CHAPTER - I

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1.1 FUZZY SETS.

Most of our traditional tools modeling, reasoning and computing are crisp, deterministic and precise in character. By crisp we mean dichotomous, that is, yes-or-no type rather than more-or-less type. Certainty eventually indicates that we assume the structures and parameters of the model to be definitely known and that there are no doubts about their occurrence.

For factual model or modeling languages two major complications arise:

1) Real situations are very often not crisp and deterministic and they cannot be described precisely.

2) The complete description of a real system often would require by far more detailed data than a human being could ever recognize, process and understand simultaneously.

In 1923, the philosopher Russell [60] referred to the first point when he wrote: “All traditional logic habitually assumes that precise
symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence”.

Zadeh [79] referred to the second point when he wrote: “As the complexity of a system increases, our ability to make precise and yet significant statement about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics”.

Among the various paradigmatic changes in science and mathematics in this century, one such change concerns the concept of uncertainty. In science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable in science. Hence, uncertainty is not only an unavoidable plague, but it has, in fact, a great utility.

Uncertainty is thus an important commodity in the modeling business, which can be traded for gains in the other essential characteristics of models. A recognition of this important role of uncertainty by some researchers, which became quite explicit in the literature of the 1960s, began the second stage of the transition from traditional view to the modern view of uncertainty. This stage is characterized by the emergence of several new theories of uncertainty, distinct from probability theory. These theories challenge the seemingly unique connection between uncertainty and probability theory.
It is generally agreed that an important point in the evolution of the modern concept of uncertainty was the publication of a seminal paper by Lotfi A. Zadeh in 1965. In his paper, he introduced a theory whose objects - fuzzy sets - are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree.

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. Fuzziness can be found in many areas of daily life, such as in Engineering (Blockley [16]), in Medicine (Vila and Delgado [73]), in Meteorology (Cao and Chen [19]) and others. It is particularly frequent, however, the meaning of a word might even be well defined but, when using the word as a label for a set, the boundaries within which objects do or do not belong to the set become fuzzy or vague.

Fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied.
Fuzziness has so far not been defined uniquely semantically and probably never will. It will mean different things, depending on the application area and the way it is measured. In the meantime, numerous authors have contributed to this theory. Fuzzy sets are having useful and interesting applications in various fields including Probability theory, Information theory [63], control [67] and optimization techniques [15].

Sostak [64] introduced a fuzzy topology as an extension of Chang's fuzzy topology [22]. It has been developed in many directions [46, 47, 48, 29, 33, 68, 64, 65]. The concept of fuzzy $\beta$-open sets was introduced by Abd.El-Monsef et al [1] and also discussed by Allam and Hakkim [2]. The concept of fuzzy t-set was introduced by Uma, Roja and Balasubramanian [71].

1.2 REVIEW OF LITERATURE.

The concept of a fuzzy set provides a natural framework for generalizing many of the concepts of a general topology. The theory of fuzzy topological space was introduced and developed by Chang [22]. Since then various notions in classical topology have been extended to fuzzy topological spaces by fuzzy topologists like Azad [5], Zadeh [79], Tomasz Kubiak [68], Tuna Hatice Yalvac [69], Brown [17], Goguen [29], Hutton and Reilly [31], Lowen [34-37], Rodabaugh [50-55], Warren [74,75], Friedler [26], Gantnet et al [28], Wong [76-78], Hohle et al [30].
In 1985, Sostak [64] introduced the fundamental concept of a fuzzy topological structures, as an extension of both classical topology and fuzzy topology, in the sense that not only objects are fuzzified, but also the axiomatic. In [65,66] Sostak gave some rules and showed how such an extension can be realized. In 1992, Ramadan [46] studied the concept of smooth fuzzy topological spaces. Chattopadhyay et al [23] have redefined the similar concept.

Various departures from the classical topology have also been observed in fuzzy topology. The method of centred system in the theory of topology was introduced in [32]. The above method is now extended to the case of fuzzy topological spaces by [72]. The concept of fuzzy hausdorff topological spaces was introduced by [49]. The concept of fuzzy compactness was found in [14]. Tietze extension theorem in L-fuzzy normal spaces was introduced by [68]. The concept of fuzzy normal spaces was introduced by Bruce Hutton [18]. The concept of Gδ-normal spaces was introduced by Roja, Uma and Balasubramanian [58].

The concept of bitopological spaces was introduced by Kandil [33]. Disconnectedness, compactness, etc., in fuzzy topology have been considered by Singal and Prakash [62]. Azad [4, 5], Mukherjee et al [38-40], Nanda [41], Bin Shahna [14], Balasubramanian [6-12], and Ganguly and Saha [27], using the concepts of fuzzy pre-open sets, fuzzy semi-open sets and fuzzy β-open sets. The concept of
$G_\delta$-connectedness and disconnectedness in fuzzy bitopological spaces was introduced by Roja, Uma and Balasubramanian [57]. The concept of fuzzy basically disconnected spaces was introduced and studied in [12].

The concepts of the upper and lower pre-irresolute multifunction was introduced by Popa, Kucuk and Noiri [44]. In 2009, Seenivasan and Balasubramanian [61] introduced upper and lower pre-irresolute fuzzy multifunctions. The concept of fuzzy $G_\delta$-continuity was introduced and studied by [59]. Dontchev [24] introduced the notion of the contra continuous mappings. The concept of fuzzy contra continuities was established by Erdal Ekici and Kerre [25].

The field of Mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Fuzzy topologists have introduced and investigated many different generalizations of fuzzy continuous functions. The concept of almost continuity for fuzzy functions has been discussed by Naseem Ajmal and Tyagi [42].

The concept of fuzzy separation axioms was investigated with the help of fuzzy $\beta$-open sets [11]. The concept of fuzzy $\beta$-$T_{1/2}$-space was introduced and studied by Roja and Balasubramanian [56]. The concept of $g$-interior, $g$-border and $g$-frontier were studied by Caldas, Jafari & Noiri [20].
1.3 OUTLINE OF THE THESIS.

The method of centred systems in the theory of topological spaces was introduced by Iliadis and Fomin[32]. In chapter II, the concept of maximal smooth fuzzy centred system, the smooth fuzzy space $\theta(\mathcal{R})$ and absolute $\omega(\mathcal{R})$ of a smooth fuzzy space $\mathcal{R}$ are introduced. The concept of fuzzy compactness was found in [14] and fuzzy Hausdorff space was found in [49]. Besides providing the preliminary results, the fundamental theorem on smooth fuzzy irreducible and smooth fuzzy perfect functions is also studied.

In chapter III, the concept of maximal smooth fuzzy $t$-centred system is introduced and studied based on the concept of method of centred systems in the theory of topology [32]. In 2007, the above concept was extended to fuzzy topological spaces by Uma, Roja and Balasubramanian [72]. In this chapter, $t$-absolute $\omega(\mathcal{R})$ is studied in the theory of smooth fuzzy topology. The fundamental theorem on smooth fuzzy $t$-irreducible and smooth fuzzy $t$-perfect function is also studied.

In chapter IV, the method of $\beta$-centred system is studied in the theory of smooth fuzzy topology. The concept of extremally $\beta$-disconnectedness in maximal structure $\theta(\mathcal{R})$ of maximal smooth fuzzy $\beta$-centred system is introduced and its properties are studied.

In 1989, Kandil [33] introduced the concept of fuzzy bitopological spaces and since then many concepts in classical
topology have been extended to fuzzy bitopological spaces. The concept of $G_δ$-connectedness and $G_δ$-disconnectedness in fuzzy bitopological spaces was introduced by Roja, Uma and Balasubramanian [57]. The purpose of chapter V is to introduce pairwise smooth fuzzy $t$-connected spaces, pairwise smooth fuzzy $t$-disconnected spaces and pairwise smooth fuzzy extremally $t$-disconnected spaces. The concept of pairwise smooth fuzzy basically $t$-disconnected spaces is also defined. Characterizations of the above spaces are given besides giving several examples. Interrelations among the spaces introduced are studied with relevant examples.

The concepts of the upper and lower pre-irresolute multifunction was introduced by Popa, Kucuk and Noiri [44]. In 2009, Seenivasan and Balasubramanian [61] introduced upper and lower pre-irresolute fuzzy multifunctions. In chapter VI, new classes of multifunction called upper $t$-irresolute and lower $t$-irresolute ( upper $t$-continuous, quasi $t$-continuous ) smooth fuzzy multifunction in smooth fuzzy topological spaces are introduced. In this chapter, the concept of smooth fuzzy contra $t$-continuity in the sense of Sostak [64] is introduced and some of its properties are also discussed with suitable examples. Also some properties concerning smooth fuzzy $t$-compactness, almost smooth fuzzy $t$-compactness and smooth fuzzy $t^*$-closed spaces are studied.
The purpose of this chapter VII is to introduce the concept of smooth fuzzy pre-$T_{1/2}$ space in the sense of Ramadan [47]. Generalization of smooth fuzzy continuous functions are introduced and studied by making use of $r$-fuzzy pre-open sets. Some interesting properties and interrelations among the concepts introduced are established. Many examples are given in connection with the above functions and spaces.

In chapter VIII, the concepts of $r$-generalized fuzzy $\beta$-border, $r$-generalized fuzzy $\beta$-exterior, $r$-generalized fuzzy $\beta$-frontier in the sense of Ramadan [47] are introduced. In this chapter, some interesting properties and interrelations among the concepts introduced are established with relevant examples. Also, some properties concerning generalized smooth fuzzy $\beta$-continuous functions and smooth fuzzy gc $\beta$-irresolute functions are studied.

**1.4 BASIC CONCEPTS IN SMOOTH FUZZY TOPOLOGICAL SPACES.**

In this section, some basic definitions like fuzzy continuous functions, fuzzy interior and fuzzy closure of a fuzzy set have been recalled. Also, related results, important theorems and propositions are collected from various research papers.

Throughout this thesis, let $X$ be a non-empty set, $I = [0, 1]$ and $I_0 = (0, 1]$. For $\in I$, $T(x) = $ for all $x \in X$. For $\alpha \in I$, $\bar{\alpha}(x) = \alpha$ for all $x \in X$. 

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A fuzzy set in $X$ is an element of the set $I^X$ of all functions from $X$ to $I$ [22].

**Definition 1.4.1. [64]**

A function $T : I^X \rightarrow I$ is called a smooth fuzzy topology on $X$ if it satisfies the following condition:

1) $T(\overline{0}) = T(\overline{1}) = 1$.

2) $T(\mu_1 \land \mu_2) \geq T(\mu_1) \land T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$.

3) $T(\bigvee_{i \in I} \mu_i) \geq \bigwedge_{i \in I} T(\mu_i)$ for any $\{\mu_i\}_{i \in I} \in I^X$.

The pair $(X, T)$ is called a smooth fuzzy topological space.

**Remark 1.4.1. [22]**

Let $(X, T)$ be a smooth fuzzy topological space. Then, for each $r \in I_0$, $T_r = \{\mu \in I^X : T(\mu) \geq r\}$ is Chang's fuzzy topology on $X$.

**Definition 1.4.2. [22]**

Let $\lambda$ and $\mu$ be any two fuzzy sets in $(X, T)$. Then $\lambda \lor \mu : X \rightarrow [0, 1]$ and $\lambda \land \mu : X \rightarrow [0, 1]$ are defined as follows:

1) $(\lambda \lor \mu)(x) = \max\{\lambda(x), \mu(x)\}$, for all $x \in X$.

2) $(\lambda \land \mu)(x) = \min\{\lambda(x), \mu(x)\}$, for all $x \in X$.

**Proposition 1.4.1. [23]**

Let $(X, T)$ be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $C_T : I^X \times I_0 \rightarrow I^X$ is defined as follows:

$$C_T(\lambda, r) = \bigwedge\{\mu : \mu \geq \lambda, T(\overline{1} - \mu) \geq r\}.$$
For \( \lambda, \mu \in \mathbb{I}^X \) and \( r, s \in I_0 \), it satisfies the following conditions:

1) \( C_T(\overline{0}, r) = \overline{0} \).

2) \( \lambda \leq C_T(\lambda, r) \).

3) \( C_T(\lambda, r) \lor C_T(\mu, r) = C_T(\lambda \lor \mu, r) \).

4) \( C_T(\lambda, r) \leq C_T(\lambda, s) \), if \( r \leq s \).

5) \( C_T(\overline{C_T(\lambda, r)}, r) = C_T(\lambda, r) \).

**Proposition 1.4.2. [23]**

Let \((X, T)\) be a smooth fuzzy topological space. For each \( \lambda \in \mathbb{I}^X \), \( r \in I_0 \), an operator \( I_T : \mathbb{I}^X \times I_0 \to \mathbb{I}^X \) is defined as follows:

\[
I_T(\lambda, r) = \bigvee \{ \mu : \mu \leq \lambda, T(\mu) \geq r \}.
\]

For \( \lambda, \mu \in \mathbb{I}^X \) and \( r, s \in I_0 \), it satisfies the following conditions:

1) \( I_T(\overline{1} - \lambda, r) = \overline{1} - C_T(\lambda, r) \).

2) \( I_T(\overline{1}, r) = \overline{1} \).

3) \( \lambda \geq I_T(\lambda, r) \).

4) \( I_T(\lambda, r) \land I_T(\mu, r) = I_T(\lambda \land \mu, r) \).

5) \( I_T(\lambda, r) \geq I_T(\lambda, s) \), if \( r \leq s \).

6) \( I_T(\overline{I_T(\lambda, r)}, r) = I_T(\lambda, r) \).

**Definition 1.4.4. [45]**

A fuzzy point \( x_t \) in \( X \) is a fuzzy set taking value \( t \in I_0 \) at \( x \) and zero elsewhere, \( x_t \in \lambda \) if and only if \( t \leq \lambda (x) \). A fuzzy set \( \lambda \) is
quasi-coincident with a fuzzy set $\mu$, denoted by $\lambda \sqcap \mu$, if there exists $x \in X$ such that $\lambda (x) + \mu (x) > 1$ Otherwise $\lambda \sqcap \mu$.

**Theorem 1.4.1.** \[22\]

Let $f$ be a function from $X$ to $Y$. Then

(a) $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ for any fuzzy set $\lambda$ in $Y$.

(b) $1 - f(\lambda) \leq f(1 - \lambda)$ for any fuzzy set $\lambda$ in $X$.

(c) $\mu_1 \leq \mu_2 \Rightarrow f^{-1}(\mu_1) \leq f^{-1}(\mu_2)$ where $\mu_1$ and $\mu_2$ are fuzzy sets in $Y$.

(d) $\lambda_1 \leq \lambda_2 \Rightarrow f(\lambda_1) \leq f(\lambda_2)$ where $\lambda_1$ and $\lambda_2$ are fuzzy sets in $X$.

(e) $f(f^{-1}(\mu)) \leq \mu$ for any fuzzy set $\mu$ in $Y$.

(f) $\lambda \leq f^{-1}(f(\mu))$ for any fuzzy set $\mu$ in $X$.

(g) Let $f$ be a function from $X$ to $Y$ and $g$ be a function from $Y$ to $Z$.

Then $(g \circ f)^{-1}(\eta) = f^{-1}(g^{-1}(\eta))$ for any fuzzy set $\eta$ in $Z$ where $g \circ f$ is the composition of $g$ and $f$.

**Definition 1.4.6.** \[47\]

Let $(X, T)$ and $(Y, S)$ be any two smooth fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be a mapping. Then

1) $f$ is called fuzzy continuous iff $S(\mu) \leq T(f^{-1}(\mu))$ for each $\mu \in I^Y$.

2) $f$ is called fuzzy open iff $T(\lambda) \leq S(f(\lambda))$ for each $\lambda \in I^X$.

3) $f$ is called fuzzy closed iff $T(I - \lambda) \leq S(I - f(\lambda))$ for each $\lambda \in I^X$.

**Definition 1.4.7.** \[3\]

Let $(X, T)$ be a smooth fuzzy topological space. For $\lambda \in I^X$, $r \in I_0$,

$\lambda$ is called
1) an r-fuzzy $G_\delta$-set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in I^X$ is such that $T(\lambda_i) \geq r$.

2) an r-fuzzy $F_\sigma$-set if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where each $(\bar{1} - \lambda_i) \in I^X$ is such that $T(\bar{1} - \lambda_i) \geq r$.

**Definition 1.4.8. [3]**

Let $(X, T)$ be a smooth fuzzy topological space. Its $Q^*$-neighborhood structure is a mapping $Q^*: X \times I^X \rightarrow I$ ($X$ denotes the totality of all fuzzy points in $X$), defined by

$$Q^*(x_0, \lambda) = \sup \{ \mu : \mu \text{ is a r-fuzzy } G_\delta\text{-set, } \mu \leq \lambda, x_0^i \in \mu, r \in I_0 \}$$

and

$$\lambda = \inf_{x_0^i \in x_0} Q^*(x_0^i, \lambda) \text{ is a r-fuzzy } G_\delta\text{-set.}$$

**Definition 1.4.9. [72]**

Let $R$ be a fuzzy Hausdorff space. A system $p = \{ X \}$ of fuzzy open sets of $R$ is called fuzzy centred if any finite collection of fuzzy sets of the system has a non-zero intersection. The system $p$ is called maximal fuzzy system or a fuzzy end if it cannot be included in any larger fuzzy centred system of fuzzy open sets.

**Definition 1.4.10. [72]**

Let $\theta (R)$ denote the collection of all fuzzy ends belonging to a Hausdorff space $R$. We introduce a fuzzy topology in $\theta (R)$ in the following way: Let $P_\lambda$ be the set of all fuzzy ends that include $\lambda$ as an element, where $\lambda$ is a fuzzy open set of $R$. Now $P_\lambda$ is a fuzzy
neighborhood of each fuzzy end contained in $P_{\lambda}$. Thus, to each fuzzy open set $\lambda$ of $R$, there corresponds a fuzzy neighborhood $P_{\lambda}$ in $\emptyset (R)$.

**Definition 1.4.11.** [72]

A fuzzy Hausdorff space $R$ is extremally disconnected if the closure of an open set is open.

**Definition 1.4.12.** [28]

The L-fuzzy real line $R(L)$ is the set of all monotone decreasing elements $\lambda \in L^R$ satisfying $\lor \{ \lambda (t) : t \in R \} = 1$ and $\land \{ \lambda (t) : t \in R \} = 0$, after the identification of $\lambda$, $\mu \in L^R$ iff $\lambda (t-) = \mu (t-)$ and $\lambda (t+) = \mu (t+)$ for all $t \in R$, where $\lambda (t-) = \land \{ \lambda (s) : s < t \}$ and $\lambda (t+) = \lor \{ \lambda (s) : s > t \}$. The natural L-fuzzy topology on $R(L)$ is generated from the sub basis $\{ L_t, R_t : t \in R \}$ where $L_t(\lambda) = \lambda (t-)'$ and $R_t(\lambda) = \lambda (t+)$. 

**Definition 1.4.13.** [31]

The L-fuzzy unit interval $I(L)$ (Hutton [18]) is a subset of $R(L)$ such that $[\lambda] \in I(L)$ if $\lambda (t) = 1$ for $t < 0$ and $\lambda (t) = 0$ for $t > 1$.

**Definition 1.4.14.** [44]

Let $(X, \tau)$ be a topological space in the classical sense and $(Y, S)$ be an fuzzy topological space. $F : X \rightarrow Y$ is called a fuzzy multifunction if and only if for each $x \in X$, $F(x)$ is a fuzzy set in $Y$.
Definition 1.4.15. [43]

For a fuzzy multifunction \( F : X \rightarrow Y \), the upper inverse \( F^+(\lambda) \) and lower inverse \( F^-(\lambda) \) of a fuzzy set \( \lambda \) in \( Y \) are defined as follows:

\[
F^+(\lambda) = \{ x \in X / F(x) \leq \lambda \} \quad \text{and} \quad F^-(\lambda) = \{ x \in X / F(x) \geq \lambda \}.
\]

Lemma 1.4.1. [43]

For a fuzzy multifunction \( F : X \rightarrow Y \), we have \( F(1-\lambda) = X - F^+(\lambda) \), for any fuzzy set \( \lambda \) in \( Y \).

Definition 1.4.16. [61]

A fuzzy multifunction \( F : (X, \tau) \rightarrow (Y, S) \) is said to be

1) fuzzy upper pre-irresolute (briefly f.u.p.i) at a point \( x \in X \), if for each \( \lambda \in \text{FPO}(Y) \) containing \( F(x) \) (i.e., \( F(x) \leq \lambda \)), there exists \( U \in \text{PO}(X, x) \) such that \( F(U) \leq \lambda \).

2) fuzzy lower pre-irresolute (briefly f.l.p.i) at a point \( x \in X \), if for each \( \lambda \in \text{FPO}(Y) \) with \( F(x) \geq \lambda \), there exists \( U \in \text{PO}(X, x) \) such that \( U \subseteq F^-(\lambda) \).

3) fuzzy upper pre-irresolute (fuzzy lower pre-irresolute) if it has the fuzzy upper pre irresolute (fuzzy lower pre irresolute) property at each point \( x \in X \).

Definition 1.4.17. [13, 5, 7, 71]

Let \( (X, T) \) be a topological space on fuzzy sets. A fuzzy set \( \lambda \) of \( (X, T) \) is said to be

1) fuzzy t-set if \( \text{int} \lambda = \text{int} \text{cl} \lambda \).
2) fuzzy pre-open if $\lambda \leq \text{int cl } \lambda$.
3) fuzzy regular-open if $\lambda = \text{int cl } \lambda$.
4) fuzzy $\beta$-open if $\lambda \leq \text{cl int cl } \lambda$.

**Definition 1.4.18. [21]**

A fuzzy bitopological space $(X, T_1, T_2)$ is said to be pairwise fuzzy extremally disconnected if $T_1$-closure of each $T_2$-fuzzy open set is $T_2$-fuzzy open and $T_2$-closure of each $T_1$-fuzzy open set is $T_1$-fuzzy open.

**Definition 1.4.19. [49]**

A fuzzy topological space $(X, Y)$ is said to be fuzzy Hausdroff iff for any two distinct fuzzy points $p, q \in X$, there exists $U, V \in T$ such that $U \cap V = \emptyset$ with $p \in U$ and $q \in V$.

**Definition 1.4.20. [12]**

A fuzzy bitopological space $(X, T_1, T_2)$ is said to be pairwise fuzzy basically disconnected if $T_1$-closure of each $T_2$-fuzzy open, $T_2$-fuzzy $F_\delta$ is $T_2$-fuzzy open and $T_2$-closure of each $T_1$-fuzzy open, $T_1$ fuzzy $F_\delta$ is $T_1$-fuzzy open.

**Definition 1.4.21. [20]**

Let $X$ be a topological space and $A$ be a subset of $X$. Then, $\text{Fr}_g (A) = \text{Cl}_g (A) \setminus \text{Int}_g (A)$ is said to be the $g$-frontier of $A$.

**Definition 1.4.22. [20]**

Let $X$ be a topological space and $A$ be a subset of $X$. Then,
Ext\(_g\) (A) = Int\(_g\) (X \setminus A) is said to be the g-exterior of A.

**Definition 1.4.23.** [20]

Let X be a topological space and A be a subset of X. Then,

b\(_g\) (A) = A \setminus \text{Int}\(_g\) (A) is said to be the g-border of A.

**Definition 1.4.24.** [72]

Let \(R_1\) and \(R_2\) be any two fuzzy Hausdorff spaces. A mapping \(f : R_1 \rightarrow R_2\) is called fuzzy irreducible function if there is no proper fuzzy closed set \(\lambda\) of \(R_1\) such that \(f(\lambda) = 1_{R_1}\).

**Definition 1.4.25.** [65]

Let \((X, T)\) be an \((L, K)\)-fuzzy (pre)-fuzzy topological space. Its Q-neighbourhood structure is a function \(N : \mathcal{X} \times L^X \rightarrow I\) (\(\mathcal{X}\) denotes the totality of all L-fuzzy points in \(X\)), defined by

\[
N(p, U) = \sup \{ T(U) : U \leq V, V(x_0) > t \}
\]

and

\[
T(U) = \inf_{p \in U} N(p, U)
\]