Chapter VI
CHAPTER – 6

ANALYSIS OF COST FUNCTION OF FUZZY QUEUES
IN SOLVING ASSEMBLY LINE PROBLEM

In this chapter, the optimal solutions of Assembly Line Problem are studied under Fuzzy Queueing System, to determine the number of operations for manufacturing an Assembly Line Balancing Problem. Optimal allocation of manpower increases production efficiency, thus increasing profits of the companies. Since there is a possibility to assign different numbers of machines to each operator in a variety of situation for machine assignment to operators will occur. Uncertainty is an inevitable factor of the manufacturing environments, some inputs of the model, such as arrival rate, service rate are Fuzzy Numbers, and profit of the model will be a fuzzy number. So Fuzzy ranking method is used for prioritizing situations.

In modern era since the growth of Industrial Automation System in manufacturing plants, manual machinery that requires work and continual monitoring of operators is replaced by semi-automatic and fully automatic machines. In such situations, an operator will be able to manage multiple machines in order to increase their efficiency. But if the managers assign a large number of machines to each operator, it will lead to reduce the cost of recruitment and deployment of manpower, the efficiency of machinery, and efficiency of
assembly line reduce and consequently, it reduces the profit of companies due to lost production.

The main aim of this chapter is to choose the best possible decision and in each of possible scenarios to develop the using of the queuing model under Fuzzy environment. Since uncertainty is an important and inevitable part of manufacturing systems, a fuzzy logic method is proposed to consider the uncertainty in problem. The service rate, arrival rate and profit of the model as fuzzy numbers are considered.

6.1 Proposed model:

**Assembly Line Problem**

The fundamental of Assembly Line Problem is to assign the tasks to an ordered sequence of stations such that the procedure relations satisfies and some measurements of effectiveness are optimized (minimize the waiting time or minimize the number of work stations etc). An Assembly Line Problem consists of work stations \( k = 1, 2, 3, \ldots, m \). Usually arranged along a conveyor belt or a similar material handling equipment. The jobs are connectively launched down the line and are moved from station to station, certain operations are repeatedly performed regarding the cycle time. In general the Assembly Line Balancing Problem consists of optimally balancing the assembly work among all stations with respect to some objective. For this purpose, the total number of work
necessary to assemble a work place (job) in split up into a set $V=\{1,2,\ldots,n\}$ of elementary operations (tasks).

The total work load necessary for assembling a work place is measured by the sum of task times $\sum t$. These elements can be summarized by the following procedure diagram. It contains a node for each task, node weights for the task times, arcs the direct and paths for the indirect procedure constraints. Fig 6.1 shows a procedure diagram with $n=9$ tasks having task times between 2-9

![Procedure Diagram](image)

**Figure 6.1 Procedure Diagram**

### 6.2 A Short-term Assignment of Operators

In short-term assignment of operators who are associated with the issue that each operation will be done by who and where, but the time study of operators deals with the time of performing the operations.

### 6.3 A Long-term Assignment of Operators

Generally production resources can be categorized into 3 main categories
including (i). Machinery (ii). Raw materials (iii). Operators. Management of human resources is an important issue when one faces when there are expensive equipment in Assembly Line. Decision – making and planning in the field of human resource management are done at two levels.

6.4 Ranking Fuzzy Numbers

The main feature of the Ranking Fuzzy Numbers may be the Centroid, area under the membership function or the intersection points between sets. A method of Ranking Fuzzy Numbers considers special features and Fuzzy Numbers will be ranked based on these features. So it is reasonable to expect the results of different Ranking methods for Ranking Fuzzy Numbers of the same data to be different. Therefore such complexities make the method of Ranking Fuzzy Numbers to be a relatively difficult process.

Here Lee and Lie method is applied of prioritizing fuzzy numbers. By this method the Fuzzy Numbers are compared using two criterions

(i) Fuzzy Number Mean

(ii) Fuzzy Number Dispersion.

The dispersion is calculated by then using standard deviation. It is assumed that a Fuzzy Number with greater mean and less standard deviation has higher priority for the decision maker.

Mean and standard deviation of a fuzzy number \( \tilde{A} \) are obtained as follows,
The above equations would be transformed as the following Fuzzy numbers. If \( \tilde{A} \) is a Triangular Fuzzy Number, then

\[
\tilde{A} = (a_1, a_2, a_3)
\]

\[
X(\tilde{A}) = \frac{1}{4} [a_1 + 2a_2 + a_3]
\]

\[
\delta(\tilde{A}) = \frac{1}{80} [3a_1^2 + 4a_2^2 + 3a_3^2 - 2a_1a_3 - 4a_1a_2 - 4a_2a_3]
\]

<table>
<thead>
<tr>
<th>Comparison of Mean Values</th>
<th>Comparison of Standard Deviation</th>
<th>Prioritization Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{X}(\tilde{A}_i) &gt; \hat{X}(\tilde{A}_j) )</td>
<td>( \sigma(\tilde{A}_i) &lt; \sigma(\tilde{A}_j) )</td>
<td>((\tilde{A}_i) &gt; (\tilde{A}_j))</td>
</tr>
<tr>
<td>( \hat{X}(\tilde{A}_i) = \hat{X}(\tilde{A}_j) )</td>
<td>( \sigma(\tilde{A}_i) &lt; \sigma(\tilde{A}_j) )</td>
<td>((\tilde{A}_i) &gt; (\tilde{A}_j))</td>
</tr>
</tbody>
</table>

Table 6.1. Ranking Fuzzy Numbers
After calculating mean and standard deviation of Fuzzy Numbers $\tilde{A}_i$ and $\tilde{A}_j$, prioritization is done by the rules stated in above table.

6.4.1 Model Description

In the proposed model, the production or assembly line is designed by using Fuzzy Queueing Models. Various scenarios of assigning operators to workstations are designed. Then a fuzzy profit is calculated for each stage using unconstrained Fuzzy Programming Method. Fuzzy Profit obtained from different stages is ranked using a Fuzzy ranking method. The stage with the maximum profit is chosen as the best stage of assigning operators to the work stations.

6.4.2 Mathematical Model Formulation

Assumptions

(i). The sequence of workstations is in a series system and the production process starts from the first station and it continuous till the last unit station.

(ii). Operators have the same abilities and service rates.

(iii). The average time required to perform the operation is assumed to be the same for all machines. Since the machines are in series, this assumption can be satisfied.
Notations

\( C_w \): Cost for work in process/unit of time.

\( C_e \): Cost for the employment of each operator/unit of time.

\( T_I \): The total income earned/unit of time.

\( P_p \): The profit of producing each unit of product.

\( n \): Number of operators.

\( T_h \): Time horizon of decision-making

\( P_C \): Production cycle time.

\( L_1 \): The Average number of products in the system (waiting or in the service)

\( L_{si} \): The Average number of products at station \( S_i \).

First, the products of assembly line are allocated as customers of queueing systems and operators are assumed as servers in the queueing system. Further it is also assumed that the arrival of items into the assembly line as according to Poisson process distribution with rate \( \tilde{\lambda} \) and the service rate \( \tilde{\mu} \) exponentially distributed.

Next the number of required machines is to be determined and should be assigned to each operator and which will lead to obtain the optimum number of operators to the production line. Now, 1,2,…….n is assigned to each operator, but
each of these assignments will result in different costs and revenues for the company. So, the total profit function which consists of income and costs arising from the implementation of that case is calculated and according to the values obtained from different stages, the best stage of assigning operators to the assembly line will be determined.

The profit function in calculated as

\[ T_i = P_R \times \left( \frac{T_h}{L_1} \right) - (C_w \times L_1) - (C_E \times n) \] ........................(1)

In this model, a constant number of machines are assigned to each operator in each stage.

In the first stage, one machine is assigned to each operator and in the second stage, two machines are assigned to each operator and it is obvious in the last stage, one or two machines are assigned to an operator with regard to the odd or even number of machines.

(i.e) N different stages for assignment of machines to the operators exist. It is noted that apart from first and the last stages, the assignment is not fully specified in the stage and depending on the number of machines in the production line, assignment in the last station changes.

In the second stage, if the number of machines in the line is a multiple of 2

(i.e) n=2k,

(i.e) We have M/M/1 model or M/E\textsubscript{1}/1 model at the station.
The method to calculate the profit function is as follows for some cases has been investigated. We consider Triangular Fuzzy Number for the arrival rate of items.

(i.e) \( \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3) \) ..............................................(2)

\[ \tilde{\mu}_1 = \tilde{\mu}_2 \ldots \ldots \ldots = \tilde{\mu}_n = (\mu_1, \mu_2, \mu_3 \ldots \ldots \mu_n) \] ..............................................(3)

The parameter \( C_w \), the cost of work in process/unit of time includes such as deterioration cost of products which are waiting between work stations, cost of the space needed to store work in process and costs of lost opportunity, \( C_w \) is also a Fuzzy Triangular Number.

(i.e) \( \tilde{C}_w = (C_1, C_2, C_3) \) .........................................................(4)

The profit \( \tilde{P}_w = (P_1, P_2, P_3) \) .........................................................(5)

is also a Triangular Fuzzy number. Since product prices are not constant and it is vary in different time intervals.

The traffic intensity \( \rho = \frac{\lambda}{\mu} \) for each work station it is \( \rho < 1 \).

(i.e) \( \rho_i = \frac{\lambda_i}{\mu_i} < 1 \) ............................................................(6)

In the first stage, the value of \( L \) is given by,

\[ L = Ls_1 + Ls_2 + Ls_3 + \ldots \ldots + Ls_m \] ............................................(7)
Given each box is M/M/1 model, then $L_s_i$ formulated as follows.

$$L_s_i = \frac{\tilde{\lambda}_i}{\tilde{\mu}_i - \tilde{\lambda}_i} = \frac{\tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}}$$ ...(8)

In this, the value of L is given by

$$\tilde{L} = \sum_{i=1}^{m} L_s_i = \frac{m \tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}}$$ ...(9)

Here $\tilde{L}$ is also Triangular Fuzzy Number, and it is calculated as,

$$\tilde{L} = \frac{m \tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}} = (L_1, L_2, L_3)$$ ...(10)

$$\tilde{L}_1 = \min[\tilde{L} / \tilde{\lambda}, \tilde{\mu}] = \frac{m \tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1}$$ ...(11)

$$\tilde{L}_2 = \frac{m \tilde{\lambda}_2}{\tilde{\mu}_2 - \tilde{\lambda}_2}$$ ...(12)

$$\tilde{L}_3 = \frac{m \tilde{\lambda}_3}{\tilde{\mu}_3 - \tilde{\lambda}_3}$$ ...(13)

Also in the stage, $n = m$, thus $\tilde{P}_c$ is obtained by the following formula,

$$\tilde{P}_c = \text{Production cycle time} = \max\{\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_m\}$$

$$= \max\left\{\frac{1}{\tilde{\mu}_1}, \frac{1}{\tilde{\mu}_2}, \ldots, \frac{1}{\tilde{\mu}_n}\right\} = \frac{1}{\tilde{\mu}} = (\frac{1}{\tilde{\mu}_1}, \frac{1}{\tilde{\mu}_2}, \frac{1}{\tilde{\mu}_3})$$ ...(14)
Therefore the Profit function $T_i$ is calculated as follows,

\[ T_i = P_R \times \left( \frac{T_h}{P_c} \right) - (C_w \times L_i) - (C_E \times m) = (T_{i_1}, T_{i_2}, T_{i_3}) \] \hspace{1cm} (15)

\[ (T_i)_1 = (P_{R_1} \times T_h \times \mu_1) - (C_{w_1} \times L_3) - (C_E \times m) \] \hspace{1cm} (16)

\[ (T_i)_2 = (P_{R_2} \times T_h \times \mu_2) - (C_{w_2} \times L_2) - (C_E \times m) \] \hspace{1cm} (17)

\[ (T_i)_3 = (P_{R_3} \times T_h \times \mu_3) - (C_{w_1} \times L_1) - (C_E \times m) \] \hspace{1cm} (18)

Next, the second stage two machines are considered which can be assigned to one operator. Their calculations are given by,

\[ \rho_i = \frac{\bar{\lambda}_i}{\bar{\mu}_i} < 1 \Rightarrow \bar{\lambda}_1 = \bar{\lambda}_2 = \cdots \bar{\lambda}_{\left[\frac{m}{2}\right]+1} = \bar{\lambda} \]

\[ \bar{\lambda} = (\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3) \] \hspace{1cm} (19)

Similarly, \[ \tilde{\mu}_1 = \tilde{\mu}_2 = \cdots = \tilde{\mu}_{\left[\frac{m}{2}\right]+1} = \tilde{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3) \] \hspace{1cm} (20)

\[ \tilde{L} = \begin{cases} \sum_{i=1}^{\frac{m}{2}} \tilde{L}_{s_i}, & m = 2k \\ \sum_{i=1}^{\frac{m}{2}} \tilde{L}_{s_i} + \tilde{L}_{s_{\left[\frac{m}{2}\right]+1}}, & m = 2k+1 \end{cases} \] \hspace{1cm} (21)

\[ n = \begin{cases} \left[\frac{m}{2}\right], & m = 2k \\ \left[\frac{m}{2}\right]+1, & m = 2k+1 \end{cases} \] \hspace{1cm} (22)

\[ \tilde{P}_c = \begin{cases} \max \left\{ \tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_{\left[\frac{m}{2}\right]} \right\}, & m = 2k \\ \max \left\{ \tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_{\left[\frac{m}{2}\right]+1} \right\}, & m = 2k+1 \end{cases} \] \hspace{1cm} (23)
$\tilde{P}_i$ is calculated as follows

$$\tilde{P}_i = \begin{cases} \frac{1}{\tilde{\mu}_1} + \frac{1}{\tilde{\mu}_2} = \frac{2}{\tilde{\mu}}, \\ \frac{1}{\tilde{\mu}}, \end{cases} \quad 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor, \quad i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \quad \text{(24)}$$

In this case, according to the formula, the cycle time is

$$\tilde{P}_c = \max\left\{ \frac{2}{\tilde{\mu}}, \frac{2}{\tilde{\mu}}, \ldots, \frac{2}{\tilde{\mu}} \right\} = \frac{2}{\tilde{\mu}}, \quad m = 2k \quad \text{(25)}$$

(i.e.)

If the number of machines in this stage is $m = 2k$, then $\left\lfloor \frac{m}{2} \right\rfloor$ stations with two machines where all these stations follows M/E$_2$/1 model. In this model M/Er/1, the expected mean of Queue length $L$ is obtained as follows.

$$(i.e.) \text{ M/E}_r/1 \Rightarrow L = \left(\frac{r+1}{2r}\right) \times \left[ \frac{\lambda^2}{\mu \times (\mu - \lambda)} \right] + \frac{\lambda}{\mu} \quad \text{.................(26)}$$

The value of $\tilde{L}_S_i$ and $\tilde{L}$ are calculated as follows

$$\tilde{L}_S_i = \frac{3}{4} \times \left[ \frac{\tilde{\lambda}^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda})} \right] + \frac{\tilde{\lambda}}{\tilde{\mu}} \quad \text{.........................(27)}$$

$$\tilde{L} = \sum_{i=1}^{\left\lfloor \frac{m}{2} \right\rfloor} \tilde{L}_S_i = \binom{m}{2} \times \left[ \frac{\tilde{\lambda}^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda})} \right] + \frac{m \tilde{\lambda}}{2 \tilde{\mu}}$$
\[ \tilde{L} = \frac{3m}{8} \times \left[ \frac{\tilde{\lambda}^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda})} \right] + \frac{m \tilde{\lambda}}{2 \tilde{\mu}} \]

\[ = (\tilde{L}_1, \tilde{L}_2, \tilde{L}_3) \] .........................................................(28)

Now, \( \tilde{L}_1 = \min\left[ \tilde{L} / \tilde{\lambda}, \tilde{\mu} \right] = \frac{3m}{8} \times X \left[ \frac{\tilde{\lambda} 1^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda} 1)} \right] + \frac{m \tilde{\lambda}}{2 \tilde{\mu}} \) ................................(29)

\[ \tilde{L}_2 = \frac{3m}{8} \times \left[ \frac{\tilde{\lambda}^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda} 2)} \right] + \frac{m \tilde{\lambda}}{2 \tilde{\mu}} \] .........................................................(30)

\[ \tilde{L}_3 = \max\left[ \tilde{L} / \tilde{\lambda}, \tilde{\mu} \right] = \frac{3m}{8} \times \left[ \frac{\tilde{\lambda} 3^2}{\tilde{\mu} \times (\tilde{\mu} - \tilde{\lambda} 3)} \right] + \frac{m \tilde{\lambda}}{2 \tilde{\mu}} \] .........................................................(31)

Therefore when \( n= \left[ \frac{m}{2} \right] = \frac{m}{2}, \tilde{T}_1 and \tilde{P}_c \) are obtained as follows.

\[ \tilde{T}_1 = \tilde{P}_R \times \left( \frac{\tilde{T}_h}{\tilde{P}_c} \right) - (C_w \times \tilde{L}_1) - (C_E \times m) \]

\[ = (T_{l_1}, T_{l_2}, T_{l_3}) \] .........................................................(32)

\[ \therefore (T_1)_1 = \left( P_{R_1 \times T_h} \times \frac{\mu_1}{2} \right) - (C_{w_3} \times L_3) - (C_E \times m) \] .............(33)

\[ (T_1)_2 = \left( P_{R_2 \times T_h} \times \frac{\mu_2}{2} \right) - (C_{w_2} \times L_2) - (C_E \times m) \] .............(34)

\[ (T_1)_3 = \left( P_{R_3 \times T_h} \times \frac{\mu_3}{2} \right) - (C_{w_1} \times L_1) - (C_E \times m) \] .............(35)

\[ \tilde{P}_c = \frac{2}{\tilde{\mu}} = \left( \frac{2}{\mu_1}, \frac{2}{\mu_2}, \frac{2}{\mu_3} \right) \] .........................................................(36)
The calculations for optimal values are done similarly for stages up to the last stage, where the entire machines are assignment to one operator. Then the model M/E/r/1 is calculated. The calculations for the last stage are as follows.

\[ \tilde{\lambda}_i = \text{fuzzy arrival rate} = \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3) \] ............(37)

\[ \tilde{\mu}_i = \text{fuzzy service rate} = \tilde{\mu} = (\mu_1, \mu_2, \mu_3) \] ............(38)

\[ \tilde{L} = \tilde{L}_{si} = \frac{m+1}{2n} \left[ \frac{\tilde{\lambda}^2}{\mu \times (\tilde{\mu} - \tilde{\lambda})} \right] + \frac{\tilde{\lambda}}{\tilde{\mu}} \]

\[ = (\tilde{L}_1, \tilde{L}_2, \tilde{L}_3) \] ..........................................................(39)

Now,

\[ L_1 = \frac{m+1}{2m} \left[ \frac{\lambda_1^2}{\mu_3 \times (\mu_3 - \lambda_1)} \right] + \frac{\lambda_1}{\mu_3} \] ......................(40)

\[ L_2 = \frac{m+1}{2m} \left[ \frac{\lambda_2^2}{\mu_2 \times (\mu_2 - \lambda_2)} \right] + \frac{\lambda_2}{\mu_2} \] ......................(41)

\[ L_3 = \frac{m+1}{2m} \left[ \frac{\lambda_3^2}{\mu_1 \times (\mu_1 - \lambda_3)} \right] + \frac{\lambda_3}{\mu_1} \] ......................(42)

In this stage all machine are assigned to one operator. Thus in this stage m=1 and \( \tilde{T}_t, \tilde{P}_c \) can be as follows.

\[ \tilde{T}_t = \tilde{P}_R \times \left( \frac{\tau_h}{\tilde{P}_c} \right) - \left( C_w \times \tilde{L} \right) - (C_E \times m) \] .................(43)
\[ (T_l)_{1} = \left( \frac{P_{R_1 \times T_h} \times \mu_1}{m} \right) - (C_{W_3} \times L_3) - (C_E \times m) \] (45)

\[ (T_l)_{2} = \left( \frac{P_{R_2 \times T_h} \times \mu_2}{m} \right) - (C_{W_2} \times L_2) - (C_E \times m) \] (46)

\[ (T_l)_{3} = \left( \frac{P_{R_3 \times T_h} \times \mu_3}{m} \right) - (C_{W_1} \times L_1) - (C_E \times m) \] (47)

\[ \bar{p}_c = \frac{1}{\tilde{\mu}_1} + \frac{1}{\tilde{\mu}_2} + \ldots + \frac{1}{\tilde{\mu}_i} = \sum_{i=1}^{m} \frac{1}{\tilde{\mu}_i} = \frac{m}{\tilde{\mu}} \]

\[ = \left( \frac{m}{\mu_3}, \frac{m}{\mu_2}, \frac{m}{\mu_1} \right) \] (48)

After calculating the profit function \( T_l \), for all the m stages, the stage that has the largest value of profit is chosen as the optimal assignment. But since the values obtained for profit function in different stage are fuzzy numbers they should be compared using priority techniques.

**Numerical Illustration**

Suppose in an Assembly line of a heavy injection moulding Industry with 10 machines is located in a queue along together then the products according to Poisson distribution with an approximate rate of 300 unit/month will be entered into a production line. The time required for the operation in each machine in order to produce a single product has exponential distribution with parameter (2, 2.5, 3). In the factory there are 25 working days and 8hrs duty per day operation. The work cost in the system is (800,900,1000) in the month and the
employment cost each operator is 6000 in the month and the profits generated per unit of product are (150,170,200), the objective is to determine optimal allocation of manpower to the production line.

Solution Procedure

In this case, totally there are 10 different assignment stages, because 10 machines are available in the production line. Calculations of the profit functions have been performed using MATLAB. The outputs of MATLAB calculations for each of the 10 stages are in the following Table.

Let \( c_w = (950, 1050, 1100) \)

\[ C_E = 8000 \]

\[ P_R = (200, 220, 250) \]

\[ T_h = \text{The available time during a month} \]

\[ = 25 \text{ working days} \times 8 \text{ hr/day} = 200 \]

\[ \lambda = (450, 500, 550) \text{ arrival rate/month} \]

\[ \tilde{\mu} = \text{fuzzy operator service rate/hr.} \]

\[ = (3, 3.25, \text{ and } 3.5) \]

\[ = (3 \times 200, 3.25 \times 200, 3.5 \times 200/\text{month}) \]

\[ = (600, 650, 700) \]
According to the Lee and Lie approach and by the MATLAB codes these 10 stages are ranked. The MATLAB outputs, resulting in ranking of the different assignment stages are given by

<table>
<thead>
<tr>
<th>Stages</th>
<th>$T_M$</th>
<th>$T_1$ = Total Income Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>$T_{I1} = (21,000, 62,300, 1,15,100)$</td>
</tr>
<tr>
<td>2</td>
<td>$[10/2] = 5$</td>
<td>$T_{I2} = (24,500, 73,000, 1,20,000)$</td>
</tr>
<tr>
<td>3</td>
<td>$[10/3] + 1 = 4$</td>
<td>$T_{I3} = (14,500, 58,000, 98,670)$</td>
</tr>
<tr>
<td>4</td>
<td>$[10/4] + 1 = 3$</td>
<td>$T_{I4} = (19,850, 61,000, 11,350)$</td>
</tr>
<tr>
<td>5</td>
<td>$[10/5] = 2$</td>
<td>$T_{I5} = (22,000, 58,550, 1,01,250)$</td>
</tr>
<tr>
<td>6</td>
<td>$[10/6] + 1 = 2$</td>
<td>$T_{I6} = (10,050, 54,150, 78,000)$</td>
</tr>
<tr>
<td>7</td>
<td>$[10/7] + 1 = 2$</td>
<td>$T_{I7} = (16,240, 54,100, 98,105)$</td>
</tr>
<tr>
<td>8</td>
<td>$[10/8] + 1 = 2$</td>
<td>$T_{I8} = (12,250, 24,580, 43,850)$</td>
</tr>
<tr>
<td>9</td>
<td>$[10/9] + 1 = 2$</td>
<td>$T_{I9} = (9,500, 53,180, 75,400)$</td>
</tr>
<tr>
<td>10</td>
<td>$[10/10] = 1$</td>
<td>$T_{I10} = (1,200, 21,040, 46750)$</td>
</tr>
</tbody>
</table>

Table 6.2. Total Income Values
From the above table values of Total Expected Income at 10 different stages, we observe that

$$\bar{T}_{I_{10}} > \bar{T}_{I_8} > \bar{T}_{I_9} > \bar{T}_{I_6} > \bar{T}_{I_7} > \bar{T}_{I_5} > \bar{T}_{I_4} > \bar{T}_{I_2} > \bar{T}_{I_1}.$$  

From the above 10 stages, the best stage is to assign five operators to the production line. In other words, it is optimal to assign two operators to each machine. Also, if it is not possible to assign 5 operators to assembly line because of some reasons and constraints, then the first scenario, the 4th and 5th respectively, are the next property for the assignment of operators to production line. It is worth noting that the amount of parameters $C_W$, $C_E$, $P_R$ and $\lambda$ are influential in determining the best scenario, so that in the numerical applications, if the cost of employing worker $C_E$ increases to a sufficiently large value, then the first stage will not be definitely selected as the superior stage. Therefore, a sensitive analysis of input parameters is needed to identify sensitive incentive parameters.

### 6.4.3 Sensitivity Analysis

In the above calculations each parameter is increased by 10% in each step, separately and then the variations in the final ranking of assignment stages are analyzed. In the following table 6.3, increasing of the amount of parameter $P_R$ results in significant changes in the ranking of stages. In fact, increasing of $P_R$ leads to these stages which assign more operators and they would have a higher
priority in the final ranking. This is because of increase in the number of operators leads to decrease in the cycle time, thus increasing the final profit.

<table>
<thead>
<tr>
<th>Percentage of change in parameter $P_R$</th>
<th>Prioritization stages</th>
<th>The amount of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0%</td>
<td>$\bar{T}_2 &gt; \bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_3 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>-------</td>
</tr>
<tr>
<td>+10%</td>
<td>$\bar{T}_2 &gt; \bar{T}_1 &gt; \bar{T}_3 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>Medium</td>
</tr>
<tr>
<td>+20%</td>
<td>$\bar{T}_1 &gt; \bar{T}_2 &gt; \bar{T}_4 &gt; \bar{T}_3 &gt; \bar{T}_5 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>Medium</td>
</tr>
<tr>
<td>+30%</td>
<td>$\bar{T}_1 &gt; \bar{T}_2 &gt; \bar{T}_3 &gt; \bar{T}_5 &gt; \bar{T}_4 &gt; \bar{T}_6 &gt; \bar{T}_7 &gt; \bar{T}_8 &gt; \bar{T}<em>9 &gt; \bar{T}</em>{10}$</td>
<td>High</td>
</tr>
<tr>
<td>+40%</td>
<td>$\bar{T}_1 &gt; \bar{T}_2 &gt; \bar{T}_3 &gt; \bar{T}_5 &gt; \bar{T}_4 &gt; \bar{T}_6 &gt; \bar{T}_7 &gt; \bar{T}_8 &gt; \bar{T}<em>9 &gt; \bar{T}</em>{10}$</td>
<td>High</td>
</tr>
<tr>
<td>+50%</td>
<td>$\bar{T}_1 &gt; \bar{T}_2 &gt; \bar{T}_3 &gt; \bar{T}_5 &gt; \bar{T}_4 &gt; \bar{T}_6 &gt; \bar{T}_7 &gt; \bar{T}_8 &gt; \bar{T}<em>9 &gt; \bar{T}</em>{10}$</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 6.3. Percentage of change in parameter $P_R$
Percentage of change in parameter $P_W$ | Prioritization scenarios | The amount of change
--- | --- | ---
+0% | $T_{I_2} > T_{I_1} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | ------
+10% | $T_{I_2} > T_{I_1} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | ------
+20% | $T_{I_2} > T_{I_1} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | ------
+30% | $T_{I_2} > T_{I_1} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | ------
+40% | $T_{I_1} > T_{I_2} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | Low
+50% | $T_{I_1} > T_{I_2} > T_{I_4} > T_{I_5} > T_{I_3} > T_{I_7} > T_{I_6} > T_{I_9} > T_{I_8} > T_{I_{10}}$ | Low

Table 6.4. Percentage of change in parameter $P_W$

According to Table 6.4 indicates that by increasing $C_w$, the final ranking of the stages does not change substantially. So, this parameter of the model 1.

In the Table 6.5, sensitivity analysis of the parameter $C_E$ indicates that although this parameter could not be assumed as sensitive in $P_R$, accurate estimation of this parameter is important in comparison $C_W$ because it has caused drastic changes in the final ranking of some cases. Also increasing this parameter results in higher priority for those stages with less assigned number of operators in the final ranking.
According to the Table 6.6, the results of sensitivity analysis for the parameter $\lambda$. This parameter affects the amounts of $L_i$. Also $L_i$ affects the final profit considering the equation (1). Therefore, $\lambda$ affects the final profit of the firm indirectly. But in the table 6.6, this parameter is an insensitivity analysis for the parameter $\mu$ in Table 6.7, which indicates that this parameter is also a sensitivity parameter of the model and has a significant effect on the final ranking of the stages.

<table>
<thead>
<tr>
<th>Percentage of change in parameter $P_E$</th>
<th>Prioritization scenarios</th>
<th>The amount of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0%</td>
<td>$I_2 &gt; I_1 &gt; I_4 &gt; I_5 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10}$</td>
<td>--------</td>
</tr>
<tr>
<td>+10%</td>
<td>$I_2 &gt; I_1 &gt; I_4 &gt; I_5 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10}$</td>
<td>--------</td>
</tr>
<tr>
<td>+20%</td>
<td>$I_4 &gt; I_2 &gt; I_5 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10} &gt; I_1$</td>
<td>Medium</td>
</tr>
<tr>
<td>+30%</td>
<td>$I_4 &gt; I_2 &gt; I_5 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10} &gt; I_1$</td>
<td>Medium</td>
</tr>
<tr>
<td>+40%</td>
<td>$I_4 &gt; I_2 &gt; I_5 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10} &gt; I_1$</td>
<td>Medium</td>
</tr>
<tr>
<td>+50%</td>
<td>$I_5 &gt; I_4 &gt; I_2 &gt; I_3 &gt; I_7 &gt; I_6 &gt; I_9 &gt; I_8 &gt; I_{10} &gt; I_1$</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 6.5. Percentage of change in parameter $P_E$
<table>
<thead>
<tr>
<th>Percentage of change in parameter $\lambda$</th>
<th>Prioritization scenarios</th>
<th>The amount of change</th>
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<tbody>
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<td>$\bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_3 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>-------</td>
</tr>
<tr>
<td>+10%</td>
<td>$\bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_3 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>-------</td>
</tr>
<tr>
<td>+20%</td>
<td>$\bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_3 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>-------</td>
</tr>
<tr>
<td>+30%</td>
<td>$\bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_3 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>-------</td>
</tr>
<tr>
<td>+40%</td>
<td>$\bar{T}_1 &gt; \bar{T}_4 &gt; \bar{T}_3 &gt; \bar{T}_5 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>Low</td>
</tr>
<tr>
<td>+50%</td>
<td>$\bar{T}_1 &gt; \bar{T}_3 &gt; \bar{T}_4 &gt; \bar{T}_5 &gt; \bar{T}_7 &gt; \bar{T}_6 &gt; \bar{T}_9 &gt; \bar{T}<em>8 &gt; \bar{T}</em>{10}$</td>
<td>Low</td>
</tr>
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</table>

Table 6.6 Percentage of change in parameter $\lambda$
<table>
<thead>
<tr>
<th>Percentage of change in parameter $\mu$</th>
<th>Prioritization scenarios</th>
<th>The amount of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0%</td>
<td>$\tilde{T}<em>{l_2} &gt; \tilde{T}</em>{l_1} &gt; \tilde{T}<em>{l_4} &gt; \tilde{T}</em>{l_5} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_7} &gt; \tilde{T}<em>{l_6} &gt; \tilde{T}</em>{l_9} &gt; \tilde{T}<em>{l_8} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>--------</td>
</tr>
<tr>
<td>+10%</td>
<td>$\tilde{T}<em>{l_1} &gt; \tilde{T}</em>{l_2} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_4} &gt; \tilde{T}<em>{l_5} &gt; \tilde{T}</em>{l_6} &gt; \tilde{T}<em>{l_7} &gt; \tilde{T}</em>{l_8} &gt; \tilde{T}<em>{l_9} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>Medium</td>
</tr>
<tr>
<td>+20%</td>
<td>$\tilde{T}<em>{l_1} &gt; \tilde{T}</em>{l_2} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_5} &gt; \tilde{T}<em>{l_4} &gt; \tilde{T}</em>{l_6} &gt; \tilde{T}<em>{l_7} &gt; \tilde{T}</em>{l_8} &gt; \tilde{T}<em>{l_9} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>High</td>
</tr>
<tr>
<td>+30%</td>
<td>$\tilde{T}<em>{l_1} &gt; \tilde{T}</em>{l_2} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_5} &gt; \tilde{T}<em>{l_4} &gt; \tilde{T}</em>{l_6} &gt; \tilde{T}<em>{l_7} &gt; \tilde{T}</em>{l_8} &gt; \tilde{T}<em>{l_9} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>High</td>
</tr>
<tr>
<td>+40%</td>
<td>$\tilde{T}<em>{l_1} &gt; \tilde{T}</em>{l_2} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_5} &gt; \tilde{T}<em>{l_4} &gt; \tilde{T}</em>{l_6} &gt; \tilde{T}<em>{l_7} &gt; \tilde{T}</em>{l_8} &gt; \tilde{T}<em>{l_9} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>High</td>
</tr>
<tr>
<td>+50%</td>
<td>$\tilde{T}<em>{l_1} &gt; \tilde{T}</em>{l_2} &gt; \tilde{T}<em>{l_3} &gt; \tilde{T}</em>{l_5} &gt; \tilde{T}<em>{l_4} &gt; \tilde{T}</em>{l_6} &gt; \tilde{T}<em>{l_7} &gt; \tilde{T}</em>{l_8} &gt; \tilde{T}<em>{l_9} &gt; \tilde{T}</em>{l_{10}}$</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 6.7. Percentage of change in parameter $\mu$

From the above observation including costs and revenues resulted from the implementation of each of these scenarios was determined given that some of the problem inputs were Fuzzy. The amount of profit function for different scenarios is obtained as a Fuzzy number thus we have applied Fuzzy prioritization techniques to select the best assignment. It follows the results of the data analysis by which this method can provide additional information for decision makers.