SYNOPSIS

The thesis entitled "Some aspects of Bimetric Geometry proposed by Karade & Nahatkar" incorporates six chapters and is mainly devoted to the study of some of the mathematical aspects of bimetric quantities with regard to five dimensional plane symmetric, spherically symmetric, and cylindrically symmetric space-times. The first chapter is introductory and gives brief information which is relevant and necessary for understanding the work done in the subsequent chapters.

In chapters II, III and IV we have studied the behaviour of bimetric quantities $K^h_{nm}, A^h_{nm}, K^{nm}, A^{nm}, B^{nm}, D^{nm}, K_g & K_0$ in five dimensional plane symmetric, spherically symmetric, and cylindrically symmetric space-times.

In the chapter II, we consider a canonical form of Spherically Symmetric Space-Times (SSST's) of higher dimension,

$$ds^2 = -e^{2\alpha} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2) + e^{2\beta} (dt^2 + du^2),$$

where $\alpha$ and $\beta$ are functions of $r$ and $t$, which was rigorously derived by Takeno (1966) from the very definition of spherical symmetry. The expressions for the various bimetric quantities starting from the Christoffel tensor to $K_g & K_0$ have been computed. Their expressions are put in compact forms and consequently the theorems are derived. It is established that the tensors $K_{nm}, A_{nm}, B_{nm}$ & $D_{nm}$ are spherically symmetric, in the light of definition of Takeno (1966). The tensor $A_{nm}$ is

(i)
expressed in terms of six functions $b_i, i = 1, ..., 6$, whereas $K_{nm}, B_{nm}$ & $D_{nm}$ are given in terms of five functions of $r$ and $t$.

The principal invariants of bimetric covariant tensors $K_{nm}, A_{nm}, B_{nm}$ & $D_{nm}$ are investigated.

The five eigen values of the bimetric spherically symmetric tensor $K_{nm}$ are spherically symmetric scalars and are of the form $(v_1, v_2, v_3, v_4)$. The form of eigen values of bimetric spherically symmetric tensor $K_{nm}$ with $\alpha = \alpha(t)$ and $\beta = \beta(t)$ is $(0, 0, v_2, v_3, v_4)$.

The five eigen values of the bimetric spherically symmetric tensor $A_{nm}$ & $D_{nm}$ are spherically symmetric scalars and are of the form $(v_1, v_2, v_3, v_4, v_5)$. The form of eigen values of bimetric spherically symmetric tensor $A_{nm}$ with $\alpha = \alpha(t)$ and $\beta = \beta(t)$ is $(v_1, v_2, 0, v_4, v_5)$.

The five eigen values of the bimetric spherically symmetric tensor $B_{nm}$ are spherically symmetric scalars and are of the form $(v_1, v_1, v_2, v_3, v_4)$ where we get a pair of double eigen values.

The spherically symmetric five dimensional space-time of various categories are as follow:

The spherically symmetric static canonical space-times of categories C(II) and C(III) are established in the form of

$$ds^2 = -dr^2 - r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) + (dt^2 + du^2).$$

The spherically symmetric non-static canonical space-time of category C(I) is of the form,

$$ds^2 = e^{2kt} (du^2 + dt^2) - dr^2 - r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2).$$

(ii)
The spherically symmetric non-static canonical space-times of categories C(II), C(III) & C(IV) reduces to line element of special relativity,

\[ ds^2 = -dr^2 - r^2 \left( d\theta^2 + \sin^2\theta \: d\phi^2 \right) + \left( dt^2 + du^2 \right) . \]

Chapter III is devoted to the study of cylindrically symmetric bimetric quantities by considering the Einstein-Rosen five dimensional space-times

\[ ds^2 = e^{2A-2B} \left( dt^2 - d\rho^2 \right) - \rho^2 e^{-2B} \: d\phi^2 - e^{2B} \left( dz^2 + du^2 \right) \]

where A & B are functions of \( \rho \) & \( t \). The form of the various bimetric quantities \( K^b_{nmp}, A^b_{nmp}, K_{nm}, A_{nm}, B_{nm}, D_{nm} \), \( K_g & K_a \) are investigated and expressed in the compact forms which are expressed in the form of theorems.

Moreover the principal invariants of bimetric covariant tensors \( K_{nm}, A_{nm}, B_{nm} & D_{nm} \) are determined. The investigation carried out result as:

The cylindrically symmetric bimetric tensors \( K_{nm} & A_{nm} \) are specified by five functions of \( \rho \) and \( t \) whereas those of \( B_{nm} & D_{nm} \) are specified by the six functions of \( \rho \) and \( t \).

The eigenvalues of the bimetric tensor \( K_{nm}, A_{nm}, B_{nm} \) & \( D_{nm} \) of cylindrically symmetric five dimensional Einstein-Rosen space-times are cylindrically scalars and are of the form \( (\nu_1, \nu_2, \nu_3, \nu_4) \).

Further it is established that, in the bimetric geometry the cylindrically symmetric non-static space-times of categories C(I), C(II) & C(III) are the space-times of special relativity.

The static cylindrically symmetric space-time of categories C (II) & C(III) reduces to space-time of special relativity.

(iii)
Chapter IV deals with the behaviour of bimetric quantities $K_{nm}^{k}, \Lambda_{nm}^{k}, K_{nm}, A_{nm}, B_{nm}, D_{nm}, K_g & K_a$ with regard to five dimensional plane symmetric space-time where the line element has the form

$$ds^2 = e^{2A} (dt^2 - dx^2) - e^{2B} (dy^2 + dz^2 + du^2),$$

where $A$ and $B$ are functions of $x$ & $t$. The values of the bimetric quantities $K_{nm}^{k}, \Lambda_{nm}^{k}$ are expressed in the form of functions of $x$ & $t$ and results are given in the form of theorems in the text.

The plane symmetric bimetric covariant tensors of order two $K_{nm}, A_{nm}$, & $B_{nm}$ are also specified by four functions of $x$ and $t$, where as the bimetric covariant tensors $D_{nm}$ is specified by five functions of $x$ & $t$. The investigations are stated in the form of theorems. The bimetric scalar curvature $K_g$ & $K_a$ are also computed.

The study of the principal invariants of second rank tensors with regard to five dimensional plane symmetric space-times leads to the following facts:

The tensors $K_{nm}, A_{nm}, B_{nm}$ & $D_{nm}$ all have the eigen values of the form $(v_1, v_1, v_1, v_2, v_3)$, where we get triple eigen value. The form of eigen values of the bimetric plane symmetric tensor $K_{nm}$ with $A = A(t)$ and $B = B(t)$ or $A = A(x)$ and $B = B(x)$ is $(v_1, v_2, v_2, v_2, v_3)$.

The plane symmetric static space-time of category C(I) is of the form,

$$da^2 = e^{2au} (dt^2 - dx^2 - du^2 + dz^2 + du^2).$$

It is conformal to the flat space-time.

The plane symmetric static space-time C(II) has the form

(iv)
\[ ds^2 = e^{2A(x)}(dt^2 - dx^2) - dy^2 - dz^2 - du^2. \]

The plane symmetric static space-time \( C(III) \) is the flat space-time of special relativity.

The plane symmetric static space-time \( C(I) \) & \( C(II) \) are conformally related.

The plane symmetric static space-time \( C(IV) \) is of the form
\[ ds^2 = e^{f(x)}(dt^2 - dx^2) - e^{-f(x)}(dy^2 + dz^2 + du^2). \]

The plane symmetric non-static space-time \( C(I) \) is of the form
\[ ds^2 = e^{2\lambda x}(dt^2 - dx^2 - dy^2 - dz^2 - du^2). \]

It is conformal to the flat space-time.

The plane symmetric non-static space-time of category \( C(II) \) can be put in the form of space-time
\[ ds^2 = e^{2\lambda x}(dt^2 - dx^2 - dy^2 - dz^2 - du^2). \]

The plane symmetric non-static space-time \( C(III) \) has the form
\[ ds^2 = e^{g(t)}(dt^2 - dx^2) - e^{-g(t)}(dy^2 + dz^2 + du^2). \]

The plane symmetric non-static space-time \( C(IV) \) is the space-time of special relativity.

Chapter V is devoted to the study of behaviour of bimetric quantities \( K_{nmp}^*, A_{nmp}^*, K_{nm}, A_{nm}, B_{nm}, D_{nm} \) & \( K_{inmp} \) obtained for Bianchi type-I space-time \( V_4 \). Special attention is paid to the behaviour of bimetric covariant curvature tensor \( K_{inmp} \).

It is investigated that \( K_{inmp} \) is not antisymmetric in its first two indices and not symmetric in pair as compared to its counterpart \( R_{inmp} \) in Riemannian geometry.
The expressions for various bimetric quantities have been computed and their expression are put in compact forms and consequently the theorems are derived. It is investigated that each of the bimetric tensors $A_{nm}$, $B_{nm}$ & $D_{nm}$ corresponding to Bianchi type-I space-time are specified by five functions of $x$ & $t$ where as the tensor $K_{nm}$ is specified by four functions of $x$ & $t$.

The principal invariants of bimetric covariant tensors $K_{nm}$, $A_{nm}$, $B_{nm}$ & $D_{nm}$ are investigated.

The four eigen values of the bimetric covariant tensor $K_{nm}$ with reference of Bianchi type-I metric are scalars and are of the form $(v_1, -v_1, v_2, v_3)$.

And that those of $A_{nm}$, $B_{nm}$ & $D_{nm}$ are of the form $(v_1, v_2, v_3, v_4)$.

The Bianchi type-I static space-time of categories C(I) & C(II) has the form of flat space-time,

$$ds^2 = dt^2 - e^{2A(x)} dx^2 - (dy^2 + dz^2).$$

Further it is established that Bianchi type-I non-static space-time of categories C(II), C(III) & C(IV) reduces to flat space-times of special relativity.

In chapter VI we have presented five dimensional and n-dimensional static plane symmetric vacuum solutions in bimetric theory of gravitation proposed by Rosen (1973). This work is an extension, in higher dimensional space-time, of investigation obtained earlier by Mohanty and Sahoo (2001).

It is investigated that the higher dimensional static plane symmetric solution do not exist in bimetric theory of relativity when the source is
either perfect fluid, mesonic massive scalar field or their coupling.

Five dimensional static plane symmetric vacuum solution in bimetric relativity leads to

\[ ds^2 = e^{2c_1x} (dt^2 - dx^2) - e^{2c_2x} (dy^2 + dz^2 + du^2). \]

It is interesting to note that, this model is free from singularity. At \( c_1 = c_2 \) it reduces to a conformally flat. For \( c_1 = c_2 = 0 \) it reduces to flat model.

\( n \)-dimensional static plane symmetric vacuum space-time in bimetric theory leads to

\[ ds^2 = e^{2\lambda x} \left\{ dt^2 - \left( dx^1 \right)^2 \right\} - e^{2\mu x} \left\{ \sum_{i=2}^{n-1} \left( dx^i \right)^2 \right\}, \]

where \( \lambda \) & \( \mu \) are the constants.

This model is also free from singularity. For \( \lambda = \mu \), the model becomes a conformal. For \( \lambda = \mu = 0 \), it reduces to flat one.

Here we like to point out that the five dimensional static plane symmetric vacuum space-time in bimetric relativity has closed resemblance to the five dimensional static plane symmetric space-time of category \( \text{C(I)} \) in bimetric geometry. Former one reduces to later for \( c_1 = c_2 \).

\[ \text{(vii)} \]