Preface

The present thesis entitled “Studies in the Differential Geometry of Special Finsler Spaces” is an outcome of investigations carried out by me in the Department of Mathematics and Statistics, D.D.U. Gorakhpur University, Gorakhpur under the supervision of Prof. H. S. Shukla.

The thesis consists of six chapters and each chapter has been further subdivided into a number of articles. References to the equation are of the form (C.A.E) where C denotes the number of the chapter, A stands for the number of the article and E stands for the number of the equation in the article. The numbers in the square brackets in a chapter correspond to the references given at the end of the chapter. The symbol $\partial_i$ and $\delta_i$ denote the partial derivatives with respect to $x^i$ and $y^i$ respectively. Small and long vertical lines ($\mid$ and $\mid$) stand for the h- and v-covariant derivatives respectively.

The first chapter is introductory and it begins with brief sketch of the history of Finsler geometry. It also deals with basic definitions of related words such as line-element, Finsler space, metric tensor, connections, geodesics, special Finsler spaces etc. which are used in this thesis.

The second chapter entitled “Conformal Kropina change of a Finsler space with $(\alpha, \beta)$–metric of Douglas type” is the study of several conditions under which a Finsler space with $(\alpha, \beta)$–metric is of Douglas type which has been introduced by M. Matsumoto. In this chapter the
condition that conformal Kropina change of a Finsler space with 
\((\alpha, \beta)\)-metric of Douglas type yields a space of Douglas type is found.

In the third chapter entitled “Kropina–Randers change of Finsler 
metric” the necessary and sufficient conditions under which Kropina–
Randers change becomes a projective change have been found. In this 
chapter the conditions under which a Kropina–Randers change of 
Douglas space becomes a Douglas space have also been investigated. The 
Kropina–Randers change of a Riemannian space has been discussed as a 
particular case.

The fourth chapter entitled “Matsumoto change of Finsler metric” 
is devoted to find the necessary and sufficient conditions under which a 
Matsumoto change becomes a projective change. The conditions under 
which a Matsumoto change of Douglas space becomes a Douglas Space 
have also been found. Matsumoto change of a Riemannian space has been 
discussed as a particular case.

The fifth chapter entitled “Relation between imbedding class 
numbers of tangent Riemannian spaces of 
\((M^n, L)\) and \((M^n, L^*)\)” is the 
study of the relation between imbedding class numbers of tangent 
Riemannian spaces of \((M^n, L)\) and \((M^n, L^*)\), where the Finsler metric \(L^*\) is 
obtained from \(L\) by \(L^* = \frac{L^2}{\beta} + \beta\), \(M^n\) is the differentiable manifold and 
\(\beta(x, y) = b_i(x) y^i\). It has been proved that if \((M^n, L^*)\) is a locally 
Minkowskin n-space obtained from a locally Minkowskin n-space \((M^n, L)\) 
by the change \(L^* (x, y) = \frac{L^2(x, y)}{\beta(x, y)} + \beta(x, y)\), and the tangent Riemannian n-
space \((M^n_x, g_x)\) to \((M^n, L)\) is of imbedding class \(r\), then tangent Riemannian n-space \((M^n_x, g^*_x)\) to \((M^n, L^*)\) is of imbedding class at most \(r + 2\).

The sixth chapter entitled “The exponential change of Finsler metric by h-vector and relation between imbedding class numbers of their tangent spaces” is devoted to the study of the relation between imbedding class numbers of tangent Riemannian spaces of \((M^n, L)\) and \((M^n, L^*)\), where the Finsler metric \(L^*\) is obtained from \(L\) by \(L^* = L \cdot e^{\beta/L}\), \(M^n\) is the differentiable manifold, \(\beta(x, y) = b_i(x, y) y^i\) and \(b_i(x, y)\) is an h-vector. The theorem proved in the fifth chapter for imbedding class number holds good for this change of Finsler metric also.
Research Activities

(A) Published / Communicated Research Papers


5. The Exponential Change of Finsler Metric by \(h\)-vector and Relation between Imbedding Class Numbers of their Tangent Spaces (communicated).

(B) Conference attended

1. 1\textsuperscript{st} U.P. Science Congress organized by D.D.U. Gorakhpur University, Gorakhpur on March 2, 3 & 4, 2013.