CHAPTER II

NEW STRUCTURES ON

IMAGINABLE FUZZY SOFT ISOMORPHISM

Introduction: Researchers in economics, engineering, environmental science, sociology, medical science and many other fields deal daily with the complexities of modeling uncertain data. Classical methods are not always successful, because the uncertainties appearances in these domains may be of various types.

Most of the existing mathematical tools for formal modeling, reasoning and computing are Crisp, deterministic and precise in character. But in real life situation, the problems in Economics, Engineering, environment, Social science, Medical science etc., do not always involve crisp data. Consequently, we cannot successfully by using the traditional classical methods because of various types of uncertainties in this problem.

Section 2.1: Previous research and contributions:

While Probability theory, fuzzy sets [Zadeh, 1965], rough sets [Pei and Miac, 2005], and other mathematical tools are well-known and often useful approaches to describe uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsov [1999].

Jun et. al. [2009] analyzed the applications of soft sets in d-algebra. Aygunouglu and Aygun [2009] introduced fuzzy soft (normal) groups. Xu et. al. [2010] introduced the notion of vague soft set, which is an extension to the soft set and vague set, discussed the basic properties of vague soft sets.

Dengfeng and Chuntian [2002] introduced the concept of the degree similarity between intuitionistic fuzzy sets, and he presented several new similarity measure for measuring the degree of similarity between intuitionistic fuzzy sets, which may be finite or continuous, and he gave corresponding proofs of these similarity measure, and discussed applications of the similarity measure between intuitionistic fuzzy sets to recognize the pattern problems.

Burillo and Bustince [1996] showed the notion of vague sets coincides with that of intuitionistic fuzzy sets, and considered the fuzzy sets in ordered groupoids. De et. al. [2001] studied the Sanchez's approach for medical diagnosis and extended this concept with the notion of intuitionistic fuzzy set theory.

Kim et. al. [2000] introduced the notion of an intuitionistic fuzzy subquasigroup of a quasigroup. Kehayopulu and Tsingelis [2002] first considered the fuzzy sets in ordered groupoids. Atin and Lee [2004] discussed the concepts of fuzzy sub-algebra of BG-algebra, and studied union, intersection and other basic properties of fuzzy sub-algebra of BG-algebra.

In case of intuitionistic fuzzy sets, there were several attempts to define intuitionistic fuzzy rings (Jun, et. al., [1999]; Li-mei Yan, [2008]) by generalizing the approach unused by Liu [1982] to define fuzzy ring. Dib et. al. [1996] obtained another new formulation for fuzzy rings and fuzzy ideals.

Mohammed F. Marashdeh and Abdul Razak Salleh [2011] presented a new formulation of intuitionistic fuzzy rings based on the notion of intuitionistic fuzzy spaces, and a relation between intuitionistic fuzzy ring based on intuitionistic fuzzy space & ordinary ring is obtained in terms of induction and correspondence.

There are several theories, for example, theory of fuzzy sets [Zadeh, 1965], theory of intuitionistic fuzzy sets [Atannassov, 1986], vague sets [Gau and Buehrer, 1993], Interval sets [Yang et. al. 2009], and rough sets [Pawlek, 1982], which can be considered as mathematical tools for dealing with uncertainties. But all these theories have this inherent difficulties pointed out by Molodtsov in [1999]. The reason for these difficulties is possibly the inadequacy of the parameterization tool of the theories.

Molodtsov [1999] initiated the Novel Concept of soft set theory which is completely new approach for modeling vagueness and Uncertainties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in [1999]. After his works, few different applications of soft set theory were studied in [Acar et.al., 2010].

Furthermore Maji et. al. [2003] worked on soft set theory. Also Maji et. al. [2001a, 2001b] presented the definition of fuzzy soft set and Maji et.al. [2002a, 2002b, 2002c] presented some applications of these notion to decision making problems. The algebraic structures of set theories dealing with uncertainties have also been studied by some authors.

.Jun et. al. [2004] introduced the concept on fuzzy h- ideals in hemi rings and investigated the idea of anti fuzzy left h- ideals in hemi rings. Some structure properties of Q-fuzzy left h- ideal are established in a hemi ring. Similar results in the intuitionistic anti fuzzy ideal in a hemi rings are obtained.

Consequently, Molodtsov [1999] proposed a completely new approach for modeling vagueness and uncertainty. This is so-called soft theory that is free from the difficulties affecting existing methods. In soft set theory, the problem of setting the membership function among other related problems simply does not arise. This makes the theory very convenient and easy to apply in practice soft set theory that has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Person integration, Probability theory and Measurement theory. Most of these applications have already been demonstrated in Molodtsov's book [1999].

At present, work on the soft set theory is progressing rapidly. Maji et al [2002a] described the application of soft set theory to a decision making problem using rough sets. The same authors have also published a detailed theoretical study on soft sets. The algebraic structure of set theories dealing with uncertainties has also been studied by some authors.

Rough ideal were discussed by Hosseini et. al. [2012], and some other authors like as Atanassov [1986] and [Maji. et. al., 2002]) have studied the algebraic properties of soft sets as well. The main purpose of the work is to introduce a basic version of soft group theory, which extends the notion of a group to include the algebraic structure of soft set.

Our definition of soft groups is similar to the definition of rough groups, but is constructed using different methods. This chapter begins by introducing the basic concepts of fuzzy soft set theory, and then a basic version of fuzzy soft group theory is discussed, and it is extended the notion of a group to the algebraic structures of fuzzy soft sets. A fuzzy soft group is a parameterized family of fuzzy subgroups.

Rosenfield [1971] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Rough groups were defined by Biswas and some authors have studied the algebraic properties of rough sets as well. Recently the many authors have discussed the soft set. Research on the soft set theory is progressing rapidly. For example, the concept of soft semi ring, soft group, soft BCK / BCI algebras, soft BL-algebras and fuzzy soft groups.

This chapter begins by introducing the basic concept of fuzzy soft set theory, then we introduce the basic version of fuzzy soft ring theory, which extends the notion of the ring to the algebraic structure of fuzzy soft set. In this discussion, Molodtsov notion of soft set is studied and fuzzy soft set is considered so that the parameters are mostly fuzzy hedges or fuzzy parameters. The algebraic structures of fuzzy soft sets are discussed, and given the definition of fuzzy soft ring. Operations are defined on fuzzy soft rings and proved some results on them. Finally image, pre-image, and fuzzy soft homomorphism on fuzzy soft set are discussed with their algebraic properties.

Section 2.2: Preliminaries and definitions:

Throughout this chapter, R denotes a commutative ring and all fuzzy soft sets are considered over R.

Muthuraj et. al. [2010] redefined the notion of a fuzzy HX group and then defined a new algebraic structure of Q-fuzzy HX group and anti Q-fuzzy HX subgroup, and analyzed some related properties. Nagarajan et. al. [2012] considered the bi Q-fuzzification of the concepts of several ideals in a semigroup and analyzed some properties of such ideals.

Jianming Zhan and Zhisong Tan [2004] introduced the concept of intuitionistic M-fuzzy groups and obtained few algebraic properties on their structures.

Jiehua Zhou et.al. [2011] applied the concept of Intuitionistic fuzzy soft sets to semigroup theory, and the notion of Intuitionistic fuzzy soft ideals over semigroup. Some lattice structures of the set of all Intuitionistic fuzzy soft ideals of a semigroup were derived.

Sharma [2011b], 2012 analyzed a relation between intuitionistic fuzzy set and its image under a homomorphism, and its properties on (α, β) cut of intuitionistic fuzzy sets. He analyzed a relation between intuitionistic fuzzy normal subgroups and its algebraic properties on them, and its properties on Klein groups.

Jun [2005] got characterizations of intuitionistic fuzzy bi-ideals are given, and with some additional conditions, equivalence relation on intuitionistic fuzzy bi-ideals is investigated. The fuzzification of the notion on a bi-ideal in ordered semigroups is considered. He proved every intuitionistic fuzzy bi-ideal of an ordered semigroup is an intuitionistic fuzzy subsemigroup, and every intuitionistic fuzzy bi-ideal is constant in a regular, left & right simple ordered semigroup. Further he gave conditions for an ordered semigroup to be completely regular in terms of intuitionistic fuzzy set, and provided characterizations of intuitionistic fuzzy bi-ideal in ordered semigroup.

Li Xiaoping and Wang Guijun [2000] discussed union and intersection properties of intuitionistic fuzzy groups, and intuitionistic fuzzy normal subgroups.

Palaniappan et. al. [2009] obtained properties on intuitionistic anti fuzzy subgroups with homomorphic images & preimages and anti-homomorphic images & primages. Prince Williams [2007, 2010] introduced a notion of intuitionistic fuzzy n-ary subgroups in an n-ary group, and its various level cut sets, images and preimages n-ary homomorphism, & some properties.

Few definitions are first given:

Definition 2.2.1. A fuzzy subset μ in a non-empty set X is a function $\mu : X \to [0,1]$. The complement μ^c of μ is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 2.2.2: A mapping μ: X **(**an arbitrary non – empty set) → [0, 1] is a fuzzy set is X. For any fuzzy set μ is X and any X ∈ [0, 1], the set $L(\mu: \alpha) = \{ x \in X: \mu(x) \geq \alpha\}$ is a lower level cut of μ.

Definition 2.2.3: Let Q and G be a set and a group respectively. A mapping $A: G \times Q \to [0,1]$ is a Q-fuzzy set in G. If a Q- fuzzy set A is upper Q-fuzzy subgroup of G if (QFG1): $A(xy, q) \leq \max \{ A(x, q), A(y, q)\}$; (QFG2): $A(x^{-1},q) = A(x, q)$; (QFG3) $A(e, q) = 1$ for all $x, y \in G$ and $q \in Q$.

Definition 2.2.4: A pair (f, A) is called a soft set over X, if f: A → P (X), where X is the initial universe and E is a set (consists of parameters); and P(X) denotes the power set X.

Definition 2.2.5: A pair (f, A) is called a fuzzy soft set over X, where f: $A \to l^x$ and l^x denote the set of all fuzzy sets on $X \Rightarrow f(a) = f_a : X \to [0, 1]$ is a fuzzy set in X for each $a \in A$.

Definition 2.2.6: A pair (f. A) is called a soft set over the lattice L, If f: A→ P(L). Here L be the initial universe and E be the set of parameters; P(L) denotes the power set of L;

Definition 2.2.7: Let (f, A) be a non-null soft set over a ring R. Then (f, A) is said to be a soft ring over R if and only if f(a) is sub ring of R for each a ∈A.

Definition: 2.2.8: Let (f, A) be a non-null fuzzy soft set over a ring R. Then (f, A) is called a fuzzy soft ring over R if and only if f(a) = f_a is a fuzzy sub ring of R,

(FSR1) $f_a(x+y) \geq T(f_a(x), f_a(y))$

(FSR2) $f_a(-x) \geq f_a(x)$

(FSR3) $f_a(xy) \geq T\{f_a(x), f_a(y)\}$, $for\ all\ x, y \in R$, and for each a ∈A.

Definition 2.2.9: Let (f, A) be a fuzzy soft set over L. The soft set $(f, A)_\alpha = \{(f_a)_\alpha / a \in A\}$ for each $\alpha \in [0, 1]$ is called α - level soft set.

Definition 2.2.10: Let $\varphi: X \to Y\ and\ \psi: A \to B$ be two functions, where A and B are parameter sets for the crisp sets X and Y respectively. Then the pair (φ, ψ) is called a fuzzy soft function from X to Y.

Definition 2.2.11: The pre-image of (g, B) under the fuzzy soft function (φ, ψ) denoted by $(\varphi, \psi)^{-1}$ is the fuzzy soft set defined by $(\varphi, \psi)^{-1}$ (g, B) = $(\varphi^{-1}(g),\ \psi^{-1}(B))$

Definition 2.2.12: Let (φ, ψ): $X \to Y$ is a fuzzy soft function. If ϕ is a homomorphism from $X \to Y$ then (φ, ψ) is said to be fuzzy soft homomorphism. If ϕ is a isomorphism from $X \to Y$ and ψ is 1-1 mapping from A onto B, then (φ, ψ) is said to be fuzzy soft isomorphism.

Definition 2.2.13: Let f_a be a fuzzy soft ring in R. Let $\theta: R \to R^1$ be a map and define $f_a^\theta(x) = f_a(\theta x)$ by defining $f_a^\theta: R \to [0,\ 1]$.

Section 2.3: Properties of fuzzy soft ring

Introduction: Molodtsov [1999] initated the concept of soft set theory as a new approach for modeling uncertainties. Then Maji et.al [2001a, 2001b] expanded this theory to fuzzy soft set theory. The algebraic structures of soft set theory have been studied increasingly in recent years.

Aktas and Ca'gman [2007] defined the notion of Soft groups. Feng et.al [2008] initiated the study of soft semi rings and finally soft rings are defined by Acar et.al [2010].

In this study, homomorphic image of fuzzy soft ring is introduced, and it is a generalization of soft rings introduced by Acar et.al [2010]. Some of their properties are studied. The contributions are given as follows:

Theorem 2.3.1: Let R and R^1 be two rings and $\theta: R \to R^1$ be a soft homomorphism. If f_b is a fuzzy soft ring of R, then the pre-image $\theta^{-1}(f_b)$ is a fuzzy soft ring of R.

Proof: Assume that f_b is fuzzy soft ring of R^1. Let $x, y \in R$

(FSR1): $\mu_{\theta^{-1}(f_b)}(x+y) = \mu_{f_b}\theta(x+y = \mu_{f_b}(\theta x + \theta y)$

$\geq \{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq \left\{\mu_{\theta^{-1}} \frac{(x)}{(f_b)}, \mu_{\theta^{-1}} \frac{(y)}{(f_b)}\right\}$

(FSR2): $\mu_{\theta^{-1}} \frac{(-x)}{(f_b)} = \mu_{f_b}\theta(-x)$

$\geq \mu_{f_b}\theta(x)$

$= \mu_{\theta^{-1}} \frac{(x)}{(f_b)}$

(FSR3): $\mu_{\theta^{-1}} \frac{(xy)}{(f_b)} = \mu_{f_b}\theta(xy)$

$= \mu_{f_b}((\theta x)(\theta y))$

$\geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\left\{\mu_{\theta^{-1}} \frac{(x)}{(f_b)}, \mu_{\theta^{-1}} \frac{(y)}{(f_b)}\right\}$

Therefore $\theta^{-1}(f_b)$ is a fuzzy soft ring of R^1

Theorem 2.3.2: Let $\theta: R \to R^1$ be an epimorphism and f_b be fuzzy soft set in R^1. If $\theta^{-1}(f_b)$ is a fuzzy soft ring of R^1, then f_b is a fuzzy soft ring of R.

Proof: Let $x, y \in R$ Then there exist $k, l \in R$ such that $\theta(k) = x,\ \theta(l) = y$ It follows that

(FSR1): $\mu_{[f_b]}(x+y) = \mu_{f_b}\theta(k+l)$

$= \mu_{\theta^{-1} f_b}(k+l)$

$$\geq T\{\mu_{\theta^{-1}f_b}(k), \mu_{\theta^{-1}f_b}(l)\}$$

$$\geq T\{\mu_{f_b}(x), \mu_{f_b}(y)\}$$

(FSR2): $\mu_{f_b}(-x) = \mu\, f_b\, \theta(-k)$

$$\geq \mu_{\theta^{-1}f_b}(-\theta(k))$$

$$\geq \mu_{\theta^{-1}f_b}(\theta(k))$$

$$\geq \mu_{f_b}(x)$$

(FSR3): $\mu_{f_b}(xy) = \mu_{f_b}(\theta(k)\,\theta(l))$

$$= \mu_{f_b}\,\theta(kl)$$

$$= \mu_{\theta^{-1}f_b}(\mathrm{kl})$$

$$\geq T\{\mu_{\theta^{-1}f_b}(k), \mu_{\theta^{-1}f_b}(l)\}$$

$$\geq T\{\mu_{f_b}(x), \mu_{f_b}(y)\}$$

$$\geq T\{\mu_{f_b}(x), \mu_{f_b}(y)\}$$

Thus (f_b) is a fuzzy soft ring of R.

Theorem 2.3.3: If f_a is a fuzzy soft ring of R and $\theta: R \to R^1$ be a soft homomorphism of R, then the fuzzy soft set $f_a^{\theta} = \{(x, f_a^{\theta}(x)) | x \in R\}$ is a fuzzy soft ring of R.

Proof: Let $x, y \in R$. It gives the following implications:

(FSR1): $f_a^\theta(x+y) = f_a\theta(x+y)$

$$= f_a(\theta x + \theta y)$$

$$\geq T\{f_a(\theta x), f_a(\theta y)\}$$

$$\geq T\{f_a^\theta(x), f_a^\theta(y)\}$$

(FSR2): $f_a^\theta(-x) = f_a\big(\theta(-x)\big)$

$$= f_a(\theta(x))$$

$$\geq f_a^\theta(x)$$

(FSR3): $f_a^\theta(xy) = f_a(\theta(xy))$

$$= f_a\big((\theta x)(\theta y)\big)$$

$$\geq T\{f_a(\theta x), f_a(\theta y)\}$$

$$\geq T\{f_a^\theta(x), f_a^\theta(y)\}$$

Thus f_a^θ is a fuzzy soft ring of R.

Section 2.4: Level cut-set of fuzzy soft set over lattice

Theorem 2.4.1: Let f_a be a fuzzy soft set over L, then f_a is fuzzy soft ring over L if and only if for all $a \in A$ and for arbitrary $\alpha \in [0,1]$ with $(f_a)_\alpha \neq 0$, - level soft set $(f_a)_\alpha$ is a fuzzy soft ring over L.

Proof: Let f_a be fuzzy soft ring over L. Then for each $a \in A$ f_a is a fuzzy sub ring of L. For arbitrary $\alpha \in [0,1], (f_a)_\alpha \neq 0$ Let $x, y \in (f_a)_\alpha$ Then $f_a(x) \geq \alpha$ and $f_a(y) \geq \alpha$.

(FSR1): $(f_a)_\alpha(x+y) \geq T\{(f_a)_\alpha(x), (f_a)_\alpha(y)\}$

$\geq T\{f_a(x), f_a(y)\}$

$\geq T(\alpha, \alpha)$

$\geq \alpha$

$\Rightarrow x + y \in (f_a)_\alpha$

(FSR2): $(f_a)_\alpha(-x) \geq (f_a)_\alpha(x)$

$\geq (f_a)(x)$

$\geq \alpha$

It gives that $-x \in (f_a)_\alpha$

(FSR3): $(f_a)_\alpha(xy) \geq T\{(f_a)_\alpha(x), (f_a)_\alpha(y)\}$

$\geq T\{f_a(x), f_a(y)\}$

$\geq T\{\alpha, \alpha\}$

$\geq \alpha$

It becomes that $xy \in (f_a)_\alpha$.

So $(f_a)_\alpha$ is a fuzzy soft ring of R.

Theorem 2.4.2: Every imaginable fuzzy soft ring μ of R is a fuzzy soft ring of R.

Proof: Assume that μ is imaginable fuzzy soft ring of R.

Then we have $\mu(x+y) \geq T\{\mu(x), \mu(y)\}$

and $\mu(-x) \geq \mu(x),\ and \ \ \mu(xy) \geq T\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

Since μ is imaginable, it implies that

$$\min\{\mu(x), (\mu(y))\} = T\ \{\min\{\mu(x), \mu(y)\}, \min\{\mu(x), \mu(y)\}\}$$

$$\geq T\{\mu(x), \mu(y)\}$$

$$\geq \min\{\mu(x), \mu(y)\}$$

and so $T\ \{\mu(x), \mu(y)\} = \min\ \{\mu(x), \mu(y)\}$.

It follows that $\mu(x+y) \geq T\ \{\mu(x), \mu(y)\}$ for all $x, y \in R$.

Hence μ is a fuzzy soft ring of R.

Theorem 2.4.3: If μ is a fuzzy soft ring of R and θ is endomorphism of R, then $\mu[\theta]$ is a fuzzy soft ring of R.

Proof: For any x , $y \in R$, it follows that

(FSR1): $\mu([\theta](x+y)) = \ \mu\left(\theta(x+y)\right)$

$$= \ \mu\ (\theta x + \theta y)$$

$$\geq T\ \{\mu\ (\theta x), \mu(\theta y)\}$$

$$\geq T\ \{\mu\ [\theta](x), \mu\ [\theta](y)\}$$

(FSR2): $\mu\,([\theta](-x)) = \mu\,\theta\,(-x)$

$$\geq \mu\theta(x) \geq \mu\,[\theta](x)$$

(FSR3): $\mu\,[\theta](xy) \quad = \mu\theta(xy)$

$$= \mu\,(\theta x)(\theta y)$$

$$\geq T\,\{\mu(\theta x), \mu(\theta y)\}$$

$$\geq T\,\{\mu\theta(x), \mu\theta(y)\}$$

$$\geq T\,\{\mu(\theta)(x), \mu(\theta)(y)\}.$$

Thus $\mu\,[\theta]$ is a fuzzy soft ring of R.

Theorem 2.4.5: Let T be continuous t-norms and f be a soft homomorphism on R. If μ is a fuzzy soft ring of R, then μ^f is a fuzzy soft ring of f(R).

Proof: $A_1 = f^{-1}(y_1)\ and\ A_2 = f^{-1}(y_2)\ and\ A_{1\,2} = f^{-1}(y_1 + y_2)$ where $y_1, y_2 \in f(R)$

Consider the set: $A_1 + A_2 = \{x \in R\colon\ x = a_1 + a_2\}$ for some $a_1 \in A_1\ and\ a_2 \in A_2$.

If $x \in A_1 + A_2$, then $x = x_1 + x_2$ for some $x_1 \in A_1\ and\ x_2 \in A_2$.

So that it follows that $(x) = f(x_1) + f(x_2) = y_1 + y_2$, and $x \in f^{-1}(y_1 + y_2) = A_{1\,2}$.

Thus $A_1 + A_2 \in A_{12}$.

(FSR1): $\mu^f(y_1 + y_2) = \sup\{\mu(x) | x \in f^{-1}(y_1 + y_2)\}$

$$= \sup\{\mu(x) | x \in A_{1\,2}\}$$

$$\geq \sup\{\mu(x) | x \in A_1 + A_2\}$$

$$\geq \sup\{\mu(x_1 + x_2) |\ x_1 \in A_1, x_2 \in A_2\}$$

Since T is continuous, therefore it sees that for every $\in > 0$,

$$\sup\{\mu(x_1)|x_1 \in A_1\} + x_1^* \leq \delta \ \ and \ \sup\{\mu(x_2)|x_2 \in A_2\} + x_2^* \leq \delta$$

$$T\,\{\sup\{\mu(x_1)|x_1 \in A_1\} \sup\{\mu(x_2)|x_2 \in A_2\}\} + T(x_1^*, x_2^*) \leq \in$$

Choose $a_1 \in A_1 \ and \ a_2 \in A_2$ such that

$\sup\{\mu(x_1)|x_1 \in A_1\} + \mu(a_1) \leq \delta; \sup\{\mu(x_2)|x_2 \in A_2\} + \mu(a_2) \leq \delta$

$$T\,\{\sup\{\mu(x_1)|x_1 \in A_1\}, \sup\{\mu(x_2)|x_2 \in A_2\}\} + T(\mu(a_1), \mu(a_2)) \leq \in$$

Consequently, it finds that

$\mu^f(y_1 + y_2) \geq \sup\{T\{\mu(x_1), \mu(x_2)|x_1 \in A_1, x_2 \in A_2\}\}$

$$\geq T\{\sup \mu(x_1)|x_1 \in A_1, \sup\{\mu(x_2)|x_2 \in A_2\}\} \geq T\{\mu^f(y_1), \mu^f(y_2)\}.$$

Similarly, $\mu^f(-x) \geq \mu^f(x) \ and \ \mu^f(xy) \geq T\{\mu^f(x), \mu^f(y)\}$. So μ^f is fuzzy soft-ring of f(R).

Theorem 2.4.6: Onto homomorphic image of a fuzzy soft ring with sup property is a fuzzy soft ring.

Proof: Let $f: R \rightarrow R^1$ be an onto homomorphism of rings.

Also μ has the sup-property of fuzzy soft ring of R.

Let $x^1, y^1 \in R^1 \ and \ x_0 \in f^{-1}(x^1) \ \ and \ \ \ y_0 \in f^{-1}(y^1)$ be such that

$$\mu(x_0) = \sup_{h \in f^{-1}(x^1)} \mu(h) \ \ and \ \ \mu(y_0) = \sup_{h \in f^{-1}(y^1)} \mu(h)$$

Respectively, then it can deduce that

(FSR1): $\mu^f(x^1 + y^1) = \sup\limits_{z \in f^{-1}(x^1+y^1)} \mu(z)$

$$\geq T\{\mu(x_0), \mu(y_0)\} = T\{\sup \mu(h), \sup \mu(h), h \in f^{-1}(x^1), \mu \in f^{-1}(y^1)\}$$

$$\geq T\{\mu^f(x^1), \mu^f(y^1)\}$$

(FSR2): $\mu^f(-x^1) = \sup\limits_{z \in f^{-1}(-x^1)} \mu(z)$

$$\geq \mu(x_0)$$

$$\geq \mathit{sup}_{h \in f^{-1}(x^1)} \mu(h)$$

$$\geq \mu^f(x^1)$$

(FSR3): $\mu^f(x^1 y^1) = \sup\limits_{z \in f^{-1}(x^1 y^1)} \mu(z)$

$$\geq T\{\mu(x_0), \mu(y_0)\}$$

$$\geq T\left\{\sup_{h \in f^{-1}(x^1)} \mu(h), \sup_{h \in f^{-1}(y^1)} \mu(h)\right\}$$

$$\geq T\{\mu^f(x^1), \mu^f(y^1)\}$$

Therefore μ^f is a fuzzy soft ring of R^1

Theorem 2.4.7: Let f_a be a fuzzy soft ring over R and (ϕ, ψ) be a fuzzy soft homomorphism from R to R^1. Then $(\phi, \psi)\ f_a$ is a fuzzy soft ring over R^1.

Proof: Let $k \in (\psi)f_a$ and $y_1, y_2 \in y$ If $\phi^{-1}(y_1) = \phi\ or\ \phi^{-1}(y_2) = \phi$

Let us assume that, there exist $x_1, x_2 \in X$ such that $\phi(x_1) = y_1\ and\ \phi(x_2) = y_2$

$$\phi(f_a)_k(y_1 + y_2) = V \underset{\phi(t)=y_1+y_2}{V} \quad \underset{\psi(a)=k}{V} f_a(f)$$

$$\geq \underset{\psi(a) = k}{V} \quad f_a(x_1 + x_2)$$

$$\geq \underset{\psi(a) = k}{V} \quad \{T\{f_a(x_1), f_a(x_2)\}\}$$

$$\geq T\{V f_a(x_1), V f_a(x_2), \psi(a) = k\ \ \psi(a) = k\}$$

This inequality satisfies for each x_1, $x_2 \in X$ for which $\phi(x_1) = y_1, \phi(x_2) = y_2$.

Then it becomes that

(FSR1): $\phi(f_a)_k(y_1 + y_2) \geq T\left\{\left(\underset{\phi(t_1)=y_1}{V} \ \underset{\psi(a)=k}{V f_{a_1}(t_1)}\right), \left(\underset{\phi(t_2)=y_1}{V} \ \underset{\psi(a)=k}{V f_{a_2}(t_2)}\right)\right\}$

$$\geq T\{\phi(f_a)_k(y_1), \phi(f_a)_k(y_2)\}$$

and similarly, it implies that $\phi(f_a)_k(-y) \geq \phi(f_a)_k(y)$;

Thus $\phi(f_a)_k(y_1 y_2) \geq T\{\phi(f_a)_k(y_1), \phi(f_a)_k(y_2)\}$. So $(\phi, \psi)f_a$is fuzzy soft ring of R_1.

Theorem 2.4.8: Let g_b be a fuzzy soft ring over R^1 and (ϕ, ψ) be a fuzzy soft homomorphism from R to R_1. Then $(\phi\psi)^{-1}g_b$ is fuzzy soft ring over R.

Proof: Let $a \in \psi^{-1}(B)\ and\ x_1, x_2 \in X$

(FSR1): $\phi^{-1}(g_a)(x_1 + x_2) = g_{\psi(a)}(\phi(x_1x_2))$

$$= g_{\psi(a)}\,(\phi(x_1)\phi(x_2)$$

$$\geq T\{g_{\psi(a)}\phi(x_1), g_{\psi(a)}\phi(x_2)\}$$

$$= T\{\phi^{-1}(g_a)(x_1), \phi^{-1}(g_a)(x_2)\}$$

and similarly, it finds that $\phi^{-1}(g_a)(-x) \geq \phi^{-1}(g_a)(x)\ \ and$

$$\phi^{-1}(g_a)(x_1x_2) \geq T\{\phi^{-1}(g_a)(x_1), \phi^{-1}(g_a)(x_2)\}$$

Thus $(\phi, \psi)^{-1}g_b$ is a fuzzy soft ring over R.

Conclusions:-This chapter summarized the basic concept of fuzzy soft set. A detailed theoretical study of fuzzy soft set is presented, which led to the definition of new algebraic structures in ring structures. This work focused on fuzzy soft rings homomorphism of fuzzy soft rings and pre-image of fuzzy soft rings. To extend this work, one could study the properties of fuzzy soft sets in other algebraic structures such as near rings, groups, ideals, fields and G-modules. A part of this chapter has published in International Journal of Fuzzy Mathematics and System, Volume 4, Number 3, (2014), 305 – 311.