SYNOPSIS

The theory of univalent and multivalent functions, during the current century, has been one of the most fertile fields of investigation by eminent mathematicians who have enriched it by way of introduction of numerous new concepts, ingenious and powerful techniques. The settlement of the Bieberbach conjecture by Louis de Branges not only created a landmark in the history of the subject but also aroused the curiosity and interest of the non-specialist in this branch of study. In fact, mathematicians, have started wondering for some more provocative and challenging problems in this area of research.

The present thesis which is devoted to the study of certain subclasses of univalent and multivalent functions consists of six chapters. Chapter I being introductory in nature, consists of some known definitions and results that are used in the succeeding chapters.

Chapter II is devoted to the study of a subclass $V_{n,p}^\lambda (A,B,\lambda)$ of $p$-valent analytic functions. We determine a sufficient condition and some coefficient inequalities for functions in $V_{n,p}^\lambda (A,B,\lambda)$. We obtain comparable results, convolution type results for functions belonging to this class and also study the class preserving

Chapter III deals with the study of a subclass of $T$, namely $T_n^*(\alpha, \beta)$ involving Ruscheweyh derivatives. We determine some coefficient inequalities for functions belonging to this class. These results are then used to derive a number of inclusion relations. We, further, deduce substantially more general distortion theorems involving fractional derivatives and fractional integral operators for functions belonging to the class $T_n^*(\alpha, \beta)$. The class preserving integral transforms and certain results on quasi-Hadamard product of functions in $T_n^*(\alpha, \beta)$ are established. The results obtained in this chapter along with a number of new results give the corresponding work of Gupta and Jain [Bull. Austral. Math. Soc., 14 (1976), 409-416] and Silverman [Proc. Amer. Math. Soc., 51 (1975), 109-116].
In Chapter IV, we introduce the classes $P_n(a;b,M)$, $F_n(a;b,M)$ and $G_n(a;b,M)$ respectively, and investigate how the second coefficient in the power series expansion of functions belonging to these classes affects certain properties such as distortion, $\gamma$-spiral radius and radius of starlikeness of these classes. The results of this chapter along with certain new results yield the results of Ahuja [The Yokohama Math. Journ., 34 (1986), 3-13], Aouf [Int. J. Math. and Math. Sci., 12 (1989), 113-118], Kapoor and Mishra [Houston J. Math., 8 (1982), 85-92] and Mogra and Ahuja [The Yokohama Math. Journ., 29 (1981), 145-156].

In Chapter V, we define the class $S_n^*(A,B)$ and give a criteria for normalized analytic functions to be in $S_n^*(A,B)$. We also obtain results for the image domain of $(D^n f(z)/z)^\mu$, where $\mu \neq 0$ is a complex constant, $f \in S_n^*(A,B)$ and $D^n f$ denotes the nth-order Ruscheweyh derivative of $f$. Finally, an estimate for the real part of the functions $(D^n f(z)/z)$ is obtained for functions $f$ satisfying the condition $\text{Re}\{D^n f(z) \cdot D^{n+1} f(z)/z^2\} > \alpha$ ($0 \leq \alpha < 1$) in the unit disc. The results found here give the corresponding work of Obradovic [Mat. Vesnik, 36 (1984), 226-270] and Obradovic and Owa [J. Math. Anal. Appl., 145 (1990), 357-364] and also yield new results.
Finally, in Chapter VI, we study the class \( S^*_n(\alpha) \) (\( 0 \leq \alpha < 1 \)) which is defined in terms of the differential operator studied by Salagean [Lecture notes in Math., Springer-Verlag, 1013 (1983), 362-372]. We derive some results on quasi-Hadamard product of two or more functions for functions belonging to the class \( S^*_n(\alpha) \). Further, distortion theorems for this class is established in terms of a general class of fractional integral operators involving hyper geometric series. The results obtained here besides generalizing some of the work of Silverman [Proc. Amer. Math. Soc., 51 (1975), 109-116] improves the work of Kumar [J. Math. Anal. Appl., 126 (1987), 70-77] and Owa [Tamkang J. Math., 14 (1983), 15-21].