CHAPTER V

INTUITIONISTIC L-FUZZY SUBALGEBRA OF BG-ALGEBRA WITH RESPECT TO TS NORM

Introduction: Since then its application have been growing rapidly over many disciplines. As a generalization of this, intuitionistic fuzzy subset was defined by Atanassov [1986]. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings.

Liu [1982] introduced the concepts of fuzzy subrings and fuzzy ideals. Jun and Park [2008] and Jun [[2008] introduced the notion of BG-algebra, and soft BG-algebra which are the generalization of B-algebra. With these ideas, fuzzy sub-algebra of BG-algebra were developed by Ahn and Lee [2004].

Section 5.1: Preliminaries and previous works

The notion of a fuzzy set was introduced Zadeh [1965] and since then this has been applied to various algebraic structures. The idea of "an intuitionistic fuzzy set" was introduced by Atanassov [2000] as a generalization of the notion of fuzzy set. The concept Γ- near ring, a generalization of both the concepts near ring and Γ- ring. Later several author such as Booth [1988] who studied the real theory of Γ- near rings. Also Jun and Park [2008] considered the fuzzification of left (respectively right) ideals of Γ-near rings.

Molodtsav [1999] proposed an approach for modeling vagueness and uncertainty called soft set theory. Since its inception work on soft set theory has been applied to many different fields, such as function smoothness, Riemann integration, Pearson integration, Measurement theory, Game theory and decision making. Maji et.al [2003] defined some operations on soft sets. Aktas and Cagman [2007] generalized soft sets by defining the concept of soft groups. After them, Sun.et.al [22] gave soft modules.

Atagun and Sezgin [2011] defined the concepts of soft sub rings of a ring, soft sub ideals of a field and soft sub-modules of a module and studied their relative properties with respect to soft set operations. Atagun and Sezgin [2011] defined soft N-subgroups and soft N-ideals of an N-group.

Solairaju, Sarangapani, & Nagarajan [2013b] studied the idea of union fuzzy soft n-group. Basic version of properties of maximal Intuitionistic Fuzzy Soft N-ideals are investigated, and properties of maximal intuitionistic fuzzy soft N-ideals.

Sharma [2012] proved that (1) If A = (μA , νA) be intuitionistic fuzzy ideal of a near ring N, then (i) $\mu A(x) \leq \mu A(0)$ and $\nu A(x) \geq \nu A(0)$ for all $x \in N$; (ii) $\mu A(-x) = \mu A(x)$ and $\nu A(-x) = \nu A(x)$ for all $x \in N$; **(iii)** $\mu A(x + y) = \mu A(y + x)$ and $\nu A(x + y) = \nu A(y + x)$ for all $x, y \in N$; (2) Let N and N′ be two near rings and let $f : N \to N'$ be near ring homomorphism. If B = (μB , νB) is an IFI in N′ , then the pre-image $f^{-1}(B)$ of B under f is an IFI of N; (3) Let N and N′ be two near rings and let $f : N \to N'$ be epimorphism If A = (μA , νA) is IFI of N , then $f(A)$ is IFI of N′.

P. K. Sharma [2011c] got (1) Let R and R′ be two rings with identities 0 and 0′. Let f : R → R′ be ring homomorphism. Then (i) f (0) = 0′ , f (1) = 1′ , where 0 , 1 and 0′ , 1′ are identities of R and R′; (ii) f (-x) = - f (x) and f (x -1) = { f (x)} -1 , for all x ∈ R; (2) Let R and R| be any two rings. Then the homomorphic image of an IFMT of an IFI of R is an IFI of R|.

Arul Selvaraj and Sivakumar [2011a] introduce the concept of *t*-norm (λ, μ)-fuzzy quotient near-ring and establish the isomorphism theorem of *t*-norm (λ, μ)-fuzzy quotient near-ring, and also they introduce the concept of *t*-norm (λ, μ)-fuzzy quasi-ideals of near-rings. Further they obtained the characterization of *t*-norm (λ, μ)-fuzzy quasi-ideal of near-rings and discuss some related properties

Arul Selvaraj and Sivakumar [2011a] analyzed (1) Let A be a t-norm (λ, μ)-fuzzy ideal of N, then for all x, y, a, b ∈N. It follows that x + A = a + A and y + A = b + A ⇒ (x + y) + A = (a + b) + A and xy + A = ab + A; (2) Let A be a t-norm (λ, μ)-fuzzy ideal of N. Then N/A = {a + A: a ∈ N} form a near-ring with the null element 0 + A, where (x + A) + (y + A) = (x + y) + A and (x + A)(y + A) = xy + A.

Arul Selvaraj and Sivakumar [2011b] introduce the concept of *t*-norm (λ, μ)-fuzzy sub near-ring which can be regarded as a generalisation of *t*-norm (λ, μ)-fuzzy sub ring and *t*-norm (λ, μ)-fuzzy ideal of a near-ring. They found (1) Let f : N1 → N2 be a homomorphism of near-rings and let A be a t-norm (λ, μ)-fuzzy sub near-ring of N1. Then f(A) is a t-norm (λ, μ)-fuzzy sub near-ring of N2. If A is a t-norm (λ, μ)- fuzzy ideal of N1 and f is onto, then f(A) is a t-norm (λ, μ)- fuzzy ideal of N2, where f(A)(y) = sup {A(x)/f(x) = y} , ∀y ∈ N2; (2) Let f : N1 → N2 be a homomorphism of near-rings and B be a t-norm (λ, μ)-fuzzy subnear-ring (fuzzy ideal) ofN2. Then f^{-1}(B) is a t-norm (λ, μ)-fuzzy sub-near-ring (fuzzy ideal) of N1, where f^{-1}(B)(x) = B(f(x))∀x ∈ N1.

Tariq Shah and Muhammad Saeed [2012] gave (1) Let f: R → R^1 be a surjective ring anti-homomorphism. If μ is a strongly irreducible fuzzy ideal of R^1, then $f^{-1}(\mu)$ is a strongly irreducible fuzzy ideal of R; (2) For a ring anti-homomorphism f : R → R^1, if μ is an finvariant strongly irreducible fuzzy ideal of R, then f(μ) is a strongly irreducible fuzzy ideal of R^1; (3) For a surjective ring anti-homomorphism f : R → R^1, if every fuzzy ideal of R is f-invariant and has a fuzzy primary (respectively, strongly primary) decomposition in R, then every fuzzy ideal of R^1 has a fuzzy primary (respectively, strongly primary) decomposition in R^1. They investigated anti- homomorphic images and pre images of semiprime, strongly primary, irreducible and strongly irreducible fuzzy ideals of a ring. They also proved that: For a surjective anti-homomorphism f:: R → R^1, if every fuzzy ideal of R is f-invariant and has a fuzzy primary (respectively, strongly primary) decomposition in R, then every fuzzy ideal of R/ has a fuzzy primary (respectively, strongly primary) decompsition in R^1.

Young Bae Jun and Hee Sik Kim [2002] found (1) Let f : R → S be an onto homomorphism of near-rings and let A be an f-invariant fuzzy prime ideal of R. Then f (A) is a fuzzy prime ideal of S; **(2)** Let f : R → S be an onto homomorphism of near-rings. If B is any fuzzy prime ideal of S, then $f^{-1}(B)$ is a fuzzy prime ideal of R;

Sung Min Hong [1998] contributed another proof of a theorem without using the sup-property. For the homomorphic image and preimage of fuzzy left (right) ideals, he established the chains of level left (right) ideals. Moreover, he proved that a necessary condition for a fuzzy ideal of a near-ring R to be prime is that, is two-valued.

In this chapter, the intuitionistic L- fuzzy sub-algebras of BG-algebras is investigated with respect to TS-norm and established some of their basic properties.

Section 5.2: Basic definitions

In this section, the basic definitions of a BG-algebra, L-fuzzy subset and intuitionistic L-fuzzy subset are recalled. The following definition is first started.

Definition 5.2.1. A non-empty set X with a constant 0 and a binary operation $*$, is said to be a BG-algebra if it satisfies the following:

1. $x * x = 0$;
2. $x * 0 = x$;
3. $(x * y) * (0 * y) = x$ for all $x, y \in X$.

Definition 5.2.2. A binary relation $<$ on BG-algebra X is defined as $x < y$ if and only if $x * y = 0$.

Definition 5.2.3. A non-empty subset S of a BG-algebra X is said to be a sub-algebra if $x * y \in S$ for all $x, y \in S$.

Definition 5.2.5. Let $(L, <)$ be a complete lattice with least element 0 and greatest element 1. A L-fuzzy subset B in non-empty set X is defined as a function $B : X \rightarrow L$.

Example 5.2.6. Let Z be the ring of integers. Define $A : Z \rightarrow L$ by

$$A(x) = \begin{cases} 1 & \text{if } x = 0 \\ 1/3 & \text{if } x \in Z - \{0, 2\} \\ 0, & \text{if } x = 2 \end{cases}$$

Then A is a L- fuzzy subset of Z.

Definition 5.2.7. An intuitionistic fuzzy subset (IFS) A in a non-empty set X is defined as an object of the form A = {< x, $\mu_A(x)$, $\nu_A(x)$ > / x $\in$ X} where $\mu_A : X \rightarrow [0,1]$ is the degree of membership and $\nu_A : X \rightarrow [0, 1]$ is the degree of non-membership of the element x $\in$ X satisfying $0 < \mu_A(x) + \nu_A(x) < 1$.

Definition 5.2.8. Let (L, <) be a complete lattice with least element 0 and greatest element 1 and an involution order reversing operation N: L $\rightarrow$ L. Then an intuitionistic L-fuzzy subset (ILFS) A in a non-empty set X is defined as A = {< x, $\mu_A(x)$, $\nu_A(x)$ > : x $\in$ X} where $\mu_A : X \rightarrow L$ is the degree of membership and $\nu_A : X \rightarrow L$ is the degree of non-membership of the element x $\in$ X satisfying $\mu_A(x) < N(\nu_A(x))$.

Example 5.2.9. Let L be a complete lattice and Z be the ring of integers. Let A: Z $\rightarrow$ L be the subset A = {< x, $\mu_A(x)$, $\nu_A(x)$ > / x $\in$ X} of X defined by for all x $\in$ Z. Then A is an intuitionistic L fuzzy subset of Z.

Definition 5.2.10. Let A and B be two intuitionistic L-fuzzy subsets of a non-empty set X. Then the following properties hold:

1. $A \subset B$ iff $\mu_A(x) < \mu_B(x)$ and $\nu_A(x) > \nu_B(x)$ for all x $\in$ X.
2. A = B if and only if $A \subset B$ and $B \subset A$.
3. $A \cup B$ = {< x, $\mu_A(x) \vee \mu_B(x)$, $\nu_A(x) \wedge \nu_B(x)$ >} for all x $\in$ X.

4. A fl B = $\{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)>\}$ for all $x \in X$.

5. A = $\{< x, \nu_A(x), \mu_A(x) >\}$ for all $x \in X$.

6. $*A = \{< x, \mu_A(x), \mu_A(x) >\}$ for all $x \in X$.

7. $\#A = \{< x, \nu_A(x), \nu_A(x) >\}$ for all $x \in X$.

Definition 5.2.12. Let $\{A_i : i \in I\}$ be an arbitrary family of intuitionistic L-fuzzy set with respect to TS-norm in a non-empty set X. Then min (Ai) = T(x, min μ_{Ai} (x), min ν_Ai (x)) for all $x \in X$.

Section 5.3: Intuitionistic L-fuzzy sub-algebra of BG-algebra

In this section, the notion of intuitionistic L fuzzy sub-algebra in a BG-algebra X is introduced. Hereafter, X denotes a BG-algebra unless otherwise specified.

Definition 5.3.1. A fuzzy subset μ in a BG-algebra X is said to be a T fuzzy sub-algebra of X if $\mu(x * y) \geq T\{\mu(x),\mu(y)\}$ for all $x, y \in X$.

Definition 5.3.2. An intuitionistic fuzzy subset A in a BG-algebra X is said to be an intuitionistic fuzzy sub-algebra with respect to TS-norm of X if

1. $\mu(x * y) \geq T\{\mu(x),\mu(y)\}$

2. $\nu(x * y) \leq S\{\nu(x), \nu(y)\}$ for all $x, y \in X$.

Definition 5.3.3. A L-fuzzy subset A in a BG-algebra X is said to be an L-fuzzy sub-algebra with respect to T-norm of X if $A(x * y) \geq T\{A(x), A(y)\}$ for all $x, y \in X$.

Definition 5.3.4. An intuitionistic L-fuzzy subset A in a BG-algebra X is said to be an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if

1. $\mu_A(x * y) \geq T\{\mu_A(x), \mu_A(y)\}$

2. $\nu_A(x * y) \leq S\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in X$.

Lemma 5.3.5. In an intuitionistic L-fuzzy sub-algebra with respect to TS-norm A of X,

1. $\mu_A(0) \geq \mu_A(x)$

2. $\nu_A(0) \leq \nu_A(x)$ for all $x \in X$.

Proof: $\mu_A(0) = \mu_A(x * x) \geq T\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$.

Similarly, $\nu_A(0) = \nu_A(x * x) \leq S\{\nu_A(x), \nu_A(x)\} = \nu_A(x)$

Theorem 5.3.6. Intersection of any two intuitionistic L-fuzzy sub-algebras of X is again an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

Proof. Let A and B be Intuitionistic L-fuzzy sub-algebras with respect to TS-norm of X.

Let $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$.

Take $C = A \cap B = \{< x, \mu_C(x), \nu_C(x) > / x \in X\}$

where $\mu_C(x) \geq T\{\mu_A(x), \mu_B(x)\}$ and

It follows that $\nu_C(x) \leq S\{ \nu_A(x), \nu_B(x)\}$.

Let $x, y \in X$.

Claim: $\mu_C(x * y) \geq T\{ \mu_C(x), \mu_C(y) \}$ and $\nu_C(x * y) < S\{ \nu_C(x), \nu_C(y)\}$ for all $x, y \in X$.

Now it gives that

$$
\begin{aligned}
\mu_C(x * y) &= \mu_{A \cap B}(x * y) \text{ for every } x, y \in X \\
&= T\{\mu_A(x * y), \mu_B(x * y)\} \\
&\geq T\{ T((\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y)) \\
&= T\{ \min(\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y) \} \\
&= T\{\mu_C(x), \mu_C(y)\}
\end{aligned}
$$

and

$$
\begin{aligned}
\nu_C(x * y) &= \nu_{A\cap B}(x * y) \\
&= S\{\nu_A(x * y), \nu_B(x * y)\} \\
&\leq S(S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y)) \\
&= S(\min(\nu_A(x), \nu_B(x)), \min(\nu_A(y), \nu_B(y)) \\
&= S\{\nu_C(x), \nu_C(y)\}
\end{aligned}
$$

Hence, C is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

This proves that the intersection of any two intuitionistic L-fuzzy sub-algebras with respect to TS-norm of X is again an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

The above theorem can be generalized as

Theorem 5.3.8. Intersection of any family intuitionistic L-fuzzy sub-algebras with respect to TS-norm of X is again an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

In the same way and by the definition of (5.3.2), the theorem (5.3.8) is followed

Theorem 5.3.9. If A is an intuitionistic L-fuzzy sub-algebra with respect to TS Norm of X, then so is $*A$.

Theorem 5.3.10. An intuitionistic L-fuzzy subset A of X is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if and only if the L-fuzzy subsets μ_A and ν_A are L-fuzzy sub-algebras with respect to TS-norm of X.

Proof: Let $A = \{(x, \mu_A(x), \nu_A(x)) \,/\, x \in X\}$ be an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X. Clearly, μ_A is a L-fuzzy sub-algebra with respect to TS-norm of X. For all x, y $\in$ X, it gives that

$$\underline{\nu}_A(x * y) = 1 - \nu_A(x * y)$$

$$\geq 1 - T[\nu_A(x), \nu_A(y)]$$

$$= T\{(1 - \nu_A(x)), (1 - \nu_A(y))\}$$

$$= T(\underline{\nu}(x), \underline{\nu}(y)).$$

This proves that ν is a L-fuzzy sub-algebra with respect to TS-norm of X.

Conversely, assume that μ_A and $\underline{\nu}_A$ are L-fuzzy sub-algebras with respect to TS-norm of X.

$\mu_A(x * y) \geq T\{ \mu_A(x), \mu_A(y) \}$and

$\nu_A(x * y) \geq T\{ \nu_A(x), \nu A(y)\}$ for all x, y $\in$ X.

Hence, it is to prove that A = {(x, $\mu_A(x)$, $\nu_A(x)$) / x $\in$ X} is an intuitionistic L-fuzzy sub-algebra with respect to TS Norm of X, it is enough to verify that $\nu_A(x * y) \leq T\{\nu_A(x), \nu_A(y)\}$ for all x, y $\in$ X.

Since $\underline{\nu}_A$ is an L-fuzzy sub-algebra of X, it implies that

$\underline{\nu}_A(x * y) \geq S\{\underline{\nu}(x), \nu(y)\}$

$1 - \nu_A(x * y) \leq (1 - \nu_A(x)) \vee (1 - \nu A(y))$

$= 1 - S[\nu_A(x), \nu A(y)]$

Then

$\nu_A(x * y) < S\{\nu_A(x), \nu_A(y)\}$ for all x, y $\in$ X.

Theorem 5.3.11 An intuitionistic L-fuzzy subset A of X is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if and only if the $*$A and #A are L-fuzzy sub-algebras with respect to TS-norm of X.

Using this theorem and by definition (5.2.11) , the theorem (5.3.11) is followed.

Conclusion: In this article, the fuzzy sub-algebra of BG-algebra is extended into intuitionistic L-fuzzy sub-algebra with respect to TS-norm of BG-algebra. These concepts can further be generalized.

Section 5.4: Bi polar anti Q-fuzzy left R-subgroup of near-ring under T,S - norm

Introduction 5.4.1: Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1].

Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property and its counter property. In a bipolar valued fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0, 1] indicate that elements some what satisfy the property, and the membership degrees on [-1, 0) indicate that elements somewhat satisfy the implicit counter property. In the definition of bipolar-valued fuzzy sets, there are two kinds of representations so called canonical representation and reduced representation.

In this chapter, the canonical representation of bipolar valued Q- fuzzy sets is used. In this chapter, the theory of fuzzy sets which was introduced by Zadeh [1975] is applied to many mathematical branches. Abou-Zoid [1991] introduced the notion of a fuzzy sub near-ring and studied fuzzy ideals of near-ring.

This concept discussed by many researchers among Cho and Jun [2005], Davvaz et. al. [2006], Jun et. al. [2004], Chengyi [1998], and Dib et. al. [1996]. Kim and Jun [2001], considered the intuitionistic fuzzification of a right (resp left) R- subgroup in a near-ring. Also Cho and Jun [2005] the notion of normal intuitionistic fuzzy R- subgroup in a nearing is introduced and related properties are investigated. The notion of intuitionistic Q-fuzzy semi primality in a semi group is given by Kim [2006].

In this section, the notion of Q- fuzzification of bipolar left R- subgroup is introduced in a near ring and investigate some related properties. Characterization of bipolar Q- fuzzy left R- subgroups are given.

Preliminaries and basic concepts

Definition 5.4.2: A non-empty set with two binary operations '+' and '.' is called a near-ring if it satisfies the following Axioms (i) (R ,+) is a group; (ii) (R, .) is a semi group.

(iii) $x . (y + z) = x .y + x . z$ for all $x, y ,z \in R$.

Precisely speaking it is a left near-ring because it satisfies the left distributive law.

As R-subgroup of a near- ring 'R' is (i) a subset 'H' of 'R' such that (H , +) is a subgroup of (R, +); (ii) $RH \subset H$; (iii) $HR \subset H$. If 'H' satisfies (i) and (ii) and then it is called a left R subgroup of 'R'. if 'H' satisfies (i) and (iii), then it is called a right R subgroup of R. A map $f : R \to S$ is a homomorphism if $f (x + y) = f(x) + f (y)$ and $f (x .y) = f(x) . f (y)$for all x, y in R.

Definition 5.4.3: Let 'G' be a non-empty set. A bipolar-valued Q-fuzzy set A in G is an object having the form. $A = \{((x, q) \mu_A^+(x, q), \mu_A^-(x, q)) : x \in G, q \in Q \}$ where $\mu_A^+: G \times Q \to [0, 1]$ and $\mu_A^- : G \times Q \to [-1,0]$ are mappings.

The positive membership degree $\mu_A^+ (x, q)$ denotes the satisfaction degree of an element x to the property corresponding to A and the negative membership degree $\mu_A^- (x, q)$ denotes the satisfaction degree of x to some implicit counter property of A.

Definition 5.4.4: A fuzzy set ‘μ’ in R is called fuzzy sub-near ring in a near ring ‘R’ if (i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$; (ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all x, y in R.

Definition 5.4.5: An anti Q-fuzzy set ‘μ’ is called a Bi polar Q fuzzy left R- subgroup of R over Q if ‘μ’ satisfies

(i) $\mu^+(x - y, q) \leq \max\{\mu^+(x, q), \mu^+(y, q)\}$

(ii) $\mu^-(x - y, q) \geq \min\{\mu^-(x, q), \mu^-(y, q)\}$

(iii) $\mu^+(rx, q) \geq \mu^+(x\ q)$. (iv) $\mu^-(rx, q) = \mu^-(x\ q)$.

Definition 5.4.6: For a bipolar anti Q-fuzzy set ‘A’ and $(\beta, \alpha) \in [-1.0] \times [0,1]$, define $A_t^+ = \{x \in X / \mu_A^+(x, q) \leq \alpha\}$ and $A_s^- = \{x \in X : \mu_A^-(x,q) \geq \alpha\}$ which are the positive t-cut and negative s-cut of A respectively.

Definition 5.4.7: Let λ and μ be two anti Q-fuzzy subsets in X. The cartesian product of $\lambda^+ \times \mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^+ \times \mu^+(x, y) = S\{\lambda^+(x, q), \mu^+(y, q)\}$ and $\lambda^+ \times \mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^- \times \mu^-(x, y) = T\{\lambda^-(x, q), \mu^-(y, q)\}$ for all $x, y \in X$ and $q \in Q$.

Definition 5.4.8: Let $f: X \rightarrow Y$ be a mapping of groups and ‘μ’ be a bipolar anti Q- fuzzy set of Y. The map μ^f is the pre-image of μ_1 and μ_2 under f if $\mu_1^{f+}(x, q) = \mu^{f+}(x, q)$, $\mu_2^{f--}(x, q) = \mu^{f--}(x, q)$.

Aim 5.4.9: In this chapter, the notion of anti Q-fuzzification of Bi polar left R- subgroup is introduced in a near-ring and investigate some related properties. Characterization of anti Bi polar Q- fuzzy left R-subgroups with respect to S-norm are given.

Section 5.5: Properties of anti Q-fuzzy left subgroup

The following theorems mentioned below are now discussed:

Theorem 5.5.1: Every imaginable Bi polar anti Q-fuzzy left R- subgroup μ^+ of a near ring R is an anti Q-fuzzy left R subgroup of R.

Proof: Assume μ^+ is imaginable bipolar anti Q-fuzzy left R- subgroup of R. Then it gives that (i) $\mu^+ (x-y , q) \leq S\{ \mu^+ (x, q), \mu^+ (y, q) \}$ and $\mu^+ (rx, q) \leq \mu^+ (x, q)$ for all x, y in R.

Since μ^+ is imaginable, it implies that

$$\max\{\mu^+(x, q),\mu^+(y, q)\} = S\{\max\{\mu^+(x, q), \mu^+(y, q), \max\{\mu^+(x, q), \mu^+(y, q)\}\}$$

$$\leq S (\mu^+(x, q), \mu^+(y, q))$$

$$\leq \max \{ \mu^+ (x, q) , \mu^+ (y, q) \}$$

(ii) $S(\mu^+ (x, q), \mu^+ (y, q)) = \max \{ \mu^+ (x, q), \mu^+ (y, q) \}$.

It gives that $\mu^+ (x - y, q) \leq S (\mu^+ (x, q), \mu^+ (y, q))$

$$= \max \{\mu^+ (x, q), \mu^+ (y, q) \} \text{ for all } x, y \in R.$$

Hence μ^+ is a Bipolar anti Q-fuzzy left R- subgroup of R.

Also (iii) $\mu^{--} (x-y , q) \geq T \{ \mu^- (x, q), \mu^- (y, q) \}$ and $\mu^- (rx, q) \leq \mu^- (x, q)$ for all x, y in R.

Since μ^{--} is imaginable, it follows that

$$\min \{ \mu^{--} (x, q), \mu^{--} (y, q)\} = T \{\min\{ \mu^{--} (x, q), \mu^{--}(y, q), \min\{\mu^{--}(x, q), \mu^{--} (y, q)\}\}$$

$$\geq T (\mu^{--} (x, q), \mu^{--} (y, q))$$

$$\geq \min \{ \mu^{--} (x, q), \mu^{--} (y, q) \}$$

Then (iv) $T(\mu^{--} (x, q), \mu^{--} (y, q)) = \min \{ \mu^{--} (x, q) , \mu^{--} (y, q)\}$.

It follows that $\mu^{--}(x - y, q) \geq T(\mu^{--}(x, q), \mu^{--}(y, q))$

$$= \min\{\mu^{--}(x, q), \mu^{--}(y, q)\} \text{ for all } x, y \in R.$$

Hence μ^{--} is a Bi polar anti Q-fuzzy left R subgroup of R.

Theorem 5.5.2: If μ^{+} is an anti Bi polar Q- fuzzy left R subgroups of a near ring R and 'θ' is an endomorphism of R, then $\mu[\Theta]$ is Bi polar anti Q- fuzzy left R- subgroup of R.

Proof: For any $x, y \in R$, it gives that

(i) $\mu^{+}[\theta](x-y, q) = \mu^{+}(\theta(x-y, q))$

$$= \mu^{+}(\theta(x, q), \theta(y, q))$$

$$\leq S\{\mu^{+}(\theta(x, q)), \mu^{+}(\theta(y, q))\}$$

$$= S\{\mu^{+}[\theta](x,q), \mu^{+}[\theta](y,q)\}$$

(ii) $\mu^{+}[\theta](rx, q) = \mu^{+}(\theta(rx, q)$

$$\geq \mu^{+}(\theta(x, q))$$

$$\geq \mu^{+}[\theta](x, q).$$

(iii) $\mu^{--}[\theta](x-y, q) = \mu^{--}(\theta(x-y, q))$

$$= \mu^{--}(\theta(x, q), \theta(y, q))$$

$$\geq T\{\mu^{--}(\theta(x, q)), \mu^{--}(\theta(y, q))\}$$

$$= T\{\mu^{--}[\theta](x, q), \mu^{--}[\theta](y, q)\}$$

(iv) $\mu^{--}[\theta](rx, q) = \mu^{--}(\theta(rx, q)$

$$\leq \mu^{--}(\theta(x, q))$$

$$\leq \mu^{--}[\theta](x, q).$$

Hence $\mu^{--}[\theta]$ is bi polar anti Q- fuzzy left R- subgroup of R.

Theorem 5.5.3: An onto homomorphism pre-image of an anti bipolar Q-fuzzy left R- subgroup of near ring R1 is also anti bipolar Q- fuzzy left R-subgroup.

Proof: Let f : R → R1 be an onto homomorphism of near rings and let 'ξ' be an anti Q-fuzzy bipolar left R- subgroup of R1 and μ^+ be the pre image of 'ξ' under 'f'.

Then it follows that

(i) $\mu^+ (x-y , q) = \xi (f(x-y , q))$

$= \xi (f(x, q), f(y, q))$

$\leq S (\xi (f(x, q)) , \xi(f(y, q)))$

$\leq S (\mu^+ (x, q) , \mu^+ (y, q))$

(ii) $\mu^+ (rx, q) = \xi (f (rx, q))$

$\leq \xi (f(x, q))$

$\leq \mu^+ (x, q)$.

(iii) $\mu^{--}(x-y , q) = \xi (f(x-y , q))$

$= \xi (f(x, q), f(y, q))$

$\geq T (\xi (f(x, q)) , \xi(f(y, q)))$

$\geq T (\mu^{--} (x, q) , \mu^{--} (y, q))$

(iv) $\mu^{--} (rx, q) = \xi (f (rx, q))$

$\geq \xi (f(x, q))$

$\geq \mu^- (x, q)$.

Section 5.6: Bi polar Q-fuzzy left R-subgroup with sup-property

Theorem 5.6.1: An onto homomorphic image of a bipolar anti Q- fuzzy left R- subgroup with the sup-property is a bipolar anti Q-fuzzy left R'-subgroup.

Proof: Let f: R → R' be an onto homomorphism of near rings and let μ^+ be a sup-property of fuzzy left R- subgroups of R.

Let x', y' ∈ R', and $x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$ be such that $\mu^{f+}(x_0, q) = \sup \mu^+(h, q)$,

Sup $\mu^+(y_0, q)$ $(h, q) \in f^{-1}(x')$ $(h, q) \in f^{-1}(y')$ respectively.

Then it follows that

(i) $\mu^{f+}(x'-y', q) = \sup \mu^+(z, q), (z, q) \in f^{-1}(x'-y', q)$

$\leq \max\{\mu^+(x_0,q), \mu^+(y_0,q)\}$

$= \max\{\sup \mu^+(h,q), \quad \sup \mu^+(h,q) \quad \}$

$(h,q) \in f^{-1}(x',q) \quad (h,q)\ \varepsilon\ f^{-1}(y',q)$

$= \max\{\mu^+ f(x',q), \mu^+ f(y',q)\}$

(ii) $\mu^{f+}(rx,q) = \sup \quad [\mu^+(z,q)]$

$(z, q) \in f^{-1}(r'x', q)$

$\leq \mu^+(y_0, q)$

$= \sup \quad [\mu^+(h,q)]$

$(h,q) \in f^{-1}(y',q)$

(iii) $\mu^{f--}(x'-y', q) = \inf_{(z,q)\, \varepsilon\, f^{-1}(x'-y',q)} [\mu^{--}(z,q)]$

$\geq \min \{ \mu^{--}(x_0,q) , \mu^{--}(y_0,q)$

$= \min \{\inf_{(h,q)\in f^{-1}(x',q)} [\mu^{--}(h ,q)] , \inf_{(h,q)\in f^{-1}(y',q)} [\mu^{--}(h,q) \}]$

$= \min \{ \mu^{--}f(x',q) , \mu^{--}f(y',q) \}$

(iv) $\mu^{f---}(rx,q) = \inf_{(z,q)\in f^{-1}(r'x', q)} [\mu^{--}(z,q)]$

$\geq \mu^{--}(y_0 , q)\backslash$

$= \inf_{(h,q)\in f^{-1}(y',q)} [\mu^{--}(h,q)]$

$= \mu^{f--}(y',q)$.

Hence 'μ^{f+} is a bipolar anti Q-fuzzy left R- subgroup of R'.

Theorem 5.6.2: Let T, S be continuous t-norm and s norm respectively. Let 'f' be a homomorphism on a near ring R. If 'μ^{+}' is bipolar anti Q- fuzzy left R- subgroup of R, then μ^{+f} is a bipolar anti Q- fuzzy left R- subgroup of f(R).

Proof: Let $A_1 = f^{-1}(y_1, q)$, $A_2 = f^{-1}(y_1, q)$ and $A_{12} = f^{-1}(y_1 - y_2, q)$

where $y_1, y_2 \in f(R), q \in Q$.

Consider $A_1 - A_2 = \{ x \in R / (x, q) = (a_1,q) - (a_2,q) \}$ for some $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$.

If $(x, q) \in A_1 - A_2$, then $(x, q) = (x_1, q) - (x_2, q)$ for some $(x_1, q) \in A_1$ and $(x_2, q) \in A_2$

so that $f(x, q) = f(x_1, q) - f(x_2, q) = y_1 - y_2$

$(x, q) \in f^{-1}((y_1,q) - (y_2,q)) = f^{-1}(y_1 - y_2, q) = A_{12}$. Thus $A_1 - A_2 \in A_{12}$.

It follows that

(i) $\mu^{f+} (y_1 - y_2, q) = \sup \{ \mu^{+} (x, q)/ (x, q) \in f^{-1}((y_1, q) - (y_2, q)) \}$

$= \sup\{ \mu^{+} (x,q) : (x, q) \in A_{12} \}$

$\geq \sup \{ \mu^{+} (x,q): (x, q) \in A_1 - A_2\}$

$\leq \sup \{ \mu^{+} ((x_1, q) - (x_2, q)): (x_1, q) \in A_1 \text{ and } (x_2, q) \in A_2\}$

$\leq \sup\{ S (\mu^{+} (x_1, q), \mu^{+} (x_2, q)): (x_1, q) \in A_1 \text{ and } (x_2, q) \in A_2\}$

Since S is continuous. For every $\varepsilon > 0$, if

$\sup \{ \mu^+ (x_1, q): (x_1, q) \in A_1\} - (x_1^*, q) \} \leq \delta$ and

$\sup \{ \mu^+ (x_2, q) : (x_2, q) \in A_2\} - (x_2^*, q) \} \leq \delta$

$T\{\sup\{ \mu^+ (x_1, q) : (x_1,q) \in A_1\} , \sup \{ \mu+ (x_2, q) : (x_2, q) \in A_2 \} - S ((x_1^*, q), (x_2^*, q)\} \leq \varepsilon$

Choose $(a_1, q) \in A_1$ and $(a_2, q) \in A_2$ such that

$\sup\{ \mu^+ (x_1,q) : (x_1.q) \varepsilon A_1\} - \mu^+ (a1, q) \leq \delta$ and

$\sup \{ \mu^+ (x_2,q) / (x_2,q) \in A_2\} - \mu(a2,q) \leq \delta$.

Then it follows that

$S\{\sup\{ \mu^+ (x_1,q) : (x_1, q) \in A_1\}, \sup \{ \mu^+ (x_2, q): (x_2, q) \in A_2 \} - S (\mu(a1,q), \mu(a2,q)\} \leq \varepsilon$

Consequently, it gives that

$\mu^{f+(}y'-y_2, q) \leq \sup \{ S(\mu^+ (x_1,q), \mu^+ (x_2,q)) : (x_1, q) \in A_1 ,(x_2, q) \in A_2\}$

$\leq S (\inf\{ \mu^+ (x_1,q) : (x_1,q) \in A_1\}, \inf\{ \mu^+(x_2,q) : (x_2, q) \in A_2\}$

$\leq S (\mu^{f+} (y', q), \mu^+f(y2, q) \}$

(ii) $\mu^{f--} (y'-y2, q)$

$= \inf \{ \mu^{--} (x, q): (x, q) \in f^{-1} ((y', q)- (y2,q)) \}$

$= \inf\{ \mu^{--} (x,q) : (x,q) \in A_{12} \}$

$\geq \inf \{ \mu^{--} (x,q): (x, q) \in A_1- A_2\}$

$\geq \inf \{ \mu^{--} ((x_1, q) - (x_2, q)): \ (x_1, q) \in A_1$ and $(x_2, q) \in A_2\}$

$\geq \inf\{ T(\mu^{-} (x_1, q) , \mu^{-} (x_2, q)): (x_1, q) \in A_1$and $(x_2, q) \in A_2\}$

Since S is continuous. for every $\varepsilon > 0$, if

$\text{Inf} \{ \mu(x_1,q) : (x_1, q) \in A_1\} - (x_1^*, q) \} \leq \delta$

and $\text{Inf} \{ \mu(x_2,q) : (x_2, q) \in A_2\} - (x_2^*,q) \} \leq \delta$, then

$T \{\inf\{ \mu^{-} (x_1,q) / (x_1,q) \in A_1\} , \inf \{ \mu^{+} (x_2,q) / (x_2,q) \in A_2 \} - T ((x_1^*,q), (x_2^*,q) \leq \varepsilon$

Choose $(a1,q) \in A_1$ and $(a2,q) \in A_2$ such that

$\text{Inf} \{ \mu^{-} (x_1,q) / (x_1.q) \in A_1 \} - \mu^{+} (a1,q) \leq \delta$

and $\text{Inf} \{ \mu^{-} (x_2,q) / (x_2,q) \in A_2\} - \mu(a2,q) \leq \delta$.

Then $T\{\inf\{ \mu^{-} (x_1,q) : (x_1,q) \in A_1\}, \inf \{\mu^{-} (x_2,q) : (x_2, q) \in A_2 \} - T (\mu^{+} (a1,q), \mu(a2,q) \leq \varepsilon$.

Consequently, it follows that

$\mu^{f++} (y'-y2, q) \geq \inf\{ T(\mu^{-} (x',q), \mu^{-} (x_2,q)) : (x',q) \in A_1,(x_2,q) \in A_2\}$

$\geq T (\inf\{ \mu^{--} (x_1,q) : (x_1,q) \in A_1\},\inf\{ \mu^{--} (x_2,q) : (x_2,q) \in A_2\}$

$\geq T (\mu^{f+} (y_1, q) , \mu^{f-} (y_2, q) \}$.

Hence 'μ^{f}' is a bipolar anti Q- fuzzy left R- subgroup of 'f(S)'.

Conclusion: Osman Kazanciet et.al. [2007a, 2007b] introduced the intuitionistic Q-fuzzy R-subgroups of near rings. In this chapter, the notion of bipolar Q-fuzzy left R- subgroup of near ring is introduced with respect to (T, S) and characterization of them. A part of this chapter has published in American Journal of Mathematical Science and Applications, Volume 2, Number 2, (2014), 93 – 97.