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Non-existence of Five Dimensional Static Plane Symmetric Solutions in Bimetric Relativity

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Abstract:
A five dimensional static plane symmetric space - time model in Rosen's (1973) bimetric theory of gravitation is considered. It is shown that, in this theory, the geometry of the five dimensional space - time does not admit perfect fluid source or mesonic massive scalar field or their coupling.

1. Introduction:
Rosen (1973) has modified the formalism of the general relativity theory by introducing into it, besides the metric tensor $g_{ij}$ associated with the line element

$$ds^2 = g_{ij} dx^i dx^j$$

(1.1)

a second metric tensor $f_{ij}$ corresponding to flat space-time described by the line element

$$dc^2 = f_{ij} dx^i dx^j$$

(1.2)
at each point of space-time.

The tensor $g_{ij}$ describes gravitation and interacts with matter. The background metric $f_{ij}$ has no direct physical significance but appears in the field equations. Therefore, it interacts with $g_{ij}$ but not directly with matter. One can regard $f_{ij}$ as giving the geometry that would exist if there were no matter.

This bimetric theory of gravitation satisfies the covariance and equivalence principles. Rosen [1], [2], [3], [4], [5], Israelit [6], [7], Karade and Dhoble [8], Karade [9], Reddy and Venkateswarulu [10], Reddy and Rao [11], Karade et al. [12], [13], G.Mohanty et al. [14], [15], are some of the eminent authors who have studied several aspects of the bimetric theory of gravitation.

In the present paper we have extended the work of G.Mohanty and P.K.Sahoo [15] for five dimensional space - time.

2. Metrics and field equations:
We consider five dimensional static plane symmetric space - time model of the form

$$ds^2 = e^{2\alpha} (dt^2 - dx^2) - e^{2\beta} (dy^2 + dz^2 + du^2),$$

(2.1)

where $\alpha$, $\beta$ are the functions of $x$ only.

The flat metric corresponding to the metric (2.1) is

$$dc^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2,$$

(2.2)
The field equations of the bimetric theory of gravitation formulated by Rosen (1973) are

\[ N_a - \frac{1}{2} g_a N = -8\pi \kappa T_{ij} , \]  \hspace{1cm} (2.3)

where

\[ N^i = \frac{1}{2} \epsilon^{ik} (g^m_{ij} g_{mk}) i , \]  \hspace{1cm} (2.4)

\[ N = N^i , \quad f = \det (f^i_j) , \quad g = \det (g_{ij}) , \quad K = \sqrt{g} f . \]

T is the usual energy momentum tensor of the matter satisfying conservation law

\[ T^i_j = 0 , \]  \hspace{1cm} (2.5)

where (\cdot) denotes the co-variant differentiation with respect to \( g \) and (\cdot) denotes the co-variant differentiation with respect to \( f \).

3. Perfect, Fluid Matter :

The energy momentum tensor for perfect fluid matter is given by

\[ \mathbf{T}_{ij} = (\mathbf{p} + \rho) \mathbf{U}_i \mathbf{U}_j - \mathbf{g}_{ij} \]  \hspace{1cm} (3.1)

together with

\[ \mathbf{g}_{ij} \mathbf{U}^i \mathbf{U}^j = 1 , \]

where \( \mathbf{U}^i \) is the four velocity vector of the fluid having \( \mathbf{p} \) and \( \rho \) as the proper pressure and energy density respectively.

The field equations (2.3) for the metrics (2.1) and (2.2) with energy momentum tensor (3.1) in bimetric theory can be written as

\[ 3\beta_{11} = 16\pi k \mathbf{p} \]  \hspace{1cm} (3.2)

\[ 2\alpha_{11} + \beta_{11} = 16\pi k \mathbf{p} \]  \hspace{1cm} (3.3)

\[ 3\beta_{11} = -16\pi k \rho \]  \hspace{1cm} (3.4)

where \( k = e^{2\alpha + 3\beta} \)  \hspace{1cm} (3.5)

and suffix '1' after a field variable stands for differentiation with respect to co-ordinate x.

From the equation (3.2) and (3.4) we obtain

\[ \mathbf{p} + \rho = 0 . \]  \hspace{1cm} (3.6)

In fact \( \mathbf{p} > 0 , \quad \rho > 0 \), therefore equation (3.6) immediately yields

\[ \mathbf{p} = 0 , \quad \rho = 0 \text{ (vacuum)} . \]  \hspace{1cm} (3.7)

Thus, for the space-time (2.1) the perfect fluid matter does not survive in bimetric theory of gravitation. Hence we obtain vacuum model.

Vacuum Model

In this case field equations (3.2) - (3.4) becomes

\[ \alpha_{11} = 0 , \quad \beta_{11} = 0 \]  \hspace{1cm} (3.8)
which on solving yields

\[ \alpha = \alpha x + B, \quad \beta = \beta x + D, \]

(3.9)

where \( A, B, C \) and \( D \) are the constants of integration.

In view of equations (3.9), by adjusting constants the metric (2.1) takes the form

\[ ds^2 = e^{2\alpha x} (dt^2 - dx^2) - e^{2\beta x} (dy^2 + dz^2 + du^2). \]

(3.10)

This model is free from singularity. It is interesting to note that, for \( c_1 = c_2 \), the model (3.10) becomes a conformally flat. Furthermore, for \( c_1 = c_2 = 0 \), it reduces to flat model.

Thus, for the space-time (2.1), the perfect fluid matter does not exist in bimetric theory of gravitation and hence only a vacuum model is constructed.

4. Mesonic Massive Scalar Field:

In this section we consider the region of the space-time filled with mesonic massive scalar field whose energy momentum tensor is given by

\[ m^\sigma_{ij} = \sigma_i \sigma_j - \frac{1}{2} g_{ij} (\sigma_i \sigma_j - m^2 V^2) \]

(4.1)

together with the Klein-Gordon Equation

\[ \sigma = g^i_j \sigma_i \sigma_j + m^2 V, \]

(4.2)

where \( m \) is the mass parameter and \( \sigma \) is the source density of the massive scalar meson field.

The field equations (2.3) for the metric (2.1) and (2.2) with energy momentum tensor (4.1) becomes

\[ 3 \beta_{11} = -8 \pi K (e^{2\alpha} V_1^2 + m^2 V^2), \]

(4.3)

\[ 2 \alpha_{11} + \beta_{11} = -8 \pi K (e^{2\alpha} V_1^2 + m^2 V^2), \]

(4.4)

\[ 3 \beta_{11} = -8 \pi K (e^{2\alpha} V_1^2 + m^2 V^2). \]

(4.5)

Klein-Gordon equation (4.2) for the metric (2.1) can be written as

\[ \sigma = - (V_{11} + 3V_1 \beta_1) e^{2\alpha} + m^2 V. \]

(4.6)

Comparing equations (4.3) and (4.5) we obtain

\[ e^{2\alpha} V_1^2 = 0, \]

(4.7)

which implies \( V_1 = 0 \) i.e. \( V = \) constant.

Thus for the space-time (2.1) the matter field like mesonic massive scalar field does not survive in bimetric theory. In this case (4.6) shows that the source density becomes constant.

5. Coupling of Perfect Fluid and Mesonic Massive Scalar Field:

Consider the matter distribution consisting of perfect fluid \( ^nT_{ij} \) coupled with mesonic massive scalar field \( m^\sigma_{ij} \) and it is given by the energy momentum tensor

\[ T_{ij} = ^nT_{ij} + m^\sigma_{ij}. \]

(5.1)

The field equations (2.3) for the metrics (2.1) and (2.2) corresponding to the energy momentum tensor (5.1) is written as

\[ 3 \beta_{11} = -8 \pi k [-2b - e^{2\alpha} V_1^2 + m^2 V^2], \]

(5.2)
Using (5.2) and (5.4) we obtain
\[ p + c - 2a V^2 = 0. \] (5.5)
Since \( p > 0 \) and \( \rho > 0 \), it clearly implies that \( \rho = 0, \ p = 0 \) and \( V = \text{constant} \).

Thus the coupling of mesonic massive scalar field with perfect fluid matter cannot be a source of gravitation in five dimensional space-time in bimetric theory of gravitation governed by a static plane symmetric metric of the from (2.1).

6. Conclusion:

Here we have studied the existence of static plane symmetric metric (2.1) in Rosen's [1] bimetric theory of gravitation in perfect fluid and mesonic massive scalar field. It is observed that matter fields like perfect fluid or mesonic massive scalar field or coupling of them can not be a source of gravitation in the Rosen's bimetric theory. Hence, only vacuum model is presented.

References:

NON-EXISTENCE OF $n$-DIMENSIONAL STATIC PLANE SYMMETRIC SOLUTIONS IN BIMETRIC RELATIVITY THEORY

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Abstract. A problem of static plane symmetric metric in the perfect fluid, the mesonic massive scalar field and in their coupling is studied in Rosen's (1973) bimetric theory of relativity. It was found that the matter field like either perfect fluid or mesonic massive scalar field or their coupling does not survive in bimetric theory of gravitation when the space–time is governed by $n$-dimensional static plane symmetric metric.

Keywords: bimetric, static, $n$-dimensional

1. Introduction

Rosen (1973) has modified the formalism of the general relativity theory by introducing into it, besides the metric tensor $g_{ij}$ associated with the line element

$$ds^2 = g_{ij} dx^i dx^j$$

(1.1)

a second metric tensor $f_{ij}$ corresponding to flat space–time described by the line element

$$d\sigma^2 = f_{ij} dx^i dx^j$$

(1.2)

at each point of space–time.

The tensor $g_{ij}$ describes gravitation and interacts with matter. The background metric $f_{ij}$ has no direct physical significance but appears in the field equations.

Therefore, it interacts with $f_{ij}$ but no directly with matter. One can regard $f_{ij}$ as giving the geometry that would exist if there were no matter. This Rosen's bimetric theory satisfies the covariance and equivalence principles.

In our earlier paper (Katore and Thakare, 2004b), we have investigated that the static plane symmetric space–time in bimetric relativity for six dimensions does

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not exist in perfect fluid or in mesonic massive scalar field or in their coupling. Only a vacuum model is obtained. In the present paper, we carry out this study for \( n \)-dimensional space-time.

2. Metrics and Field Equations

We consider \( n \)-dimensional static plane symmetric space–time in the form

\[
ds^2 = e^{2\alpha} [dt^2 - (dx^1)^2] - e^{2\beta} \sum_{i=2}^{n-1} (dx^i)^2 \tag{2.1}
\]

where \( \alpha, \beta \) are the functions of \( x \) only.

The flat metric corresponding to the metric (2.1) is

\[
d\sigma^2 = dt^2 - (dx^1)^2 - \sum_{i=2}^{n-1} (dx^i)^2. \tag{2.2}
\]

The field equations of the bimetric theory of gravitation formulated by Rosen are

\[
N^j_i - \frac{1}{2} \delta^j_i N = -8\pi k T^i_j, \tag{2.3}
\]

where

\[
N^j_i = \frac{1}{2} f^{ab} (g^{hr} g_{hj} a) b, \tag{2.4}
\]

\[
N = N^i_i, \quad f = \text{det}(f_{ij}), \quad g = \text{det}(g_{ij}), \quad k = \sqrt{g/f}. \tag{2.5}
\]

\( T^i_j \) is the usual energy momentum tensor of the matter satisfying conservation law

\[
T^i_j = 0, \tag{2.5}
\]

where \( ; \) denotes the co-variant differentiation with respect to \( g_{ij} \) and \( (i) \) denotes the co-variant differentiation with respect to \( f_{ij} \).

3. Perfect Fluid Matter

The energy momentum tensor for perfect fluid matter is given by

\[
T^i_j = (p + \rho) U_i U_j - p g_{ij} \tag{3.1}
\]
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together with
\[ g_{ij} U^i U^j = 1, \]
where \( U^i \) is the velocity vector of the fluid for \( n \)-dimensional space–time, whereas \( p \) and \( \rho \) are the proper pressure and energy density, respectively.

Using co-moving co-ordinate system, the field Eq. (2.3) for the metrics (2.1) and (2.2) with energy momentum tensor (3.1) in bimetric theory can be written as

\[
\begin{align*}
(n - 2)\beta_{11} &= 16\pi kp, \\
2\alpha_{11} + (n - 4)\beta_{11} &= 16\pi kp, \\
(n - 2)\beta_{11} &= -16\pi kp.
\end{align*}
\]

where
\[ k = e^{2\alpha + (n - 2)p}, \]
and
\[ \alpha_1 = \frac{d\alpha}{dx}, \text{ etc.} \]

From the Eqs. (3.2) and (3.4), we obtain
\[ p + \rho = 0. \tag{3.6} \]

This equation of state is known as \( p \)-vacuum or false vacuum or degenerate vacuum (Davis, 1984). In view of reality conditions, \( p > 0 \) and \( \rho > 0 \), Eq. (3.6) immediately implies that
\[ p = 0, \quad \rho = 0 \tag{3.7} \]

Thus, it is found that the matter field, like perfect fluid in plane symmetric bimetric theory, does not survive and only vacuum model can be constructed.

3.1. VACUUM MODEL

In view of (3.7) the field Eqs. (3.2)–(3.4) yield the solutions
\[ \alpha = c_1 x + c_2, \quad \beta = c_3 x + c_4, \tag{3.8} \]
where \( c_1, c_2, c_3 \) and \( c_4 \) are the constants of integration.
Using (3.8), the metric (2.1) takes the form
\[ ds^2 = e^{2\lambda x} [dt^2 - (dx^i)^2] - e^{2\mu x} \sum_{i=2}^{n-1} (dx^i)^2 \]  
(3.9)

This model (3.9) is free from singularity. It is interesting to note that, for \( \lambda = \mu \), the model (3.9) becomes a conformal. Furthermore, for \( \lambda = \mu = 0 \), it reduces to flat one.

4. Mesonic Massive Scalar Field

In this section, we consider the region of the space-time filled with mesonic massive scalar field whose energy momentum tensor is given by
\[ T_{ij}^M = V_i V_j - \frac{1}{2} g_{ij}(V_k V^k - m^2 V^2) \]  
(4.1)

together with the Klein-Gordon equation
\[ \sigma = g^{ij} V_{ij} + m^2 V, \]  
(4.2)

where \( m \) is the mass parameter and \( \sigma \) the source density of the massive scalar meson field.

The field Eq. (2.3) for the metric (2.1) and (2.2) with energy momentum tensor (4.1) becomes
\[ (n - 2)\beta_{11} = -8\pi k (e^{-2\alpha} V_1^2 + m^2 V^2), \]  
(4.3)
\[ 2\alpha_{11} + (n - 4)\beta_{11} = -8\pi k (e^{-2\alpha} V_1^2 + m^2 V^2), \]  
(4.4)
\[ (n - 2)\beta_{11} = -8\pi k (e^{-2\alpha} V_1^2 + m^2 V^2). \]  
(4.5)

Klein-Gordon Eq. (4.2) for the metric (2.1) can be written as
\[ \sigma = -[V_{11} + (n - 2) V_1 \beta_1] e^{-2\alpha} + m^2 V. \]  
(4.6)

Comparing Eqs. (4.3) and (4.4), we obtain
\[ e^{-2\alpha} V_1^2 = 0, \]  
(4.7)

which implies \( V_1 = 0 \), i.e., \( V = \) constant.

Thus, for the space-time (2.1) the matter field, like mesonic massive scalar field, with or without mass parameter, does not survive in bimetric theory. Eq. (4.6) shows that the source density becomes constant.
5. Coupling of Perfect Fluid with Mesonic Massive Scalar Field

Consider the matter distribution consisting of perfect fluid $T_{ij}^p$ coupled with mesonic massive scalar field $T_{ij}^M$ and it is given by the energy momentum tensor

$$T_{ij} = T_{ij}^p + T_{ij}^M.$$  \hspace{1cm} (5.1)

The field Eq. (2.3) for the metrics (2.1) and (2.2) corresponding to the energy momentum tensor (5.1) is written as

$$(n - 2)\beta_{11} = -8\pi k\left[-2\rho - e^{-2\alpha}V_1^2 + m^2V_2^2\right],$$  \hspace{1cm} (5.2)

$$2\alpha_{11} + (n - 4)\beta_{11} = -8\pi k\left[-2\rho + e^{-2\alpha}V_1^2 + m^2V_2^2\right],$$  \hspace{1cm} (5.3)

$$(n - 2)\beta_{11} = -8\pi k\left[2\rho + e^{-2\alpha}V_1^2 + m^2V_2^2\right].$$  \hspace{1cm} (5.4)

Using (5.2) and (5.4), we obtain

$$(p + \rho) + e^{-2\alpha}V_1^2 = 0.$$  \hspace{1cm} (5.5)

Since $p > 0$, $\rho > 0$, it clearly implies that $\rho = 0$, $\rho = 0$ and $V = constant$.

Thus, coupling of perfect fluid with mesonic massive scalar field cannot be a source of gravitation in bimetric theory when the space–time is governed by a static plane symmetric metric of the form (2.1).

6. Conclusion

We conclude that the matter field, like either perfect fluid or mesonic massive scalar field or their coupling, does not survive in bimetric theory of gravitation when the space–time is governed by $n$-dimensional static plane symmetric metric of the form (2.1). Only a vacuum model (3.9) exists and is found to be static conformally flat for $\lambda = \mu$. Furthermore, for $\lambda = \mu = 0$, it reduces to flat one.

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References

BIMETRIC PLANE SYMMETRIC QUANTITIES IN HIGHER DIMENSIONAL SPACE-TIME
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ABSTRACT
In this paper bimetric covariant tensors of order two are investigated in bimetric geometry proposed by Karade and Nahatkar (Proc. Einstein Found., 12, 20-30, 2001) in the higher dimensional plane symmetric space-times. The space-times of various categories C(I), C(II), C(III) and C(IV) are also studied.

Key words: Relativity, Bimetric geometry, Plane symmetry, Higher dimensional.

1. INTRODUCTION
Various attempts have been made to introduce two metric at each point of space by Rosen (1940), Eisenhart (1966) and Logunov and Mestvirishvili (1989). The geometry to that effect was applied by Nathan Rosen (1940) in introducing a theory well-known as bimetric theory of relativity of Rosen or bimetric theory of gravitation, which differs from the general theory of relativity of Einstein (1912, 1915) and which was later developed by many scientists Falik and Rosen (1980, 1981), De Liebscher (1975), Yilmaz (1977), Goldman and Rosen (1977), Rosen and Rosen (1977), Israelit and Rosen (1983), Israelit (1979, 1981), Rosen (1973, 1974, 1975, 1979, 1980, 1983). This theory has some advantages over the general relativity; the quantities such as christoffel symbols and others become tensors which otherwise in Riemannian geometry they are not.

Recently M. Nahatkar (2002) have investigated the various bimetric quantities in plane symmetric space-times. Present work is the extension of the same in higher dimensional plane symmetric space-times. The behaviour of bimetric quantities $K_{mn}$, $A_{mn}$, $B_{mn}$ and $D_{mn}$ in plane symmetric higher dimensional space-time is studied. The space-time of various categories C(I), C(II), C(III) and C(IV) are also studied.

We consider the plane symmetric metric

$$ds^2 = e^{2A}(dt^2 - dx^2) - e^{2B}(dy^2 + dz^2 + du^2), \quad (1.1)$$

where $A$ and $B$ are functions of $x$ and $t$ with the convention

$x^i = (x, y, z, u, t), i = 1, 2, 3, 4, 5.$

or our purpose, we adopt $g_{ij}$ as the metric (1.1) and $\alpha$ the metric

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2. \quad (1.2)$$

For the line elements (1.1) and (1.2), we have

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Then we compute
\[ g_{11} = -e^{-2A}, \quad g_{22} = g_{33} = g_{44} = -e^{-2B}, \quad g_{55} = e^{2A} \]
and \[ \alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha_{44} = -1, \alpha_{55} = 1. \]

Then we compute
\[ g_{11} = -e^{-2A}, \quad g_{22} = g_{33} = g_{44} = -e^{-2B}, \quad g_{55} = e^{2A} \]
and
\[ \alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha_{44} = -1, \alpha_{55} = 1, \text{ where } g = \det g_{mn} = e^{4A+6B} \text{ and } \alpha = \det \alpha_{nn} = 1. \]

2. BIMETRIC CHRISTOFFEL TENSOR

The non-vanishing \( g \) and \( \alpha \) Christoffel symbols are as follows
\[
\begin{align*}
\Gamma^1_{11} &= A', \quad \Gamma^1_{12} = \dot{A}, \quad \Gamma^1_{22} = g^1_{23} = g^1_{44} = -B' e^{2(B-A)}, \quad \Gamma^1_{55} = A', \\
\Gamma^2_{12} &= B', \quad \Gamma^2_{22} = B', \quad \Gamma^2_{33} = \dot{B}, \quad \Gamma^2_{44} = B', \quad \Gamma^2_{55} = B, \\
\Gamma^3_{11} &= \dot{A}, \quad \Gamma^3_{13} = A', \quad \Gamma^3_{22} = g^3_{23} = g^3_{33} = g^3_{44} = \dot{B} e^{2(B-A)}, \quad \Gamma^3_{55} = \dot{A},
\end{align*}
\]
and \[ \alpha_{nn} = 0 \forall \ n,m,n = 1, \ldots, 5. \]

Here, the prime & dot represents derivative with respect to \( x \) & \( t \) respectively.

Then the non-vanishing components of the bimetric Christoffel tensors are expressed in terms of six functions of \( x \) and \( t \) as follows:
\[
\begin{align*}
\Gamma^1_{11} &= (f_1,0,0,0,f_2), & \Gamma^1_{12} &= (0,f_1,0,0,0), & \Gamma^1_{22} &= (0,0,f_1,0,0), \\
\Gamma^1_{44} &= (0,0,0,0,0), & \Gamma^1_{55} &= (0,0,0,0,0), \\
\Gamma^2_{12} &= (0,f_1,0,0,0), & \Gamma^2_{22} &= (f_2,0,0,0,0), & \Gamma^2_{33} &= (0,0,f_1,0,0), \\
\Gamma^3_{13} &= (0,0,0,0,0), & \Gamma^3_{13} &= (f_3,0,0,0,0), & \Gamma^3_{22} &= (0,0,0,0,0), \\
\Gamma^4_{44} &= (0,0,0,0,0), & \Gamma^4_{55} &= (f_4,0,0,0,0), & \Gamma^4_{55} &= (0,0,0,0,0), \\
\Gamma^5_{13} &= (f_5,0,0,0,0), & \Gamma^5_{22} &= (0,f_5,0,0,0), & \Gamma^5_{33} &= (0,0,0,0,0), \\
\Gamma^6_{44} &= (0,0,0,0,0), & \Gamma^6_{55} &= (f_6,0,0,0,0), & \Gamma^6_{55} &= (f_6,0,0,0,0).
\end{align*}
\]

\[ \text{ (2.1) } \]
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where

\[ f_1 = A', f_2 = A, f_3 = -B' e^{2(B-A)}, f_4 = B', f_5 = \hat{B}, f_6 = \hat{B} e^{2(B-A)}. \]  

(2.2)

The above study leads to the theorem:

**Theorem 1**: The bimetric plane symmetric Christoffel tensor in five dimensional space-time is specified by the six functions \( f_i(x, t), i = 1, \ldots, 6 \), which are given by (2.2).

3. **BIMETRIC COVARIANT TENSORS OF ORDER TWO**

(a) **The tensor** \( K_{nm} \)

By definition, \( K_{nm} = \Gamma^e_{nm} \Gamma^p_{ep} - \Gamma^e_{np} \Gamma^p_{em} \) or \( K_{nm} = K_{nm}^e \).

The bimetric plane symmetric covariant tensor \( K_{nm} \) can be expressed in the form of four functions \( I_1, \ldots, I_4 \) of \( x \) and \( t \) as

\[
\begin{align*}
K_{1k} &= (I_1, 0, 0, 0, I_2), \\
K_{2k} &= (0, I_1, 0, 0, 0), \\
K_{3k} &= (0, 0, I_2, 0, 0) \\
K_{4k} &= (0, 0, 0, I_3, 0), \\
K_{5k} &= (I_4, 0, 0, 0, I_4)
\end{align*}
\]

where

\[
\begin{align*}
I_1 &= 3(A'B' - B'^2 + \hat{A}\hat{B}), \\
I_2 &= (3\hat{A}'B' - BB'^2 + BA'), \\
I_3 &= (-2A'B' + 2\hat{A}\hat{B} - 2A\hat{B}' + 2\hat{B}A' + 2\hat{B}\hat{B}' - \hat{B}'^2 + B'^2)e^{2(B-A)}, \\
I_4 &= 3(A'B' + \hat{A}\hat{B} - \hat{B}'^2)
\end{align*}
\]

(3.1)

(b) **The tensor** \( A_{nm} \)

The non-vanishing components are

\[
\begin{align*}
A_{1k} &= (H_1, 0, 0, 0, H_2), \\
A_{2k} &= (0, H_1, 0, 0, 0), \\
A_{3k} &= (0, 0, H_1, 0, 0), \\
A_{4k} &= (0, 0, 0, H_1, 0), \\
A_{5k} &= (H_2, 0, 0, 0, H_4)
\end{align*}
\]

(3.3)
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where

\[ H_1 = 3B^* + A^* - \ddot{A}, \]
\[ H_2 = 3\dot{B}, \]
\[ H_3 = (-2A'\dot{B} + 2\dot{A}\dot{B} + 2B^2 - 2\ddot{B} + B^* - \ddot{B})e^{2(B-A)}, \]
\[ H_4 = \ddot{A} - A^* + 3\dot{B}. \]

\[(3.4)\]

\((c)\) The tensor \( B_{nm} \)

The total number of components are four and are as:

\[ B_{1k} = (J_1,0,0,0,J_2), \]
\[ B_{2k} = (0,J_3,0,0,0), \]
\[ B_{3k} = (0,0,J_4,0,0), \]
\[ B_{4k} = (0,0,0,J_1,0), \]
\[ B_{5k} = (J_2,0,0,0,J_4) \]

\[(3.5)\]

where \( J_1, \ldots, J_4 \) are the functions of \( x, t \) given as

\[ J_1 = 2A^* + 3B^* + 2A'^2 + 2A^2 + 3B^2, \]
\[ J_2 = 2\dot{A}' + 3\dot{B} + 4A' + 3\dot{B}' + 3B', \]
\[ J_3 = 2(\dot{B}^2 - B^2)e^{2(B-A)}, \]
\[ J_4 = 2\ddot{A} + 3\dot{B} + 2A'^2 + 2\dot{A}^2 + 3\dot{B}. \]

\[(3.6)\]

\((d)\) The tensor \( D_{mn} \)

There are five non-vanishing components of \( D_{mn} \)

\[ D_{1k} = (L_1,0,0,0,L_2), \]
\[ D_{2k} = (0,L_3,0,0,0), \]
\[ D_{3k} = (0,0,L_4,0,0), \]
\[ D_{4k} = (0,0,0,L_1,0), \]
\[ D_{5k} = (L_4,0,0,0,L_5). \]

\[(3.7)\]
Bimetric plane symmetric quantities in higher imenstonal space-time

Where \( L_1, \ldots, L_5 \) are the functions of \( x, t \) given by

\[
\begin{align*}
L_1 &= 2A^* + 3B^* - 2A^2 - 3AB, \\
L_2 &= 2A' + 3B' - 2AA' - 3BB', \\
L_3 &= (2A'B' + 3B'^2 - 2AB - 3B^2) e^{2(B - A)}, \\
L_4 &= 2A^* + 3B^* - 2AA' - 3BA', \\
L_5 &= 2A' + 3B' - 2A' - 3AB.
\end{align*}
\]

The investigation carried out results in the following theorem:

**Theorem 2**: Each of the plane symmetric bimetric tensors \( K_{nm}, A_{nm} \) and \( B_{am} \) are specified by four functions of \( x \) and \( t \) which are given by (3.4.2), (3.4.4) and (3.4.6) respectively whereas the tensor \( D_{nm} \) is specified by five functions of \( x \) and \( t \) given by (3.4.8)

4. **SPACE-TIME OF CATEGORIES C (I), C (II) AND C (IV)**

4.1 **Plane symmetric Static Space-times**:

We know that the space-time C (I) corresponds to \( A_{nm} = 0 \)

i.e. \( H_i = 0, i = 1, 2, 3, 4. \)

i.e. \( A^* + 3B^* = 0 \) \( \ldots \ldots (i) \)

\(-2A'B' + 2B'^2 + B^* = 0 \) \( \ldots \ldots (ii) \)

\( A^* = 0 \) \( \ldots \ldots (iii) \)

Equation (i) and (iii) leads to

\( A^* = 0, B^* = 0 \)

Which on integration gives

\( A = ax + b, B = cx + d \) \( \ldots \ldots (iv) \)

where \( a, b, c, d \) are the constants of integration.

Using (iv) in (ii), we get

\( a = c, \) or \( c = 0. \)
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Then the space-time (1.1) reduces to

\[ ds^2 = e^{2(ax+b)}(dt^2 - dx^2) - e^{2(ax+b)}(dy^2 + dz^2 + du^2), \]

by absorbing constants \( b \) and \( d \) in the differentials and \( c \neq 0 \) leads to

\[ ds^2 = e^{2ax}(dt^2 - dx^2 - dy^2 - du^2) \quad (4.1.1) \]

which is conformal to the flat.

This confirms the following theorem.

**Theorem 3**: The plane symmetric static space-time of category \( C(I) \) is of the form (4.1.1) and is conformal to the flat space-time.

**Space-time \( C(II) \)**

By definition of \( C(II) \)

\[ K_{nm} = 0 \]

i.e. \( I_j = 0, \ i = 1, 2, 3, 4. \)

i.e. \( A'B' - B'^2 = 0 \) .... (i)

\[ -2A'B' + B'^2 = 0 \] .... (ii)

\[ A'B' = 0 \] .... (iii)

These equations leads to

(i) \( B' = 0 \) and \( A \) is undetermined.

Or

(ii) \( A = \) constant, \( B = \) constant

The space-time (1.1) becomes

\[ ds^2 = e^{2A(x)}(dt^2 - dx^2) - dy^2 - dx^2 - du^2. \quad (4.1.2) \]

In the second case, it reduces to flat space-time of special relativity.

Then following the theorem:

**Theorem 4**: The plane symmetric static space-time \( C(II) \) has the form (4.1.2).

**Space-time \( C(III) \)**

By definition of \( C(III) \)

\[ B_{nm} = 0 \]

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i.e. \( J_i(x) = 0, \quad i = 1, 2, 3, 4. \)

i.e. \( 2A^2 + 3B^2 + 2A'^2 = 0 \)
\[ B'^2 = 0 \]
\[ A'^2 = 0 \]

\( \Rightarrow A = \text{constant and } B = \text{constant.} \)

Then the space-time (1.1) becomes
\[ ds^2 = e^\phi (dt^2 - dx^2 - dy^2 - dz^2 - du^2). \]

By absorbing constant in the differentials, we get
\[ ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2, \]
which is the flat space-time.

The above investigation results in the following theorem:

**Theorem 5**: The plane symmetric static space-time \( C(III) \) is the flat space-time of special relativity.

i.e. \( ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2. \)

The above discussion results in the next theorem.

**Theorem 6**: The plane symmetric static space-time \( C(I) \) and \( C(II) \) are conformally related.

**The space-time of category \( C(IV) \)**

We know that the space-time \( C(IV) \) corresponds to \( D_{nm} = 0, i = 1, \ldots, 5 \) and hence we write
\[ 2A^2 + 3B^2 = 0, \quad 2A'B' + 3B'^2 = 0. \]

Rejecting the non-trivial case \( A = \text{constant and } B = \text{constant} \) above equations are satisfied by
\[ A = a + f(x) \]
\[ B = a - f(x). \]
Then follow the result in the form of following theorem

**Theorem 7**: The plane symmetric static space-time $C_{(IV)}$ has the form

$$ds^2 = e^{f(x)}(dt^2 - dx^2) - e^{-f(x)}(dy^2 + dz^2 + du^2).$$

### 4.2 PLANE SYMMETRIC NON-STATIC SPACE-TIMES

**The space-time** $C_{(I)}$

By definition of $C_{(I)}$, we have $A_{nm} = 0$
i.e.

$$\tilde{A} = 0 \quad \cdots (4.2.1)$$

$$2\tilde{A}B - 2\tilde{B}^2 - \dot{B} = 0 \quad \cdots (4.2.2)$$

$$\dot{A} + 3\dot{B} = 0 \quad \cdots (4.2.3)$$

(4.2.1) and (4.2.3) give $A = \lambda$, $B = \mu$ where $\lambda$ and $\mu$ are constants.

Putting these values in (1.1), we get

$$ds^2 = e^{2\lambda}(dt^2 - dx^2) - e^{2\mu}(dy^2 + dz^2 + du^2).$$

Absorbing the constants in the differentials, we get

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2. \quad (4.2.1)$$

**Theorem 8**: The plane symmetric non-static space-time $C_{(I)}$ is flat space-time of special relativity.

**The space-time** $C_{(II)}$:

We know that the space-time $C_{(II)}$ corresponds to $K_{nm} = 0$ i.e. $I_i = 0, i = 1, \ldots 4$.

This leads us to

$B = \text{constant}$ and $A$ is undetermined

This confirm the following theorem.

**Theorem 9**: The plane symmetric non-static space-time of category $C_{(II)}$ can be put in the form

$$ds^2 = e^{2A(t)}(dt^2 - dx^2) - dy^2 - dz^2 - du^2).$$
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The Space-time $C(III)$:

By definition of $C(III)$, $B_{nn} = 0$ i.e. $J_i(t) = 0, i = 1,\ldots,4$.

i.e. $\dot{A}^2 = 0$

$i.e.\quad \dot{B}^2 = 0$

$2\dddot{A} + 3\dddot{B} + 3\dddot{B}^2 = 0$

i.e. $2\dddot{A} + 3\dddot{B} = 0$

On evaluation, rejecting nontrivial case $A = constant, B = constant$, the above equations are satisfied by

$A = K_1, B = K_2 (constants)$

Then follows the theorem:

**Theorem 10**: The plane symmetric non-static space-time $C(III)$ is the flat space-time of special relativity.

The Space-time $C(IV)$:

By definition of $C(IV)$, $D_{nn} = 0$, i.e. $K_i = 0, i - 1,\ldots,5$.

i.e. $2\dot{A}^2 + 2\dot{A}\dot{B} = 0$

$2\dddot{A} + 3\dddot{B}^2 = 0$

Then evaluating we get

$A = \text{Constant} = C_1, B = \text{Constant} = C_2$

Hence follow the theorem:

**Theorem 11**: The plane symmetric non-static space-time $C(IV)$ is the flat space-time of special relativity.

**CONCLUSION**

The study of bimetric covariant tensors of order two is carried out with reference to plane symmetric five dimensional space-time (1.1). The forms of bimetric quantities $K_{nm}, A_{nm}, B_{nm},\text{ and } D_{nm}$ have been computed and are expressed in the form of theorems. It is established that the plane symmetric static space-time of category $C(I)$ is conformal to the flat, the space-time of category $C(III)$ is the flat space-time of special relativity.
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plane symmetric static space-time C(I) and C(II) are conformally related. The non-static space-times of category C(I), C(III) and C(IV) are flat space-time of special relativity. The present results obtained in higher dimensional space-time are resembles with that of obtained by M. Nahatkar (2002).

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