i-1,j-1} + 2T_{i,j+1} - 2T_{i,j-1}}{2hk^2}
\]
\[ \frac{\partial^2 T_{i,j}}{\partial \eta^2} = \frac{1}{h^2 k^2} (T_{i+1,j+1} + T_{i+1,j-1} + T_{i-1,j+1} + T_{i-1,j-1} - 2T_{i,j+1} - 2T_{i,j-1} - 2T_{i+1,j} - 2T_{i-1,j} + 4T_{i,j}) \]

While approximating the boundary conditions for temperature, \( \frac{\partial T}{\partial \eta} \) is approximated by fourth order forward difference along \( \eta = 0 \) (\( j = 1 \)) and fourth order backward difference along \( \eta = 1 \) (or \( j = m + 1 \)) as follows.

\[
T(i, 1) = \frac{1}{25} [48T(i, 2) - 36T(i, 3) + 16T(i, 4) - 3T(i, 5)] \\
T(i, m + 1) = \frac{1}{25} [48T(i, m) - 36T(i, m - 1) + 16T(i, m - 2) - 3T(i, m - 3)]
\]

The algebraic system of equations obtained using the fourth order compact scheme described as above are solved using Point Gauss-Seidel method. The scheme helps to achieve convergence up to \( 10^{-6} \) even at higher values of parameters.

### 3.3 Results and Discussion

Higher order compact scheme is applied to the problem of heated cylinder which is immersed in an incompressible, viscous and electrically conducting fluid. The stream-wise magnetic field is applied in the opposite direction of the flow to control the heat transfer. The flow results of Sekhar et al [61] have been extended to higher values of \( N \) up to 20 using server machine and then forced convection is studied. The pressure drag and viscous drag coefficients increases with increase of \( N \) and hence total drag also increases with \( N \) as shown in Fig. 3.1. The separation length and angle exhibits non-monotonic behaviour as shown in Fig. 3.2. In this study, the results are presented for \( Re = 40 \) and \( Pr \) and \( N \) in the ranges \( 0 < Pr \leq 7 \) and \( 0 \leq N \leq 20 \). The heat flux \( q(\theta) \) from the cylinder to the fluid is computed using

\[
q(\theta) = \frac{k(T_s - T_\infty)}{a} \left( \frac{\partial T}{\partial \xi} \right)_{\xi=0} \tag{3.17}
\]
The local Nusselt number \( N_u(\theta) \) and the mean Nusselt number \( N_m \) are calculated as follows:

\[
N_u(\theta) = \frac{2a q(\theta)}{k(T_s - T_\infty)} = \frac{-2}{\pi} \left( \frac{\partial T}{\partial \xi} \right)_{\xi=0}
\tag{3.18}
\]

\[
N_m = \frac{-2}{\pi} \int_0^1 \left( \frac{\partial T}{\partial \xi} \right)_{\xi=0} d\eta.
\tag{3.19}
\]

In equations (3.18) and (3.19) the derivative \( \frac{\partial T}{\partial \xi} \) is approximated with fourth order finite differences.

### 3.3.1 Local Nusselt number

In the absence of the magnetic field \( (N = 0) \) the basic hydrodynamic problem is equivalent to the steady viscous flow around a cylinder. Therefore, for \( N = 0 \), the developed scheme is validated with the available theoretical results for different \( Re \) and \( Pr \) in Table 3.2. It is clear from the table that the present results agree well with the literature data [62, 17]. The angular variation of the local Nusselt number \( N_u \) on the surface of the cylinder for different Prandtl numbers and for different interaction parameters are presented in Fig. 3.3 and in Fig. 3.4 respectively. At \( Re = 40 \), in the absence of the magnetic field, it is observed that the local Nusselt number decreases along the surface of the cylinder up to the point of separation and then increases. In the upstream region (Fig. 3.4), the viscous boundary layer thickens with the application of magnetic field. All the curves in this figure meet at one critical point after which an inverse effect is exhibited, that is, the boundary layer gets thinner with magnetic field. The curves meet once again in the far downstream. These features are attributed to changes in radial and transverse velocity gradients of the fluid which resulted due to the application of magnetic field to the flow. The applied magnetic field brings changes in the local Nusselt number. While studying the dependance of \( N_u \) on \( Pr \), when external magnetic field is not present, the maximum heat transfer takes place near the front stagnation point \( \theta = \pi \) (Fig. 3.3) whereas, when magnetic field is increased, the peak heat transfer region is shifted towards \( \theta = \pi/2 \). Irrespective of the magnetic field, when \( Pr \) is increased, the
local Nusselt number increased along the surface of the cylinder. However, the heat transfer rate of a fluid with higher $Pr$ is affected more by external magnetic field when compared to a fluid with lower $Pr$. Figure 3.5 exhibits the dependence of local Nusselt number on various magnetic strengths $N = 0, N = 0.5, N = 9$ and $N = 20$ for $Re = 5, 40$ and $Pr = 0.73, 7$. It is clear that higher the $Re$ higher the heat transfer in the case of application of lower magnetic strengths ($N = 0, 0.2$) as we expected. And the increase of $Pr$ also increases the heat transfer. But when $N$ increases ($N = 9$), there is a change in this pattern in the rear stagnation point, that is, the fluid flow for $Re = 40$ with $Pr = 0.73$ produces less heat transfer when compared with that of for $Re = 5$ with $Pr = 7$. This implies the influence of $Pr$ is more than the influence of $Re$ in terms of heat transfer at higher magnetic strengths. Figs. 3.6, 3.7 explains about the heat transfer changes along particular radial lines at $\theta = 0, \pi/6, \pi/2, \pi$ with variation of $N$ for different $Pr$. It is clear that when $N$ increases $Nu$ decreases along $\theta = 0$ and $\theta = \pi$ whereas a reversal effect is observed when $\theta = \pi/2$. On the other hand the pattern is so different along $\theta = \pi/6$, that is, for $Pr = 0.065, 0.73$, $Nu$ increases as $N$ increases. Meanwhile At higher Prandtl numbers ($Pr = 5$ and $Pr = 7$) there is a reduction of $Nu$ up to a critical level and then it increases. Hence on applying magnetic field heat transfer is much controlled in the front and rear stagnation points but non-monotonically changes in the region near separation point.

### 3.3.2 Mean Nusselt number

When these local values of heat flux is surface averaged over the cylinder using equation 3.19, we get mean Nusselt number $N_m$. In order to compare with the available results of Uda et al. [34], simulations have been carried over the parameters $Re = 5$ and $Re = 40$ with $Pr = 0.065$ for different $N$ values. The plots are made on the variation of mean Nusselt number versus strength of magnetic field in Fig. 3.8. In the case of both $Re = 5$ and $40$ the non-monotonic behavior in the increase/decrease of mean Nusselt number is observed. This fact is in agreement with the observations of Uda et al. [34]. The variation of mean Nusselt number with respect to the magnetic field $N$ for $Re = 5$ in Fig. 3.9 and for $Re = 40$ in Fig. 3.10
with a variation of Prandtl numbers \( Pr = 0.065, 0.73, 5 \) and 7 are plotted. From Figs. 3.9 and 3.10, we observe that there is a degradation in mean Nusselt number when \( 0 \leq N \leq 0.2 \) for \( Re = 5 \) and when \( 0 \leq N \leq 0.4 \) for \( Re = 40 \). Beyond which it increases leading to a non-monotonic behavior. This observation is in line with the recent experimental findings of Uda et al. [34] and Yokomine et al. [35]. These figures ensure the non-monotonic behavior of \( Nm \) with respect to \( Pr \) too. But the decrease of \( Nm \) is significantly more in the case of higher \( Re(= 40) \) than \( Re = 5 \), which says the influence of \( Re \). In Fig. 3.11, the dependence of mean Nusselt number on Prandtl number \( Pr \) with variation in the strength of magnetic field \( N \) for \( Re = 5 \) and \( Re = 40 \) are presented. An increase of \( Nm \) with respect to \( Pr \) is identified for all \( N \) values which matches with the observation (in the absence of magnetic field) made by Kurdyumov & Fernandez [58]. On adding the parameters \( Re = 10, 20 \) with \( Pr = 0.73, 5 \) the comparison on \( Nm \) is made with available literature in the case when there is no magnetic field applied. The simulations have been carried out over 32 \( \times \) 32, 64 \( \times \) 64, 128 \( \times \) 128, 256 \( \times \) 256 and 512 \( \times \) 512 grids and the mean Nusselt number for \( Re = 40 \) for selected \( Pr \) and \( N \) are presented in Tables 3.1. It is clear that the solutions obtained from the present numerical scheme exhibit grid independence.

**Temperature field**

The roll of \( Re, R_m, N \) and \( Pr \) on forced convective heat transfer distribution from isothermal cylinder immersed in magnetohydrodynamic fluid flow are analyzed by means of isothermal contours from Figs. 3.12 to 3.16. Due to high thermal gradients (implied by the presence of dense layer of isotherms) near the upstream region close to the surface of the cylinder, the local Nusselt number \( Nu \) is found to be more and not so in the downstream region (Figure 3.12). In the absence of magnetic field, as \( Re \) increases from 5 to 40, there is a shift of higher heat transfer region towards the bluff body surface all along the cylinder. This shift is considerably more at the far down stream region between \( 0 \leq \theta \leq 50 \). It is due to the presence of larger recirculation region at higher \( Re \). Further, for the increase of \( Pr \), when \( N = 0 \), isotherms with higher temperature moved towards the surface of the cylinder at
Table 3.1: **Effect of grid size on the results for $Re = 40$ and $Pr = 0.73$.**

<table>
<thead>
<tr>
<th>$N$</th>
<th>$N_m$</th>
<th>32 $\times$ 32</th>
<th>64 $\times$ 64</th>
<th>128 $\times$ 128</th>
<th>256 $\times$ 256</th>
<th>512 $\times$ 512</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.237</td>
<td>3.268</td>
<td>3.265</td>
<td>3.265</td>
<td>3.265</td>
<td>3.265</td>
</tr>
<tr>
<td>1</td>
<td>3.252</td>
<td>3.278</td>
<td>3.275</td>
<td>3.275</td>
<td>3.275</td>
<td>3.275</td>
</tr>
<tr>
<td>5</td>
<td>3.322</td>
<td>3.351</td>
<td>3.349</td>
<td>3.349</td>
<td>3.349</td>
<td>3.349</td>
</tr>
</tbody>
</table>

Table 3.2: **Comparison of the mean Nusselt number with literature in the absence of the magnetic field.**

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$Pr$</th>
<th>$N_m$</th>
<th>Ref.[62]</th>
<th>Ref.[17]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.73</td>
<td>-</td>
<td>1.897</td>
<td>1.874</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.73</td>
<td>-</td>
<td>2.557</td>
<td>2.472</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.73</td>
<td>-</td>
<td>3.48</td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>4.593</td>
<td>-</td>
<td></td>
<td>4.6</td>
</tr>
</tbody>
</table>
the axis of the symmetry whereas it is still away from the cylinder in the region $10 \leq \theta \leq 30$ (Figure 3.13. But upon applying magnetic field it uniformly pushed towards the axis of symmetry (Figures 3.14 and 3.15). On fixing $Pr$, for the increase of strength of magnetic field it is observed that the heat transfer is well controlled to move towards the line of symmetry in the down stream region which is seen in figure 3.16.

### 3.3.3 Conclusions

- The non-monotonic behavior of separation length and separation angle is found with the increase of magnetic field.

- When external magnetic field is not present, the maximum heat transfer takes place near the front stagnation point $\theta = \pi$ whereas, when magnetic field is increased, the peak heat transfer region is shifted towards $\theta = \pi/2$.

- Irrespective of the magnetic field, when $Pr$ is increased, the local Nusselt number increased along the surface of the cylinder.

- The heat transfer rate of a fluid with higher $Pr$ is affected more by external magnetic field when compared to a fluid with lower $Pr$.

- We observe that there is a degradation in mean Nusselt number when $0 \leq N \leq 0.4$ beyond which it increases leading to a non-monotonic behavior. This observation is in line with the recent experimental findings of Uda et al [34] and Yokomine et al [35].
Figure 3.1: Variation of viscous drag, pressure drag and total drag coefficients as a function of $N$ for $Re = 5$ and $Re = 40$. 

\[ \text{Drag coefficients} \]

\[ N \]

\[ Re = 5 \]

\[ Re = 40 \]
Figure 3.2: Recirculation length and separation angle as a function of interaction parameter for $Re = 40$
Figure 3.3: Angular Nusselt number for Re = 40 and for different Prandtl numbers with $N = 0, N = 0.8, N = 9$ and $N = 20$. 
Figure 3.4: Angular variation of the Nusselt number for $Re = 40$ when $Pr = 0.065$, $0.73$, $5$ and $7$. 
Figure 3.5: Dependence of local Nusselt number $\textit{Nu}_\theta$ on $N = 0$, $N = 0.5$, $N = 9$ and $N = 20$ for different Prandtl numbers with $Re = 5, 40$. 
Figure 3.6: Variation of local Nusselt number (a) $Nu(\theta = 0)$, (b) $Nu(\theta = \frac{\pi}{6})$ with $N$ for $Re = 40$
Figure 3.7: Variation of local Nusselt number (a) $Nu(\theta = \pi/2)$, (b) $Nu(\theta = \pi)$ with $N$ for $Re = 40$
Figure 3.8: Variation of mean Nusselt number $N_m$ with magnetic field $N$. The case with $Pr = 0.065$ corresponds to liquid metal (Li).
Figure 3.9: Variation of mean Nusselt number $N_m$ with magnetic field $N$ for different $Pr$ for $Re = 5$. 
Figure 3.10: Variation of mean Nusselt number $N_m$ with magnetic field $N$ for different $Pr$ for $Re = 40$. 
Figure 3.11: Dependance of mean Nusselt number $N_m$ on $Pr$ and $N$ for $Re = 5, 40$. 
Figure 3.12: Isotherms of the flow in the absence of the magnetic field for $Re = 5$ and 40 with $Pr = 0.73$. 
Figure 3.13: Effect of $Pr$ in the absence of magnetic field on the temperature field for $Re = 40$. 

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Figure 3.14: Effect of $Pr$ in the presence of a weak magnetic field on the temperature field for $Re = 40$. 
Figure 3.15: Effect of $Pr$ in the presence of a strong magnetic field ($N = 20$) on the temperature field for $Re = 40$. 
Figure 3.16: Isotherms of the flow for $Re = 40$ with $Pr = 7$ and $N = 0, 0.6, 9, 18$. 