CHAPTER 8
ANALYTICAL STUDY

8.1 GENERAL

In the previous chapter, the behaviour of GFRP reinforced concrete beams was studied by experimental investigations. The results can be compared to the theoretical calculations that estimated the deflections and internal stress/strain distributions within the beam. Finite element analysis can also be used to model the behaviour numerically to confirm these calculations, as well as to provide a valuable supplement to the experimental investigations, particularly in parametric studies. The finite element analysis is an assemblage of finite elements which are interconnected at a finite number of nodal points. In this chapter, the experimental results of the beams reinforced internally with GFRP bars are validated by finite element analysis using ANSYS software. The load deflection values from the analysis are compared with experimental values. The study also presents the results of the comparison between the measured and the predicted crack width of the GFRP beams.

8.2 MODELING OF BEAMS WITH GFRP BARS

In order to compare the performance of GFRP beams, the full scale specimens were modeled and analyzed using finite element software ANSYS by assigning the element types and properties to various materials like concrete, steel and GFRP bar. Totally nine full size GFRP beams were
modeled in which the first set consisting of 3 models for M20 grade beams reinforced with 16mm, 20mm and 24mm GFRP bars. The second set of 3 models for M40 grade beams reinforced with 16mm, 20mm and 24mm GFRP bars and last set of 3 models for M60 grade beams reinforced with 16mm, 20mm and 24mm GFRP bars. In this research discrete modeling approach is used to model the behaviour of beams reinforced with GFRP bars. The models are analyzed for monotonic loadings (downward directions). The various steps involved in the modeling of the GFRP beam are discussed here.

8.2.1 Element Types

The concrete beam element is modeled by an eight-node solid element, Solid65. The solid element has eight nodes with three degrees of freedom at each node, translations in the nodal x, y, and z directions. The element is capable of plastic deformation, cracking in tension, and crushing in compression. The geometry and node locations for Solid65 element type are shown in Figure 8.1.

![Figure 8.1 Solid65 – 3D Reinforced Concrete Solid](image)
The steel and GFRP reinforcement in the beam was modeled by Link 8 elements. Two nodes are required for this element. At each node, the degrees of freedom were identical to those for the Solid 65. Each node has three degrees of freedom, translations in the nodal x, y, and z directions. The element is also capable of undergoing plastic deformation.

It was assumed that the slip between the concrete and the reinforcement was zero. By meshing the concrete and reinforcing bar with the same element size, the concrete nodes share the same nodes of reinforcing bar. Then using merging option all the nodes as well as elements of the model were merged hence perfect bond is achieved between concrete and reinforcing bar. The nonlinearity is derived from the nonlinear relationships in material models and the effect of geometric nonlinearity is not considered. The geometry and node locations for link 8 element type are shown in Figure 8.2.

Figure 8.2 Link8 – 3D Spar
8.2.2 Material Properties

Concrete compression strength was 20MPa, 40MPa and 60MPa and the yield stress of steel reinforcements was 415MPa. The modulus of elasticity of concrete was calculated from the formula, \( E_c = 5000 \sqrt{f_{ck}} \) and the Poisson's ratio was assumed as 0.2. For Steel and GFRP bar the modulus of elasticity was taken as 200GPa, and 31.6GPa respectively and the Poisson's ratio as 0.3 and 0.25 respectively.

8.2.3 Loading and Boundary Conditions

The boundary conditions were applied to ensure that the model acted in the same way as that of test specimen. The loads were applied to the GFRP beam model in the same manner as that of the experimental study. Since the Beam modeled in ANSYS has 9 numbers of nodes across its cross-section, every increment in load has to be distributed among these nodes. The GFRP beam reinforcement model, and the loading and support conditions of beam model are shown in Figures 8.3 and 8.4.

The non-linear relation between load and displacement requires an incremental-iterative solution procedure, in which the load is incrementally increased. Within each increment equilibrium is iteratively achieved. Iterations are repeated until internal equilibrium conditions are sufficiently fulfilled and the convergence is obtained. The deflection plot obtained through the analysis for the beams M20-D16, M40-D16 and M60-D16 is shown in Figures 8.5-8.7. The deflection plot and the mid deflection values for all the nine models for each increment of loading were obtained from linear and non linear analysis. A comparative study of the experimental deflection values with ANSYS results were made and shown in Figures 8.8-8.10.
Figure 8.3 Reinforcement Model of GFRP Beam

Figure 8.4 Loading and Support Conditions
Figure 8.5 Deflection Plot for M20-D16

Figure 8.6 Deflection Plot for M40-D16
8.3 DEFLECTION CALCULATION - ACI FORMULA

The magnitude of deflection of GFRP reinforced beams tend to be greater due to lower stiffness of GFRP reinforcements. The ACI 440.1R guide suggests two methods to control the deflection. Recommending minimum thickness of beams is the indirect method of deflection control whereas, the direct method followed the concept of effective moment of inertia ($I_e$) to limit the computed deflection. In the uncracked section, the moment of inertia is equal to the gross moment of inertia ($I_g$) and the cracking starts when the applied moment ($M_a$) exceeds the cracking moment ($M_{cr}$). The corresponding moment of inertia is cracking moment of inertia ($I_{cr}$) and can be calculated from the elastic analysis with the assumption that the concrete in the tension is negligible.
In order to calculate the service deflection of a flexural member, ACI code presents the concept of effective moment of inertia \((I_e)\). The service load of the beam can be assumed as the ultimate load divided by 1.6, which is the factor for the live load suggested by ACI 318. For FRP bar reinforced beams, ACI 440.1R recommends a modified form of equation for effective moment of inertia

\[
I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_d I_e + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_e
\] (8.1)

The factor \(\beta_d\) in Equation (8.1) is a reduction coefficient that taken into account for the lower modulus of elasticity and different bond characteristics of GFRP. Based on the evaluation of experimental results from several studies, ACI 440.1R suggests the relationship for \(\beta_d\) as

\[
\beta_d = \alpha \left(\frac{E_f}{E_s} + 1\right)
\]

where \(E_f\) is the modulus of elasticity of the GFRP bar, \(E_s\) is the modulus of elasticity of steel rod, and \(\alpha\) is a bond coefficient, which is 0.5 for GFRP bars.

From equation 8.1

\(I_{cr} = \) cracking moment of inertia and given by (ACI 440.1R)

\[
I_{cr} = \frac{bd^3}{3} \cdot k^3 + n_f A_f d^2 (1 - k)^2
\] (8.2)

where \(n_f\) is modular ratio between FRP reinforcement and concrete, \(A_f\) be the area of GFRP bar, and \(k\) is given by (ACI 440.1R)

\[
k = \sqrt{\left(2\rho_f n_f + (\rho_f n_f)^2\right)} - \rho_f n_f
\] (8.3)
\[ \rho_f = \frac{A_f}{b \cdot d} \]  
(8.4)

\[ M_{cr} = \frac{f_r \cdot I_s}{y_i} \]  
(8.5)

\[ f_r = 0.62\sqrt{f_c'} \]  
(8.6)

Where \( \rho_f \) = FRP reinforcement ratio, \( M_{cr} \) = cracking moment, \( f_r \) = modulus of rupture of concrete.

The midspan deflection for a simply supported beam of span ‘L’ subjected to equal concentrated loads of magnitude \( W/2 \) symmetrically placed at a distance ‘a’ from the support is given by (Rafi et al 2007)

\[ \delta = \frac{W(a^3 - a)}{48E_c I_c} \]  
(8.7)

\( E_c \) in the above equation was calculated using ACI 440.1R formula given by

\[ E_c = 4750\sqrt{f_c'} \]  
(8.8)

Table 8.1 indicates the comparison of mid span deflection of GFRP beams at service load resulted from experimental, ANSYS modeling and ACI formula. It showed that the experimental deflections and ANSYS deflections were similar for all the GFRP beams, whereas, the ACI deflection formula predicted 28% higher deflection than experimental in M20 grade concrete beam reinforced with 16mm GFRP bars. In the case of M40 grade concrete, the ACI formula resulted in 43%, 42% and 58% greater deflection in 16mm, 20mm and 24mm GFRP bars reinforced beams respectively, than experimental result. The ACI formula for beams reinforced with 16mm,
20mm and 24mm GFRP bar in M60 grade concrete overestimated the deflection of about 64%, 97% and 139% respectively compared to tested deflection.

Table 8.1 Comparison of Deflection at Service Load

<table>
<thead>
<tr>
<th>Beam</th>
<th>Service load (kN)</th>
<th>Deflection (mm)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>ANSYS</td>
<td>ACI</td>
</tr>
<tr>
<td>M20-D16</td>
<td>51</td>
<td>5.16</td>
<td>5.45</td>
<td>6.63</td>
</tr>
<tr>
<td>M20-D20</td>
<td>56</td>
<td>5.68</td>
<td>5.36</td>
<td>5.77</td>
</tr>
<tr>
<td>M20-D24</td>
<td>66</td>
<td>5.56</td>
<td>6.10</td>
<td>5.81</td>
</tr>
<tr>
<td>M40-D16</td>
<td>72</td>
<td>5.47</td>
<td>5.11</td>
<td>7.86</td>
</tr>
<tr>
<td>M40-D20</td>
<td>84</td>
<td>5.87</td>
<td>5.76</td>
<td>8.36</td>
</tr>
<tr>
<td>M40-D24</td>
<td>91</td>
<td>4.57</td>
<td>4.18</td>
<td>7.24</td>
</tr>
<tr>
<td>M60-D16</td>
<td>78</td>
<td>3.72</td>
<td>3.33</td>
<td>6.14</td>
</tr>
<tr>
<td>M60-D20</td>
<td>94</td>
<td>4.08</td>
<td>3.76</td>
<td>8.04</td>
</tr>
<tr>
<td>M60-D24</td>
<td>106</td>
<td>3.23</td>
<td>2.95</td>
<td>7.74</td>
</tr>
</tbody>
</table>

The comparison of laboratory recorded load deflection values with ACI calculation, and ANSYS modeling for their flexural behaviour are presented in Figures 8.8 - 8.10.
Figure 8.8  Comparisons of Load Deflection for Experimental, ANSYS and ACI for M20-D16, M20-D20 and M20-D24 Beams
Figure 8.9  Comparisons of Load Deflection for Experimental, ANSYS and ACI for M40-D16, M40-D20 and M40-D24 Beams
Figure 8.10 Comparisons of Load Deflection for Experimental, ANSYS and ACI for M60-D16, M60-D20 and M60-D24 Beams

In the comparison curves, the load deflection curve drawn for ACI results, predicted the original behaviour of GFRP beams subjected to loading. The curves in all the beams were initially had steep slope which denoted the stiffness of the GFRP beams before cracking. Later on, the slope of the curves got decreased and became gradual, which indicated the reduction in the stiffness of the GFRP beams. But the ACI expression underestimated the effective moment of inertia of the beams in higher grades of concrete. There was a favourable comparison between the experimental measured deflection and the deflection value from finite element analysis using ANSYS. It
indicated that the developed model is accurate and it is possible to determine the deflections of GFRP reinforced concrete beams quite accurate by using the model. Since ACI overestimated the deflection values, it needs to be modified to account for the properties of FRP materials.

8.4 CRACKWIDTH PREDICTION - ACI FORMULA

The crack width and its propagation path influence the stresses induced in the GFRP bars and stirrups crossing the cracks, and the concrete above the crack tip. The crack geometry and crack width are the critical parameters governing the contribution of stirrups, beam strength and failure modes. A reliable prediction of crack width for the assessment of flexural strength of GFRP beams is essential. Hence, the crack width of GFRP reinforced beams was calculated by ACI guide. To ensure the consistency of the recorded crack width, a comparative study was made between the measured values and ACI formula given in Equation (8.9).

\[
w = 2 \times \frac{f_{\text{f}}}{E_f} \times \beta \times k_b \times \sqrt{d_e^2 + \left(\frac{s}{2}\right)^2}
\]  \hspace{1cm} (8.9)

The comparison was made for all the grades of concrete with change in tension reinforcement ratio of the GFRP reinforced beams. The tested results compared had favourable similarity with ACI code. Figures 8.11-8.16 show the comparative plot of load crack width for M20, M40, and M60 grade of concrete.
Figure 8.11 Load Crack Width Curves for Experimental and ACI for M20-D16 and M20-D20 Beams

Figure 8.12 Load Crack Width Curves for Experimental and ACI for M20-D24 Beam
Figure 8.13 Load Crack Width Curves for Experimental and ACI for M40-D16 and M40-D20 Beams

Figure 8.14 Load Crack Width Curves for Experimental and ACI for M40-D24 Beam
In M20 and M40 grade concrete, the load crack width curves were similar for 16mm and 20mm diameter reinforced GFRP beams, whereas beams reinforced with 24mm diameter bars had higher crack width in ACI calculation. In M60 grade concrete, only beams reinforced with 16mm bar showed similar curve. ACI predicted wider crack width values when there was increase in tension reinforcement ratio in M60 grade concrete.
8.5 SUMMARY

The deflections of finite element model developed by ANSYS 12.0 software are consistent with the experimental values of deflection for each stage of loading. The ACI formula predicted the crack width similar to that of experimental values for beams with lower reinforcement ratio at tension side in normal and medium strength concrete.