CHAPTER - I

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1.1. FUZZY SETS

Among the various paradigmatic changes in Science and Technology in this century, one such change concerns the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view which insists that uncertainty is undesirable in Science and should be avoided by all means. According to the modern view, uncertainty is considered essential to Science; it is not only an unavoidable plague, but it has in fact the great utility.

The initial transition from the traditional view to the modern view began in the late 19th century. A recognition of the important role of uncertainty by some researchers, became quiet explicit in the literature of the 1960's. This stage is characterized by the emergence of several new theories of uncertainty distinct from probability theory.

The important point in the evolution of the modern concept of uncertainty was highlighted by Zadeh [92] in 1965. In his paper, Zadeh introduced a theory whose objects-fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree.
The capability of fuzzy sets to express gradual transition from membership to non-membership and vice-versa has a broad utility. It provides us not only with a meaningful and powerful presentation of the measurement uncertainties, but also with a meaningful representation of vague concepts expressed in natural language.

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing the grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is compatible with the concept represented in the fuzzy set. In general, the concept of a crisp set can be considered as a restricted case of the more general concept of a fuzzy set.

Research on the theory of fuzzy sets has been growing steadily since the inception of the theory in 1960's. Research on a broad variety of applications has also been very active and has produced results that are impressive. Fuzziness can be found in many areas of daily life such as Engineering [18], Medicine [87] and Meteorology [23]. Also, the applications of fuzzy sets in various fields include Information theory [76], Control [80], Pattern Recognition [42] and Optimization Techniques [17].

1.2 REVIEW OF LITERATURE

The concept of a fuzzy set provides a natural framework for generalizing many of the concepts of a general topology. The theory of fuzzy topological space was introduced and developed by Chang [24]. Since then various notions in classical topology have been extended to
fuzzy topological spaces by fuzzy topologists Azad [5], Zadeh [92], Tomasz Kubiak [83], Warren [88], Friedler [31], Tuna Hatice Yalvac [84], Brown [19], Goguen [35], Hutton [38], Wong [89-91], Rodabaugh [65-70], Lowen [45-48], Hohle et al. [37] and Gantner et al.[33]. Jun [41] introduced and studied the concepts of filters and ideals on WFI-algebras. He also established the topological aspects of filters in lattice implication algebras in [40].

Sostak began the study of fuzzy structures of the topological type, introducing the degree of openness for fuzzy sets [77] as an extension of both crisp topology and Chang's fuzzy topology [24]. In [78, 79], Sostak gave some rules and showed how such an extension can be realized and has developed the theory of fuzzy topological spaces. In 1992, Ramadan [60] studied the concept of smooth fuzzy topological spaces. Chattopadhyay et al. [25, 26] have redefined the similar concept. In [60, 61] Ramadan gave a similar definition namely “Smooth Fuzzy Topology” for Lattice $L = [0, 1]$.

The method of centred systems in the theory of topological spaces was introduced by Illiadis and Fomin [39]. The concept in fuzzy setting was introduced by Uma, Roja and Balasubramanian [85]. Fuzzy topologists have introduced and investigated many different generalizations of fuzzy continuous functions. The concepts of continuity, connectedness and disconnectedness in fuzzy topology have been considered by Azad [5], Mukherjee et al. [50,52,53], Nanda [54], Binshahna [15,16], Balasubramanian [6-13] and Ganguly and Saha [32] using the fuzzy semi open sets, fuzzy regular open sets and
fuzzy \( \beta \)-open sets. Azad [5] defined fuzzy regular open, fuzzy regular closed sets and studied almost fuzzy continuous and weakly fuzzy continuous functions in the fuzzy settings. The concept of almost continuity for fuzzy functions has been discussed by Naseem Ajmal and Tyagi [56]. The concept of strongly compact fuzzy topological spaces was studied by Nanda [55]. Almost compactness in fuzzy topological spaces was studied by Di Concilio et al. [27].

Defining the closure and the interior of a fuzzy set in fuzzy topological space, a different approach for the compactness structure of fuzzy topological space was introduced in [29]. In this connection, new definitions of smooth closure and smooth interior of a fuzzy set were studied by Riza Erturk and Mustafa Demirci [64].

The concept of fuzzy normal spaces was introduced by Bruce Hutton [20]. The concept of \( G_\delta \)-normal spaces was introduced by Roja, Uma and Balasubramanian [73]. Separation axioms such as pairwise \( T_i \) (\( i = 0, 1, 2, 3, 4 \)) spaces are studied by Ramadan et al. [61]. Tietze extension theorem was discussed by Tomasz Kubiak [83].

Kasahara [43] defined the concept of an operation on topological spaces. The concept of an operation \( \gamma \) on a family of semi open sets in a topological space is introduced in [74]. Caldas et al. [22] introduced and studied the topological properties of g-border, g-frontier and g-exterior of a set using the concept of g-open sets.

Caldas and Jafari [21] investigated some applications of b-open sets in topological spaces. Semi open sets and semi continuous functions were introduced by Masshour et al. [49]. Bin Shahna [15]
studied these concepts in the fuzzy setting using fuzzy sets. The
concept of fuzzy $\beta$-open sets was introduced by Abd. El Monsef et al.
[2] and also discussed by Allam and Hakkim [3]. This concept in fuzzy
setting was defined and studied by Balasubramanian [8]. Extremally
disconnectedness were defined and studied in [36]. Fuzzyfied form of
extremally disconnected spaces were defined and established by
Balasubramanian [7].

1.3 OUTLINE OF THE THESIS

This section presents a chapter wise summary of results
obtained on smooth fuzzy centred system, the absolute $\omega(R)$ of a
smooth fuzzy topological space, extremally and basically
disconnectedness in smooth fuzzy centred system, smooth fuzzy
contra $G_\delta$-continuous function, smooth fuzzy almost $G_\delta$-compactness,
smooth fuzzy $S$-closed spaces, smooth fuzzy $G_\delta$-regular and
$G_\delta$-normal spaces, smooth fuzzy $\beta$-continuous functions, smooth fuzzy
$\beta$-$T_{1/2}$ spaces, $r$-fuzzy semi (resp. $G_\delta$-$\gamma$-open set, $r$-fuzzy semi
(resp. $G_\delta$-$\gamma$-regular space, $r$-fuzzy semi (resp. $G_\delta$-$\gamma$-normal space,
$r$-fuzzy semi (resp. $G_\delta$-$\gamma$-$T_i$ spaces (i = 0, 1, 2, $\frac{1}{2}$), $r$-fuzzy semi
(resp. $G_\delta$-$\gamma$-$R_0$ space, smooth fuzzy upper b-irresolute multifunction,
smooth fuzzy b-connected space, smooth fuzzy super b-connected
space, smooth fuzzy strongly b-connected space.

The method of centred system in the theory of topological
spaces was introduced in [39]. In 2007, Uma, Roja and
Balasubramanian [85] introduced the method of centred system in the
theory of smooth fuzzy topological spaces. Motivated by these concepts, the concept of smooth fuzzy centred system, the fundamental theorem on smooth fuzzy \( G_5 \)-irreducible* and smooth fuzzy perfect function are introduced in Chapter II.

In Chapter III, the concept of extremally and basically disconnectedness in smooth fuzzy centred system is introduced. Tietze extension theorem is established as in [82].

Ekici and Kerre [28] and Thangaraj [81] studied the concept of fuzzy contra continuous functions. In Chapter IV, the concept of smooth fuzzy contra \( G_5 \)-continuity is introduced. Some interesting properties and interrelations between the concepts introduced are established. Properties concerning smooth fuzzy \( G_5 \)-compactness, smooth fuzzy \( G_5 \)-regular spaces, smooth fuzzy \( G_5 \)-normal spaces are studied.

Roja and Balasubramanian [71] introduced smooth fuzzy \( \beta \)-\( T_{1/2} \) spaces and studied its generalization. Based on this concept, the concept of smooth fuzzy \( \beta \)-\( T_{1/2} \) spaces is introduced in Chapter V. Interesting properties and interrelations among the concept introduced are established, several counter examples are also given.

In Chapter VI, the concept of \( r \)-fuzzy semi (resp. \( G_6 \))-\( \gamma \)-open set, smooth fuzzy semi (resp. \( G_6 \))-\( \gamma \)-regular space, smooth fuzzy semi (resp. \( G_6 \))-\( \gamma \)-normal spaces are introduced. Further semi (resp. \( G_6 \))-\( \gamma \)-\( T_i \) spaces (\( i = 0, 1, 2, \frac{1}{2} \)) are introduced and interrelations are discussed with relevant examples.
In Chapter VII, new classes of smooth fuzzy multifunctions are introduced as in [75]. The concept of smooth fuzzy b-connectedness in smooth fuzzy topological space is introduced as in [10] and [86]. Interesting properties and some characterizations are obtained.

1.4 BASIC CONCEPTS IN SMOOTH FUZZY TOPOLOGICAL SPACES

In this section, some basic definitions like fuzzy continuous functions, fuzzy interior and fuzzy closure of a fuzzy set have been recalled. Also related results, important theorems and propositions are collected from various research papers.

Throughout this chapter, let X be a non-empty set, I = [0,1] and I_0 = (0,1]. For α ∈ I, α(x) = α for all x ∈ X.

**Definition 1.4.1.** [77]

A function T : I^X → I is called a smooth fuzzy topology on X if it satisfies the following conditions:

1. T(0) = T(1) = 1
2. T(µ_1 ∧ µ_2) ≥ T(µ_1) ∧ T(µ_2) for any µ_1, µ_2 ∈ I^X
3. T( ∨_i∈I µ_i) ≥ _i∈I T(µ_i) for any {µ_i}_i∈I ∈ I^X

The pair (X, T) is called a smooth fuzzy topological space.

**Remark 1.4.1.**[24]

Let (X, T) be a smooth fuzzy topological space. Then, for each r ∈ I_0, T_r = {µ ∈ I^X : T(µ) ≥ r} is Chang’s fuzzy topology on X.

**Proposition 1.4.1.** [26]

Let (X, T) be a smooth fuzzy topological space. For each r ∈ I_0,
\( \lambda \in I^X \), an operator \( C_T : I^X \times I_0 \to I^X \) is defined as follows:

\[
C_T(\lambda, r) = \land \{ \mu : \mu \geq \lambda, T(\bar{I} - \mu) \geq r \}.
\]

For \( \lambda, \mu \in I^X \) and \( r, s \in I_0 \) it satisfies the following conditions:

1. \( C_T(\bar{0}, r) = \bar{0} \),
2. \( \lambda \leq C_T(\lambda, r) \),
3. \( C_T(\lambda, r) \lor C_T(\mu, r) = C_T(\lambda \lor \mu, r) \),
4. \( C_T(\lambda, r) \leq C_T(\lambda, s) \), if \( r \leq s \),
5. \( C_T(C_T(\lambda, r), r) = C_T(\lambda, r) \).

**Proposition 1.4.2.** [26]

Let \( (X, T) \) be a smooth fuzzy topological space. For each \( r \in I_0 \),

\( \lambda \in I^X \), an operator \( I_T : I^X \times I_0 \to I^X \) is defined as follows:

\[
I_T(\lambda, r) = \lor \{ \mu : \mu \leq \lambda, T(\mu) \geq r \}.
\]

For \( \lambda, \mu \in I^X \) and \( r, s \in I_0 \) it satisfies the following conditions:

1. \( I_T(\bar{I} - \lambda, r) = \bar{I} - C_T(\lambda, r) \),
2. \( I_T(\bar{I}, r) = \bar{I} \),
3. \( \lambda \geq I_T(\lambda, r) \),
4. \( I_T(\lambda, r) \land I_T(\mu, r) = I_T(\lambda \land \mu, r) \),
5. \( I_T(\lambda, r) \geq I_T(\lambda, s) \), if \( r \leq s \),
6. \( I_T(I_T(\lambda, r), r) = I_T(\lambda, r) \).
Definition 1.4.2. [62]

Let \((X, T)\) and \((Y, S)\) be any two smooth fuzzy topological spaces. Let \(f : (X, T) \rightarrow (Y, S)\) be a function. Then

1) \(f\) is called fuzzy continuous iff \(S(\mu) \leq T(T^{-1}(\mu))\) for each \(\mu \in \Gamma^Y\);

2) \(f\) is called fuzzy open iff \(T(\lambda) \leq S(f(\lambda))\) for each \(\lambda \in \Gamma^X\);

3) \(f\) is called fuzzy closed iff \(T(\overline{1} - \lambda) \leq S(\overline{1} - f(\lambda))\) for each \(\lambda \in \Gamma^X\).

Definition 1.4.3. [24]

Let \(\lambda\) and \(\mu\) be any two fuzzy sets of \((X, T)\). Then \(\lambda \lor \mu : X \rightarrow [0, 1]\) and \(\lambda \land \mu : X \rightarrow [0, 1]\) are defined as follows

1) \((\lambda \lor \mu)(x) = \max\{\lambda(x), \mu(x)\}\), for all \(x \in X\).

2) \((\lambda \land \mu)(x) = \min\{\lambda(x), \mu(x)\}\), for all \(x \in X\).

Definition 1.4.4. [44]

The characteristic function of a subset \(\lambda \in \Gamma^X\) is denoted by \(1_\lambda\) and is defined as

\[
1_\lambda(x) = \begin{cases} 
1 & \text{if } x \in 1_\lambda \\
0 & \text{if } x \notin 1_\lambda
\end{cases}
\]

Definition 1.4.5. [59]

A fuzzy point \(x_t\) in \(X\) is a fuzzy set taking value \(t \in \Gamma_0\) at \(x\) and zero elsewhere; \(x_t \in \lambda\) if and only if \(t \leq \lambda(x)\). A fuzzy set \(\lambda\) is quasi-coincident with a fuzzy set \(\mu\), denoted by \(\lambda \text{q} \mu\), if there exists \(x \in X\) such that \(\lambda(x) + \mu(x) > 1\). Otherwise \(\lambda\) is not quasi-coincident with a fuzzy set \(\mu\), denoted by \(\lambda \text{q} \mu\) if \(\lambda(x) + \mu(x) \leq 1\).
Lemma 1.4.1. [5]

Let $f : X \rightarrow Y$ be a function and let $\{ \lambda_\alpha \}$ be a family of fuzzy sets of $Y$, then

1. $f^{-1}(\vee \lambda_\alpha) = \vee f^{-1}(\lambda_\alpha)$ and

2. $f^{-1}(\wedge \lambda_\alpha) = \wedge f^{-1}(\lambda_\alpha)$.

Theorem 1.4.1. [24]

Let $f$ be a function from $X$ to $Y$. Then

1. $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ for any fuzzy set $\lambda$ in $Y$.

2. $1 - f(\lambda) \leq f(1 - \lambda)$ for any fuzzy set $\lambda$ in $X$.

3. $\mu_1 \leq \mu_2 \Rightarrow f^{-1}(\mu_1) \leq f^{-1}(\mu_2)$ where $\mu_1$ and $\mu_2$ are fuzzy sets in $Y$.

4. $\lambda_1 \leq \lambda_2 \Rightarrow f(\lambda_1) \leq f(\lambda_2)$ where $\lambda_1$ and $\lambda_2$ are fuzzy sets in $X$.

5. $f(f^{-1}(\mu)) \leq \mu$, for any fuzzy set $\mu$ in $Y$.

6. $\lambda \leq f^{-1}(f(\lambda))$, for any fuzzy set $\lambda$ in $X$.

7. Let $f$ be a function from $X$ to $Y$ and $g$ be a function from $Y$ to $Z$. Then $(g \circ f)^{-1}(\delta) = f^{-1}(g^{-1}(\delta))$ for any fuzzy set $\delta$ in $Z$.

Definition 1.4.6.[85]

Let $R$ be a fuzzy Hausdorff space. A system $\mathcal{G} = \{ \lambda_\alpha \}$ of fuzzy open sets of $R$ is called fuzzy centered system if any finite collection of fuzzy sets of the system has a non-zero intersection. The system $\mathcal{G}$ is called maximal fuzzy centered system or a fuzzy end if it cannot be included in any larger fuzzy centered system.

Definition 1.4.7.[85]

Let $\Theta(R)$ denote the collection of all fuzzy ends belonging to $R$. 
We introduce a fuzzy topology in \( \Theta(\mathbb{R}) \) in the following way: Let \( P_\lambda \) be the set of all fuzzy ends that include \( \lambda \) as an element, where \( \lambda \) is a fuzzy open set of \( \mathbb{R} \). Now \( P_\lambda \) is a fuzzy neighbourhood of each fuzzy end contained in \( P_\lambda \). Thus to each fuzzy open set of \( \mathbb{R} \), there corresponds a fuzzy neighbourhood \( P_\lambda \) in \( \Theta(\mathbb{R}) \).

**Definition 1.4.8.** [85]

A fuzzy Hausdorff space \( \mathbb{R} \) is extremally disconnected if the closure of an open set is open.

**Definition 1.4.9.** [20]

The fuzzy real line \( R(L) \) is the set of all monotone decreasing elements \( \lambda \in L^R \) satisfying \( \vee \{ \lambda(t) / t \in \mathbb{R} \} = 1 \) and \( \wedge \{ \lambda(t) / t \in \mathbb{R} \} = 0 \), after the identification of \( \lambda, \mu \in L^R \) iff \( \lambda(t-) = \mu(t-) \) and \( \lambda(t+) = \mu(t+) \) for all \( t \in \mathbb{R} \), where \( \lambda(t-) = \wedge \{ \lambda(s) : s < t \} \) and \( \lambda(t+) = \vee \{ \lambda(s) : s > t \} \). The natural \( L \)-fuzzy topology on \( R(L) \) is generated from the sub-basis \( \{ L_t, R_t \} \) where \( L_t(\lambda) = \lambda(t-) \) and \( R_t(\lambda) = \lambda(t+) \).

**Definition 1.4.10.** [20]

The \( L \)-fuzzy unit interval \( I(L) \) is a subset of \( R(L) \) such that \( [\lambda] \in I(L) \) if \( \lambda(t) = 1 \) for \( t < 0 \) and \( \lambda(t) = 0 \) for \( t > 1 \).

**Definition 1.4.11.** [8]

Let \( (X, T) \) be a fuzzy topological space and \( \lambda \) a fuzzy set in \( X \). \( \lambda \) is called a fuzzy \( G_\delta \)-set if \( \lambda = \bigwedge_{i=1}^{\infty} \lambda_i \) where \( \lambda_i \in T \) for \( i \in I \).

**Definition 1.4.12.** [8]

Let \( (X, T) \) be a fuzzy topological space and \( \lambda \) a fuzzy set in \( X \). \( \lambda \)
is called a fuzzy $F_\alpha$-set if $\lambda = \vee_{i=1}^{\infty} \lambda_i$ where $1 - \lambda_i \in T$ for $i \in I$.

**Definition 1.4.13.** [72]

Let $(X, T)$ be a fuzzy topological space and $\lambda$ a fuzzy set in $X$. Then $\text{int}_\alpha(\lambda) = \vee \{ \mu : \mu \leq \lambda, \mu \text{ is a fuzzy G}_\delta\text{-set} \}$ is called the fuzzy $G_\delta$-interior of $\lambda$ and $\text{cl}_\alpha(\lambda) = \wedge \{ \mu : \mu \geq \lambda, \mu \text{ is a fuzzy F}_\sigma\text{-set} \}$ is called the fuzzy $G_\delta$-closure of $\lambda$.

**Definition 1.4.14.** [72]

Let $(X, T)$ be a fuzzy topological space and $\lambda$ a fuzzy set in $X$. $\lambda$ is said to be a fuzzy regular $G_\delta$-set if $\lambda = \text{int}_\alpha(\text{cl}_\alpha(\lambda))$.

**Definition 1.4.15.** [72]

Let $(X, T)$ be a fuzzy topological space and $\lambda$ a fuzzy set in $X$. $\lambda$ is said to be a fuzzy regular $F_\sigma$-set if $\lambda = \text{cl}_\alpha(\text{int}_\alpha(\lambda))$.

**Definition 1.4.16.** [78]

Let $(X, T)$ be an $(L,K)$-fuzzy (pre)-fuzzy topological space. Its $Q$-neighbourhood structure is a function $\mathcal{N} : X \times L^X \to I$ ($X$ denotes the totality of all $L$-fuzzy points in $X$), defined by

$$\mathcal{N}(p, U) = \sup \{ T(U) : U \subseteq V, V(x_0) > t \} \text{ and }$$

$$T(U) = \inf_{p \in U} \mathcal{N}(p, U)$$

**Definition 1.4.17.** [1]

Let $(X, T)$ be a smooth fuzzy topological space. For $\lambda \in I^X$, $r \in I_0$

(1) $\lambda$ is called $r$-fuzzy semi open if $\lambda \leq C_T(I_r(\lambda,r),r)$.

(2) The complement of a $r$-fuzzy semi open set is $r$-fuzzy semi-closed.
Definition 1.4.18. [74]

Let $(X, T)$ be a fuzzy topological space. An operation $\gamma$ on the topology $T$ is a function from $T$ into power set $P(X)$ of $X$ such that $V \subseteq V^\gamma$ for each $V \in T$, where $V^\gamma$ denotes the value of $\gamma$ at $V$. It is denoted by $\gamma : T \rightarrow P(X)$.

Definition 1.4.19. [74]

A subset $A$ of a topological space is called a $\gamma$-open set of $(X, T)$ if for each $x \in A$ there exists an open set $U$ such that $x \in U$ and $U^\gamma \subseteq A$. The complement of a $\gamma$-open set is said to be $\gamma$-closed.

Definition 1.4.20. [74]

Let $(X, T)$ be a topological space. An operation $\gamma$ on $SO(X)$ is a function from $SO(X)$ into a power set $P(X)$ of $X$ such that $V \subseteq V^\gamma$ for each $V \in SO(X)$ and $V^\gamma$ denotes the value of $\gamma$ at $V$. It is denoted by $\gamma : SO(X) \rightarrow P(X)$. Now, $SO(X)$ denotes the family of all semi open sets of $(X, T)$.

Definition 1.4.21. [74]

Let $(X, T)$ be a topological space and $\gamma$ be an operation on $SO(X)$. Then a subset $A$ of $X$ is said to be a semi-$\gamma$-open set if for each $x \in A$, there exists a semi open set $U$ such that $x \in U$ and $U^\gamma \subseteq A$. Also $SO(X)_\gamma$ denotes the family of semi-$\gamma$-open sets in $X$.

Definition 1.4.22. [75]

Let $(X, T)$ be a topological space in the classical sense and $(Y, S)$ be an fuzzy topological space. $F : X \rightarrow Y$ is called a fuzzy multifunction if and only if for each $x \in X$, $F(x)$ is a fuzzy set in $Y$. 
**Definition 1.4.23.** [75]

For a fuzzy multifunction \( F : X \rightarrow Y \), the upper inverse \( F^*(\lambda) \) and the lower inverse \( F^-(\lambda) \) of a fuzzy set \( \lambda \) in \( Y \) are defined as follows:

\[ F^*(\lambda) = \{ x \in X / F(x) \leq \lambda \} \text{ and } F^-(\lambda) = \{ x \in X / F(x) \geq \lambda \}. \]

**Lemma 1.4.2.** [75]

Let \( F : (X, T) \rightarrow (Y, S) \) be a fuzzy multifunction. Then, \( F^-(\lambda) = X - F^*(\lambda) \), for a fuzzy set \( \lambda \) in \( Y \).

**Notation 1.4.1**

\( PO(X, x) \) denotes the family of all pre-open sets of \((X, T)\).

**Definition 1.4.24.** [75]

A fuzzy multifunction \( F : (X, T) \rightarrow (Y, S) \) is said to be

1. Fuzzy upper pre-irresolute (f.u.p.i) at a point \( x \in X \), if for each \( \lambda \in FPO(Y) \) containing \( F(x) \) (i.e., \( F(x) \leq \lambda \)), there exists an \( U \in PO(X, x) \) such that \( F(U) \leq \lambda \).

2. Fuzzy lower pre-irresolute (f.l.p.i) at a point \( x \in X \), if for each \( \lambda \in FPO(Y) \) with \( F(x) \geq \lambda \), there exists an \( U \in PO(X, x) \) such that \( U \subseteq F^-(\lambda) \).

3. Fuzzy upper pre-irresolute (Fuzzy lower pre-irresolute) if it has the f.u.p.i (f.l.p.i) property at each \( x \in X \).

**Definition 1.4.25.** [4]

Let \((X, T)\) be a topological space. Let \( A \subseteq X \), then \( A \) is said to be a b-open if \( A \subseteq cl(int(A)) \cup int(cl(A)) \).

**Definition 1.4.26.** [86]

A fuzzy topological space \( X \) is said to be fuzzy semi connected if
the only fuzzy sets which are both fuzzy semi open and fuzzy semi-
closed are $0_x$ and $1_x$.

**Definition 1.4.27.**[22]

Let $X$ be a topological space and $A$ be a subset of $X$. Then $b_g(A) = A \setminus \text{Int}_g(A)$ is said to be the $g$-border of $A$.

**Definition 1.4.28.**[22]

Let $X$ be a topological space and $A$ be a subset of $X$. Then $\text{Fr}_g(A) = \text{Cl}_g(A) \setminus \text{Int}_g(A)$ is said to be the $g$-frontier of $A$.

**Definition 1.4.29.**[22]

Let $X$ be a topological space and $A$ be a subset of $X$. Then $\text{Ext}_g(A) = \text{Int}_g(X \setminus A)$ is said to be the $g$-exterior of $A$.

**Definition 1.4.30.**[85]

Let $R_1$ and $R_2$ be any two smooth fuzzy Hausdorff spaces. $f : R_1 \rightarrow R_2$ is called a fuzzy irreducible function if there is no proper fuzzy closed set $\lambda$ of $R_1$ such that $f(\lambda) = 1_{R_2}$.