CHAPTER - III

THE METHOD OF CENTRED SYSTEM
IN SMOOTH FUZZY TOPOLOGICAL
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t-OPEN SETS.

Ever since the introduction of fuzzy sets by Zadeh [79] and fuzzy topological spaces by Chang [22] various notions in classical topology have been extended to fuzzy topological spaces. The method of centred systems in the theory of topology was introduced in [32]. In 2007, the above concept was extended to fuzzy topological spaces by Uma, Roja and Balasubramanian [72]. In this chapter, we introduce maximal smooth fuzzy t-centred system and the concept of t-absolute ω ( R ) of a smooth fuzzy topological spaces. Besides providing the preliminary results, the fundamental theorem on smooth fuzzy t-irreducible* and smooth fuzzy t-perfect functions are also studied.

Throughout this chapter R stands for a smooth fuzzy t-Hausdorff space.
3.1 THE SPACES OF MAXIMAL SMOOTH FUZZY t-CENTRED SYSTEMS.

In this section, the maximal smooth fuzzy t-centred system is introduced and its properties are discussed.

Definition 3.1.1.

A smooth fuzzy topological space \((X, T)\) is said to be smooth fuzzy t-Hausdorff iff for any two distinct fuzzy points \(x_{t_1}, x_{t_2}\) in \(X\), there exists \(r\)-fuzzy t-open sets \(\lambda, \mu \in I^X, r \in I_0\), such that, \(x_{t_1} \in \lambda\) and \(x_{t_2} \in \mu\) with \(\lambda \uplus \mu\).

Definition 3.1.2.

Let \(R\) be a smooth fuzzy t-Hausdorff space. A system \(p_t = \{\lambda_i\}\) of \(r\)-fuzzy t-open sets of \(R\) is called a smooth fuzzy t-centred system if any finite collection of \(\{\lambda_i\}\) is such that \(\lambda_i \uplus \lambda_j\) for \(i \neq j, r \in I_0\). The system \(p_t\) is called maximal smooth fuzzy t-centred system or a smooth fuzzy t-end if it cannot be included in any larger smooth fuzzy t-centred system.

Definition 3.1.3.

Let \((X, T)\) be a smooth fuzzy topological space. Its smooth fuzzy \(Q^*\)-t-neighborhood structure is a mapping \(Q^* : X \times I^X \to I\) (\(X\) denotes the totality of all fuzzy points in \(X\)), defined by

\[
Q^* (x, \lambda) = \sup \{ \mu : \mu \text{ is a } r\text{-fuzzy t-open set, } \mu \leq \lambda, x \in \mu, r \in I_0 \} \quad \text{and}
\]

\[
\lambda = \inf_{x \in X} Q^* (x, \rho) \text{ is a } r\text{-fuzzy t-open set, } r \in I_0.
\]
We note the following properties of maximal smooth fuzzy t-centred system.

1. If $\lambda_i \in p_t (i = 1, 2, 3 ... n)$, then $\bigwedge_{i=1}^{n} \lambda_i \in p_t$.  

Proof:

If $\lambda_i \in p_t (i = 1, 2, 3...n)$, then $\lambda_i \neq \lambda_j$ for $i \neq j$. If $\bigwedge_{i=1}^{n} \lambda_i \notin p_t$ then $p_t \cup \{ \bigwedge_{i=1}^{n} \lambda_i \}$ will be a larger smooth fuzzy t-end than $p_t$. This contradicts the maximality of $p_t$. Therefore, $\bigwedge_{i=1}^{n} \lambda_i \in p_t$.

2. If $0 < \lambda < \mu$, $\lambda \in p_t$ and $\mu$ is a r-fuzzy t-open set, $r \in I_0$, then $\mu \in p_t$.

Proof:

If $\mu \notin p_t$, then $p_t \cup \{ \mu \}$ will be a larger smooth fuzzy t-end than $p_t$. This contradicts the maximality of $p_t$. Therefore, $\mu \in p_t$.

3. If $\lambda$ is a r-fuzzy t-open set, $r \in I_0$, then $\lambda \notin p_t$ iff there exists $\mu \in p_t$ such that $\lambda \not\subseteq \mu$.

Proof:

Let $\lambda \notin p_t$ be a r-fuzzy t-open set, $r \in I_0$. If there exists no $\mu \in p_t$ such that $\lambda \not\subseteq \mu$, then $\lambda \not\subseteq \mu$ for all $\mu \in p_t$. That is, $p_t \cup \{ \lambda \}$ will be a larger smooth fuzzy t-end than $p_t$. This contradicts the maximality of $p_t$.

Conversely, suppose that there exists $\mu \in p_t$ such that $\lambda \not\subseteq \mu$. If $\lambda \in p_t$, then $\lambda \not\subseteq \mu$. Contradiction. Hence, $\lambda \notin p_t$. 

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4. If \( \lambda_1 \lor \lambda_2 = \lambda_3 \in p_t \), \( \lambda_1 \) and \( \lambda_2 \) are r-fuzzy t-open sets in \( R \), \( r \in I_0 \),
with \( \lambda_1 \lor \lambda_2 \), then either \( \lambda_1 \in p_t \) or \( \lambda_2 \in p_t \).

**Proof:**

Let us suppose that both \( \lambda_1 \in p_t \) and \( \lambda_2 \in p_t \). Then, \( \lambda_1 \lor \lambda_2 \).

Contradiction. Hence, either \( \lambda_1 \in p_t \) or \( \lambda_2 \in p_t \).

**Note 3.1.1.**

Every smooth fuzzy t-centred system can be extended in at least one way to a maximum one.

**3.2 THE SMOOTH FUZZY MAXIMAL STRUCTURE IN \( \theta ( R ) \).**

In this section, smooth fuzzy maximal structure in the collection of all smooth fuzzy t-ends \( \theta ( R ) \) is introduced and its properties are investigated.

Let \( \theta ( R ) \) denote the collection of all smooth fuzzy t-ends belonging to \( R \). We introduce a smooth fuzzy maximal structure in \( \theta ( R ) \) in the following way: Let \( P_{\lambda} \) be the set of all smooth fuzzy t-ends that include \( \lambda \) as an element, where \( \lambda \) is a r-fuzzy t-open set of \( R \), \( r \in I_0 \). Now, \( P_{\lambda} \) is a smooth fuzzy Q*-t-neighborhood structure of each smooth fuzzy t-end contained in \( P_{\lambda} \). Thus, to each r-fuzzy t-open set \( \lambda \) of \( R \) there corresponds a smooth fuzzy Q*-t-neighborhood structure \( P_{\lambda} \) in \( \theta ( R ) \).

**Proposition 3.2.1.**

If \( \lambda \) and \( \mu \) are r-fuzzy t-open sets, \( r \in I_0 \), then
(a) \( P_{\lambda\mu} = P_\lambda \cup P_\mu \).

(b) \( P_\lambda \cup P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} = \emptyset (R) \).

**Proof:**

(a). Let \( p_t \in P_\lambda \). That is, \( \lambda \in p_t, r \in I_0 \). Then by Property 2., \( \lambda \vee \mu \in p_t \).

That is, \( p_t \in P_{\lambda\mu} \). Hence, \( P_\lambda \cup P_\mu \subseteq P_{\lambda\mu} \). Let \( p_t \in P_{\lambda\mu} \). That is, \( \lambda \vee \mu \in p_t \).

By the definition of \( P_\lambda \), \( \lambda \in p_t \) or \( \mu \in p_t \). That is, \( p_t \in P_\lambda \) or \( p_t \in P_\mu \),

therefore, \( p_t \in P_\lambda \cup P_\mu \). This shows that \( P_\lambda \cup P_\mu \supseteq P_{\lambda\mu} \). Hence,

\[ P_{\lambda\mu} = P_\lambda \cup P_\mu. \]

(b). If \( p_t \notin P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} \), then \( \Gamma^{-t-C_{t(R)}(\lambda, r)} \notin p_t \). That is, \( \lambda \in p_t \) and \( p_t \in P_\lambda, r \in I_0 \). Hence, \( \emptyset (R) - P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} \subset P_\lambda \). If \( p_t \in P_\lambda \), then \( \lambda \in p_t \).

That is, \( \Gamma^{-t-C_{t(R)}(\lambda, r)} \notin p_t, p_t \notin P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} \). Therefore,

\[ p_t \notin \emptyset (R) - P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}}. \]

That is, \( P_\lambda \subset \emptyset (R) - P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} \). Hence, \( P_\lambda \cup P_{\Gamma^{-t-C_{t(R)}(\lambda, r)}} = \emptyset (R) \).

**Definition 3.2.1.**

\( \emptyset (R) \) with the smooth fuzzy maximal structure is said to be smooth fuzzy \( t \)-compact if every covering of \( \emptyset (R) \) of the form \( P_{\lambda\alpha} \),

where each \( P_{\lambda\alpha} \) is \( r \)-fuzzy \( t \)-open, \( r \in I_0 \), has a finite subcover.

**Proposition 3.2.2.**

\( \emptyset (R) \) with the smooth fuzzy maximal structure described above
is a smooth fuzzy t-compact space and has a base of smooth fuzzy $Q^*t$-neighborhood structure $\{ P_\lambda \}$ that are both $r$-fuzzy t-open and $r$-fuzzy t-closed, $r \in I_0$.

**Proof:**

Each $P_\lambda$ in $\theta (R)$ is $r$-fuzzy t-open, by definition and by (b) of Proposition 3.2.1., it follows that $P_\lambda$ is $r$-fuzzy t-closed. Thus $\theta (R)$ has a base of smooth fuzzy $Q^*t$-neighborhood structure $\{ P_\lambda \}$ that are both $r$-fuzzy t-open and $r$-fuzzy t-closed, $r \in I_0$. We now show that $\theta (R)$ is smooth fuzzy t-compact. Let $\{ P_{\lambda_\alpha} \}$ be a covering of $\theta (R)$ where each $P_{\lambda_\alpha}$ is $r$-fuzzy t-open. If it is impossible to pick a finite subcovering from the covering, then no set of the form $\tilde{1} - \bigvee_{i=1}^{n} t-C_{T(\theta(R))} (\lambda_{\alpha_i}, r)$ is $\tilde{0}$, since otherwise the sets $P_{\lambda_\alpha}$ would form a finite covering of $\theta (R)$. Hence, the $r$-fuzzy t-open sets $\tilde{1} - \bigvee_{i=1}^{n} t-C_{T(\theta(R))} (\lambda_{\alpha_i}, r)$ form a smooth fuzzy t-centred system. It may be extended to a maximal smooth fuzzy t-centred system $p_\alpha$. This maximal smooth fuzzy t-centred system is not contained in any $\{ P_{\lambda_\alpha} \}$ since it contains, in particular, all the $\tilde{1} - t-C_{T(\theta(R))} (\lambda_{\alpha_i}, r)$. This contradiction proves that $\theta (R)$ is smooth fuzzy t-compact.
3.3 THE ABSOLUTE $\omega ( R )$ OF A SMOOTH FUZZY TOPOLOGICAL
SPACE R.

In this section, smooth fuzzy t-absolute $\omega ( R )$ of R is defined and its properties are studied.

The maximal smooth fuzzy t-centred system of r-fuzzy t-open sets, $r \in I_0$ of R regarded as elements of the space $\theta ( R )$, fall into two classes, those smooth fuzzy t-ends each of which contain all r-fuzzy t-open sets containing a fuzzy point of R and the smooth fuzzy t-ends not containing such smooth fuzzy system of r-fuzzy t-open sets. The space of all smooth fuzzy t-ends of the first type of $\theta ( R )$ is called the smooth fuzzy t-absolute of R and is denoted by $\omega ( R )$. In $\omega ( R )$ each fuzzy point $\alpha$ of R is represented by smooth fuzzy t-ends containing all r-fuzzy t-open sets containing $\alpha$. Now, $\omega ( R ) = \cup \{ \lambda ( \alpha ) / \alpha$ is a fuzzy point of R, where $\lambda ( \alpha )$ denotes the set of all smooth fuzzy t-ends containing all r-fuzzy t-open sets containing $\alpha$, $r \in I_0 \}$. The smooth fuzzy t-absolute space $\omega ( R )$ is functioned in a natural way onto R. If $p \in \omega ( R )$, then we define $\pi_R ( p ) = \alpha$, where $\alpha$ is the fuzzy point such that all r-fuzzy t-open sets containing $\alpha$ belongs to $p$. Now, $\pi_R$ is called smooth fuzzy natural function of $\omega ( R )$ onto R.

**Definition 3.3.1.**

Let $R_1$ and $R_2$ be any two smooth fuzzy t-Hausdorff spaces. A function $f : R_1 \rightarrow R_2$ is called a smooth fuzzy t-irreducible* function if
there is no proper r-fuzzy t-closed set \( \lambda \) of \( R_1 \), \( r \in I_0 \), such that \( f(\lambda) = \bar{I} \).

**Definition 3.3.2.**

Let \( R_1 \) and \( R_2 \) be any two smooth fuzzy t-Hausdorff spaces. A function \( f: R_1 \to R_2 \) is called a smooth fuzzy t-perfect function if the image of a r-fuzzy t-closed set is r-fuzzy t-closed, \( r \in I_0 \) and the inverse image of each fuzzy point is smooth fuzzy t-compact.

**Definition 3.3.3.**

Let \( R_1 \) and \( R_2 \) be any two smooth fuzzy t-Hausdorff spaces. A function \( f: R_1 \to R_2 \) is called a smooth fuzzy t-compact function if the inverse image of each \( \lambda \) is smooth fuzzy t-compact.

**Proposition 3.3.1.**

The natural function \( \pi_R \) of \( \omega ( R ) \) onto \( R \) is smooth fuzzy t-irreducible* and smooth fuzzy t-compact.

**Proof:**

Let \( \beta \) be a fuzzy point of \( R \). If \( \pi_R ( p_t ) = \beta \), \( \pi_R^{-1} ( \beta ) \) is a set of all smooth fuzzy t-ends \( p_t \) which contain all r-fuzzy t-open sets, \( r \in I_0 \), containing \( \beta \). Since \( \theta ( R ) \) has a base of smooth fuzzy \( Q^*t \)-neighborhood structure \( \{ P_{\lambda} \} \) that are both r-fuzzy t-open and r-fuzzy t-closed, \( \pi_R^{-1} ( \beta ) \) is a r-fuzzy t-closed set in \( \theta ( R ) \). Since \( \theta ( R ) \) is smooth fuzzy t-compact, \( \pi_R^{-1} ( \beta ) \) is smooth fuzzy t-compact. Therefore, \( \pi_R \) is smooth fuzzy t-compact.
To Prove $\pi_R$ is smooth fuzzy $t$-irreducible* it is enough to show that every $r$-fuzzy $t$-open set in $\omega ( R )$ contains whole of some set $\pi_R^{-1} ( \beta )$, where $\beta$ is a fuzzy point of $R$. But this follows, because each $P_\lambda$ contains the whole of $\pi_R^{-1} ( \beta )$, where $\beta \leq \lambda$, and because $\{ P_\lambda \}$ is a smooth fuzzy $Q^*t$-neighborhood structure in $\theta ( R )$.

**Proposition 3.3.2.**

If $f$ is a smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-closed function of $R_1$ onto $R_2$ then the image of every $r$-fuzzy $t$-open set $\lambda \neq \bar{0}$ in $R_1$ is a $r$-fuzzy $t$-open set in $R_2$ with $f(\lambda) \neq \bar{0}$, $r \in I_0$.

**Proof:**

Let $\lambda$ be a $r$-fuzzy $t$-open set with $\lambda \neq \bar{0}$, $\lambda \in R_1$, $r \in I_0$. Since $f$ is a smooth fuzzy $t$-closed function, $f(\bar{1} - \lambda)$ is also a $r$-fuzzy $t$-closed set. Since $f$ is onto, $f(\bar{1} - \lambda) = \bar{1} - f(\lambda)$. Therefore, $f(\lambda)$ is a $r$-fuzzy $t$-open set, $f(\lambda) \in R_2$. Since $f$ is smooth fuzzy $t$-irreducible*, $f(\bar{1} - \lambda) \neq \bar{1}$. That is, $\bar{1} - f(\lambda) \neq \bar{1} \Rightarrow f(\lambda) \neq \bar{0}$.

**Notation:**

$t$-$\text{Int}_t(\lambda, r)$ denotes the $t$-interior of $\lambda$ throughout this chapter.

**Proposition 3.3.3.**

If $f$ is a smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-closed function of $R_1$ onto $R_2$, $\text{Int}_{\text{t}[R_1]}(f^{-1}(\lambda), r) \neq \bar{0}$ for every $r$-fuzzy $t$-open set $\lambda \neq \bar{0}$, $\lambda \in R_2$, $r \in I_0$. 

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Proof:

Since \( f \) is a smooth fuzzy \( t \)-closed and smooth fuzzy \( t \)-irreducible* function, \( f(\bar{1} - \text{Int}_{\tau(R_1)}(f^{-1}(\lambda), r)) \neq \bar{1}, \ r \in I_0 \). Since \( f \) is onto, \( f(\text{Int}_{\tau(R_1)}(f^{-1}(\lambda), r)) \neq \emptyset \). By Proposition 3.3.2., it follows that \( \text{Int}_{\tau(R_1)}(f^{-1}(\lambda), r) \neq \emptyset \).

3.4 THE FUNDAMENTAL THEOREM ON SMOOTH FUZZY \( t \)-IRREDUCIBLE* AND SMOOTH FUZZY \( t \)-PERFECT FUNCTION.

In this section, the fundamental theorem on smooth fuzzy \( t \)-irreducible* and smooth fuzzy \( t \)-perfect functions are introduced.

Theorem 3.4.1.

Let \( R_1 \) and \( R_2 \) be any two smooth fuzzy \( t \)-Hausdorff spaces. Let \( f \) be a smooth fuzzy \( t \)-continuous, smooth fuzzy \( t \)-irreducible* and smooth fuzzy \( t \)-perfect function of \( R_1 \) onto \( R_2 \). Then, there exists a smooth fuzzy \( t \)-homeomorphism \( \psi \) of \( \omega(R_1) \) onto \( \omega(R_2) \) such that \( f \circ \pi_{R_1} = \pi_{R_2} \circ \psi \).

\[
\begin{array}{ccc}
R_1 & \xrightarrow{f} & R_2 \\
\pi_{R_1} & & \pi_{R_2} \\
\omega(R_1) & \xrightarrow{} & \omega(R_2)
\end{array}
\]
**Proof:**

Let \( \{ \lambda \} \) be a maximal smooth fuzzy \( t \)-centred system of \( r \)-fuzzy \( t \)-open sets of \( R_1 \), \( r \in I_0 \). In \( R_2 \), consider the system \( \{ t\text{-Int}_{T(R_2)}(f(\lambda), r) \} \), where \( t\text{-Int}_{T(R_2)}(f(\lambda), r) \) is a \( r \)-fuzzy \( t \)-open set, by Proposition 3.3.2., each of its sets is non-zero, \( r \in I_0 \). Clearly, the system is smooth fuzzy \( t \)-centred. Extend it to a maximal smooth fuzzy \( t \)-centred system of \( r \)-fuzzy \( t \)-open sets in \( R_2 \) and prove that this extension is unique.

Suppose that there exist two \( r \)-fuzzy \( t \)-open sets \( \lambda_1, \lambda_2 \in R_2 \) with \( \lambda_1 \not\subseteq \lambda_2 \), such that \( \lambda_1 \not\subseteq t\text{-Int}_{T(R_2)}(f(\lambda), r) \) and \( \lambda_2 \not\subseteq t\text{-Int}_{T(R_2)}(f(\lambda), r) \) for every \( \lambda \) in \( \{ \lambda \} \). Now, \( t\text{-Int}_{T(R_1)}(f^{-1}(\lambda_1), r) \not\subseteq t\text{-Int}_{T(R_1)}(f^{-1}(\lambda_2), r) \) implies, \( t\text{-Int}_{T(R_1)}(f^{-1}(\lambda_1), r) \not\subseteq \lambda \) and \( t\text{-Int}_{T(R_1)}(f^{-1}(\lambda_2), r) \not\subseteq \lambda \). But this is impossible, because \( \{ \lambda \} \) is maximal smooth fuzzy \( t \)-centred system. Thus \( \{ t\text{-Int}_{T(R_2)}(f(\lambda), r) \} \) can be extended in only one way to a maximal smooth fuzzy \( t \)-centred system \( \{ y_1 \} \) where \( y_i \) is a \( r \)-fuzzy \( t \)-open set, \( r \in I_0 \).

Assume that \( \{ \lambda \} \) contains all \( r \)-fuzzy \( t \)-open sets, \( r \in I_0 \), containing the fuzzy point \( \alpha \) of \( R_1 \) and show that \( \{ y \} \) contains all \( r \)-fuzzy \( t \)-open sets containing the fuzzy point \( \beta \) of \( R_2 \) such that \( \beta = f(\alpha) \). Let \( \delta_\beta \) be a \( r \)-fuzzy \( t \)-open set containing the fuzzy point \( \beta \). Because \( f \) is a smooth fuzzy \( t \)-irreducible* and smooth fuzzy \( t \)-closed
function, \( t-\text{Int}_{T(R_1)}(f^{-1}(\delta_\beta), r) \) is a r-fuzzy t-open set containing the fuzzy point \( \alpha \), \( t-\text{Int}_{T(R_1)}(f^{-1}(\delta_\beta), r) \in \{\lambda\} \).

The set \( t-\text{Int}_{T(R_2)}(f(t-\text{Int}_{T(R_1)}(f^{-1}(\delta_\beta), r), r)) \leq \delta_\beta \) and belongs to \( \{\gamma\} \). Hence, \( q = \{\gamma\} \) is a point of \( \omega(R_2) \). Let \( \psi(p) = q \), to show that \( \psi \) is a function of \( \omega(R_1) \) onto \( \omega(R_2) \). Let \( q = \{\gamma\} \in \omega(R_2) \).

Consider the system \( \{t-\text{Int}_{T(R_1)}(f^{-1}(\gamma), r)\} \) of r-fuzzy t-open sets in \( R_1 \). The system is smooth fuzzy t-centred. We extend it to a maximal smooth fuzzy t-centred system of r-fuzzy t-open sets \( \{\lambda\} \) and consider the point \( \psi(p) \). As we show that, \( t-\text{Int}_{T(R_2)}(f(\lambda), r) \) may be extended in a unique way to a maximal system \( \{\gamma_1\} \).

To show that \( \psi(p) = q \), it is sufficient to show that \( \{\gamma\} \subseteq \{\gamma_1\} \) and for this, it is enough to show that \( \gamma \in \{\gamma\} \) \( t-\text{Int}_{T(R_2)}(f(\lambda), r) \) for each \( \gamma \in \{\gamma\} \) and each \( t-\text{Int}_{T(R_2)}(f(\lambda), r) \in \{t-\text{Int}_{T(R_2)}(f(\lambda), r)\} \).

Clearly, \( \lambda \in \{\gamma\} \) \( t-\text{Int}_{T(R_1)}(f^{-1}(\gamma), r) \). That is, \( t-\text{Int}_{T(R_2)}(f(\lambda), r) \backslash q \gamma \).

Therefore, \( \psi(p) = q \). \( \psi \) is onto. The function \( \psi \) is one to one. For if, \( p_1 \neq p_2 \) then there exist r-fuzzy t-open sets \( \lambda_1 \) and \( \lambda_2 \), \( \lambda_1 \in p_1 \) and \( \lambda_2 \in p_2 \) such that \( \lambda_1 \notin \lambda_2 \), but then \( f(\lambda_1) \notin f(\lambda_2) \). That is,

\[
t-\text{Int}_{T(R_1)}(f(\lambda_1), r) \backslash t-\text{Int}_{T(R_2)}(f(\lambda_2), r).
\]

Hence, \( \psi(p_1) \neq \psi(p_2) \). The function \( \psi \) is one - one of \( \theta(R_1) \) into \( \theta(R_2) \).
taking $\omega ( R_1 )$ onto $\omega ( R_2 )$. To prove that $\psi$ is a smooth fuzzy $t$-homeomorphism, it is enough to prove that $\psi$ is a smooth fuzzy $t$-continuous because $\theta ( R_1 )$ is smooth fuzzy $t$-compact. Let $p' = \{ \lambda \}$ be an arbitrary smooth fuzzy $t$-end in $R_1$, that is an element of $\theta ( R_1 )$ and let $q' = \psi ( p' ) = \{ \gamma \}$. Now, $\psi ( p_\lambda ) \subset p_\gamma = P_{t-\text{Int}_{\tau_2}} ( f ( \lambda ), r )$. If $p'' \in p_\lambda$, then $\lambda \in p''$. Now, $t-\text{Int}_{\tau_2} ( f ( \lambda ), r ) \in \psi ( p'' )$ which means that

$$\psi ( p'' ) \subset P_{t-\text{Int}_{\tau_2}} ( f ( \lambda ), r ).$$

This proves that $\psi$ is a smooth fuzzy $t$-homeomorphism. To prove the theorem we have to show that $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$. Consider the function $\psi$ only on $\omega ( R_1 ) \subset \theta ( R_1 )$. From the construction of $\psi$ it follows that every smooth fuzzy $t$-end containing all $r$-fuzzy $t$-open sets containing $a$ is functioned by $\psi$ into a smooth fuzzy $t$-end with $r$-fuzzy $t$-open sets containing fuzzy point $\beta$. $\psi ( \pi_{R_1}^{-1} ( a ) ) \subset \pi_{R_2}^{-1} ( \beta )$. Hence, $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$. Thus the theorem proved.

**Corollary 3.4.1.**

The smooth fuzzy $t$-absolute of $R_1$ and $R_2$ are smooth fuzzy $t$-homeomorphic if there exists a smooth fuzzy topological space $R$ such that $R$ can be functioned onto both $R_1$ and $R_2$ by smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-perfect function.
Proof:

Let $f_1$ be a smooth fuzzy t-irreducible* and smooth fuzzy t-perfect function from $\mathbb{R}$ onto $\mathbb{R}_1$ and let $f_2$ be smooth fuzzy t-irreducible* and smooth fuzzy t-perfect function from $\mathbb{R}$ into $\mathbb{R}_2$. By theorem 3.4.1., there exists a smooth fuzzy t-homeomorphism $\psi_1$ of $\omega(\mathbb{R})$ onto $\omega(\mathbb{R}_1)$ such that $f_1 \circ \pi_\mathbb{R} = \pi_{\mathbb{R}_1} \circ \psi_1$ and there exists a smooth fuzzy t-homeomorphism $\psi_2$ of $\omega(\mathbb{R})$ onto $\omega(\mathbb{R}_2)$ such that $f_2 \circ \pi_\mathbb{R} = \pi_{\mathbb{R}_1} \circ \psi_2$. Therefore, $\omega(\mathbb{R}_1)$ and $\omega(\mathbb{R}_2)$ are smooth fuzzy t-homeomorphic.