CHAPTER I

PRELIMINARIES AND BASIC DEFINITIONS

Introduction: The algebra of fuzzy spaces and fuzzy subspaces were studied earlier days. Using the concept of fuzzy space and fuzzy binary operation, a new approach can be considered as a generalization and reformulation of the Rosenfeld theory of fuzzy groups. Therefore it is an active tool to develop the theory of fuzzy groups. Rosenfeld [1971] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets.

Demirci [2001] introduced that the concepts of smooth groups and smooth homomorphism and investigated their basic properties. Dib [1994] introduced the concept of fuzzy space. It corresponds to the concept of the universal set in the ordinary case.

Dobrista and Yahhyaeva [2002] have defined and given the conditions for the early notion of homomorphism of fuzzy group. The conditions are considered for fulfillment of various properties of homomorphism of ordinary group as well as properties specific systems with fuzzy operation. It is considered as the introductory notion of preservation of fuzzy operation.

The theory of fuzzy sets has been developed in many directions and it finds applications in a wide variety of fields. Fuzzy groups were discussed, and investigated some of their structures on the concept of fuzzy group family [Bashir Humera and Zahid Raza, (2013); Bayram Ali Ersoy, (2012); Chandrasekhara & Swaminathan, (2006)].

Davvaz et.al. [2006] considered intuitionistic fuzzification of the concept of the H_v – submodules in a H_v – module. Ray [1999] discussed some properties of the product of two fuzzy subsets and fuzzy subgroups.

Osman Kazanci et. al. [2007a and 2007b] studied intuitionistic Q-fuzzy R-subgroups of near ring. Rovento [2001] proved that in the crisp environment, the notions of normal subgroup and group operation on a set are well known due to many applications.

Section 1.1: Fuzzy group, fuzzy normal subgroup, and fuzzy sub-groupoid

Rosenfeld [1971] defined fuzzy sub-groupoids, and proved that a homomorphic image of a fuzzy sub-groupoid with the sup property is a fuzzy groupoid and hence that a homomorphic image of a fuzzy subgroup with sup property is a fuzzy subgroup. This theorem needs the sup property, but it can be showed without sup property.

Moreover Mukherjee and Prabir Bhattacharya [1986, 1991] showed that if [Ậ] is a fuzzy subgroup of a finite group G such that all the level subgroups of G are normal subgroups then [Ậ] is a fuzzy normal subgroup. They can also obtained the theorem without finiteness using the transfer principle which is a fundamental tool developed here. There is a result that if A is fuzzy subgroup of G, then gAg^{-1} is also fuzzy subgroup of G for all g in G and $\cap\ gAg^{-1}$ is a normal subgroup of G (under a t- norm on A). If A and B are two fuzzy subgroups of G under the t- norms T_1 and T_2 respectively, then $A \cap B$ is also a fuzzy subgroups under any t- norm T such that $T_1, T_2 \geq T$. The intersection of any two normal fuzzy subgroups of G is also a normal fuzzy subgroup of G under any t- norm weaker than the t- norms of the two fuzzy subgroups.

Mukherjee and Battacharya [1991] also explained that if f: G → H is a group homomorphism and if A is a normal fuzzy subgroup of H, then $f^{-1}(A)$ is a normal fuzzy subgroup of G, and if f is an epimorphism, then f(A) is a normal fuzzy subgroup of H. Also they derived that $G^t = \{ x \in G : A(x) \geq t \}$ is a subgroup of G. The normalizer of a fuzzy subgroup of G is a subgroup of G. The p-level subset and p-level subgroup are introduced in the thesis followed by their algebraic properties.

Davvaz et.al. [2006] considered the intuitionistic fuzzification of the concept of sub hyperquasigroups in a hyper quasigroup. An intuitionistic Q-fuzzy R- subgroups of near rings are given by Osman kazanci et.al. [2007a, 2007b].

The notion of intuitionistic Q- fuzzy semi primarily in a semi group is given by Kim [2006]. Also Roh et.al. [2006] considered the intuitionistic Q-fuzzification of the BCK / BCI algebra.

Preliminaries of various fuzzy groups:

The classification suggested for the early notion of homomorphism of fuzzy group's conditions is considered for fulfillment of various properties of homomorphism of ordinary groups as well as properties specific to systems with fuzzy operation correctness which is discussed in the introduction notion of preservation of fuzzy operation.

Ray [1999] analyzed that in the short communication, some properties of the product of two fuzzy subsets and fuzzy subgroups. Rovento [2001] proved that in crisp environment, the notions of normal subgroup and group operating on a set are well known due to many applications.

Section 1.2: Structures on imaginable fuzzy soft isomorphism

Saravanan and Sivakumar [2012c] obtained (1) Let (R, +, .) and (R|, +, .) be any two semi-rings. The homomorphic image of an intuitionistic fuzzy normal sub-semiring of R is an intuitionistic fuzzy normal sub-semiring of R|; **(2)** Let (R, +, .) and (R|, +, .) be any two semirings. The homomorphic pre-image of an intuitionistic fuzzy normal sub-semiring of R| is an intuitionistic fuzzy normal sub-semiring of R; **(3)** Let A be an intuitionistic fuzzy sub-semiring of a semiring H and f is an isomorphism from a semiring R onto H. If A is an intuitionistic fuzzy normal sub-semiring of the semiring H, then A ◦ f is an intuitionistic fuzzy normal sub-semiring of the semiring R;

Further they [2012c] showed (4) Let A be an intuitionistic fuzzy sub-semiring of a semiring H and f is an anti- isomorphism from a semiring R onto H. If A is an intuitionistic fuzzy normal sub-semiring of the semiring H, then A ◦ f is an intuitionistic fuzzy normal sub-semiring of the semiring R. They made an attempt to study the algebraic nature of intuitionistic fuzzy normal sub-semiring of a semiring under homomorphism and anti-homomorphism.

Saravanan and Sivakumar [2012b] gave (1) Let (R, +, .) and (R|, +, .) be any two semirings. The homomorphic image of an intuitionistic fuzzy sub-semiring of R is an intuitionistic fuzzy sub-semiring of R|; (2) The homomorphic pre-image of an intuitionistic fuzzy sub-semiring of R| is a intuitionistic fuzzy sub-semiring of R;

They [2012b] verified that (3) If f : R →R| is an anti-homomorphism, then the anti-homomorphic pre-image of a level sub-semiring of an intuitionistic fuzzy sub semiring of R| is a level sub-semiring of an intuitionistic fuzzy sub-semiring of R. They made an attempt to study the algebraic nature of intuitionistic fuzzy sub-semiring of a semiring under homomorphism and anti-homomorphism.

Tariq Shah and Muhammad Saeed [2012] gave (1) Let f: R $\rightarrow$ R^1 be a surjective ring anti-homomorphism. If μ is a strongly irreducible fuzzy ideal of R^1, then $f^{-1}(\mu)$ is a strongly irreducible fuzzy ideal of R; (2) For a ring anti-homomorphism f : R $\rightarrow$ R^1, if μ is an f-invariant strongly irreducible fuzzy ideal of R, then f(μ) is a strongly irreducible fuzzy ideal of R^1; (3) For a surjective ring anti-homomorphism f : R $\rightarrow$ R^1, if every fuzzy ideal of R is f-invariant and has a fuzzy primary (respectively, strongly primary) decomposition in R, then every fuzzy ideal of R^1 has a fuzzy primary (respectively, strongly primary) decomposition in R^1.

They investigated anti-homomorphic images and pre-images of semiprime, strongly primary, irreducible and strongly irreducible fuzzy ideals of a ring. (4) For a surjective anti-homomorphism f : R $\rightarrow$ R^1, if every fuzzy ideal of R is f-invariant and has a fuzzy primary (respectively, strongly primary) decomposition in R, then every fuzzy ideal of R^1 has a fuzzy primary (respectively, strongly primary) decomposition in R^1.

Fray [1992] analyzed that the group near-ring constructed from a right near-ring R and a group G is studied in the special case where the near-ring is distributively generated. In particular, results concerning homomorphism of near-rings, groups and the augmentation ideals are obtained which resemble closely those obtained for groups and rings.

Gtjnter Pilz &Yong-Sian So [1980] investigated near-rings of polynomials and polynomial functions. After some results which belong to universal algebra the attention are turned to the familiar case of polynomials and polynomial functions over a commutative ring with identity. They studied the relation between ring-homomorphism and near-ring homomorphism, and the behavior of polynomial near-rings when the ring splits into a direct sum.

Kyung Ho Kim and Young Bae Jun [2002] formulated fuzzy characteristic right (resp. left) R-subgroups, the fuzzy same type right (resp. left) R-subgroups of a near-ring R, and gave their characterizations.

The theory of fuzzy set has application in a many directions and is finding applications in a wide variety of fields. Solairaju and Nagarajan [2010b] have given independent proof of several theorems on pseudo fuzzy cossets of fuzzy normal subgroups. They have introduced the notion of pseudo fuzzy double cossets, pseudo fuzzy middle cossets of a group and given its fundamental properties. In this chapter, some characterization of Q-fuzzy sets and some results on Q- fuzzy groups are obtained.

Solairaju and Nagarajan [2010a] studied an extension of this classical notion to the Q-fuzzy sets to define the concept of Q-cyclic fuzzy groups. By using these Q- cyclic fuzzy groups, they defined a Q-cyclic fuzzy group family and investigated its structural properties with applications.

The following definitions are needed for the discussions in chapter II:

Definition 1.2.1: A fuzzy subset μ in a non-empty set X is a function μ : X → [0,1]. The complement μ^c of μ is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all x ∈ X. A mapping μ: X **(**an arbitrary non – empty set) → [0, 1] is a fuzzy set is X. For any fuzzy set μ is X and any X ∈ [0, 1], the set L(μ: α) = { x ∈ X: μ(x) ≥ α} is a lower level cut of μ.

Definition 1.2.2: Let Q and G be a set and a group respectively. A mapping A: G×Q → [0,1] is a Q-fuzzy set in G. If a Q- fuzzy set A is upper Q-fuzzy subgroup of G if (QFG1): A(xy, q) ≤ max { A(x, q) , A(y, q)}; (QFG2): $A(x^{-1},q) = A(x, q)$; (QFG3) A(e, q) = 1 for all x, y ∈ G and q ∈ Q.

Definition 1.2.3: A pair (f, A) is a soft set over X, if f: A → P (X) where P(X) denotes the power set X. A pair (f, A) is called a fuzzy soft set over L, where f :A →I^L (for each a ∈ A, f_a : L →I is a fuzzy set in L). Let (f, A) be a non-null soft set over a ring R. Then (f, A) is said to be a soft ring over R if and only if f(a) is sub-ring of R for each a ∈ A.

Definition 1.2.4: Let (f, A) be a non-null fuzzy soft set over a ring R. Then (f, A) is called a fuzzy soft ring over R if and only if f(a) = f_a is a fuzzy sub ring of R. for each a $\in$ A,

(FSR1) $f_a (x + y) \geq T \{ f_a(x), f_a(y) \}$

(FSR2) $f_a (-x) \geq f_a (x)$

(FSR3) $f_a (x . y) \geq T \{ f_a(x), f_a(y) \}$ for all x, y$\in$ R.

Definition 1.2.5: Let (f, A) be a fuzzy soft set over L. The soft set $(f, A)_\alpha$ is defined by $\{ (f_\alpha)_a : a \in A\}$ for each $\alpha \in A$ is called α-level soft set.

Definition 1.2.6: Let ϕ: X $\rightarrow$ Y and ψ: A $\rightarrow$ B be two functions, where A and B are parameters sets for crisps sets X and Y respectively. Then $t^{(\varphi, \psi)}$ is a fuzzy soft function from X to Y.

Definition 1.2.7: The pre-image of (g, B) under the fuzzy soft function $< \phi, \psi >$ denoted by $< \phi, \psi >^{-1}$ is the fuzzy soft set defined by $< \phi, \psi >^{-1} (g, B) = < \phi^{-1}(g), \psi^{-1} (B) >$.

Definition 1.2.8: Let $< \phi, \psi > : X \rightarrow Y$ is a fuzzy soft function. If ϕ is a homomorphism from X $\rightarrow$ Y, then $< \phi, \psi >$ is fuzzy soft homomorphism; if ϕ is a isomorphism from X $\rightarrow$ Y and ψ is 1-1 mapping from A onto B.

Definition 1.2.9: Let f_a be a fuzzy soft ring in R, and θ: R $\rightarrow$ R' be a map. Define f_a^θ: R $\rightarrow$ [0, 1] by $f_a^\theta(x) = f_a (\theta x)$ for all x in R.

Properties of fuzzy soft ring

Notation: Here R and R' denotes a commutative ring.

Few contributions are mentioned here:

Theorem 1.2.10: Let R and R' be two rings and θ: R $\rightarrow$ R' be a soft homomorphism. If f_b is a fuzzy soft ring of R, then the pre-image θ^{-1} (f_b) is a fuzzy soft ring of R.

Theorem 1.2.11: Let θ: R $\rightarrow$ R' be an epimorphism and f_b be fuzzy soft set in R'. If θ^{-1} (f_b) is a fuzzy soft ring of R', then f_b is a fuzzy soft ring of R.

Theorem 1.2.12: If f_a is fuzzy soft ring of R and θ: R $\rightarrow$ R' be a soft homomorphism of R Then the fuzzy soft set f_a^{θ} = { (x, $f_a^{\theta}(x)$) : x $\in$ R} is fuzzy soft ring of R.

Theorem 1.2.13: Let f_a be a fuzzy soft set over L. Then f_a is fuzzy soft ring over L if and only if for all a $\in$ A and for arbitrary $\alpha \in [0, 1]$ with (f_a) $\neq$ 0, α-level soft set $(f_a)_\alpha$ is a fuzzy soft ring over L.

Theorem 1.2.14: Every imaginable fuzzy soft ring μ of R is a fuzzy soft ring of R.

Theorem 1.2.15: If μ is a fuzzy soft ring of R and θ is an endomorphism of R. then $\mu[\theta]$ is fuzzy soft ring of R.

Theorem 1.2.16: Let T be continuous t-norms and f be a soft homomorphism on R. If μ is a fuzzy soft ring of R, then μ^f is a fuzzy soft ring of f (R).

Theorem 1.2.17: Onto homomorphic image of a fuzzy soft ring with sup property is a fuzzy soft ring of R.

Theorem 1.2.18: Let f_a be a fuzzy soft ring over R and < ϕ, ψ > be a fuzzy soft homomorphism from R to R^1. Then < ϕ, ψ > f_a is a fuzzy soft ring over R^1.

Theorem 1.2.19: Let g_b be a fuzzy soft ring over R_1 and < ϕ, ψ > be a fuzzy soft homomorphism from R to R_1. Then $< \phi, \psi >^{-1} (g_b)$ is a fuzzy soft ring over R.

Section 1.3: Q-fuzzy soft normal subgroups of Hemi ring

Nagarajan et. al. [2012a] showed (1) if μ is fuzzy soft ring of r and θ is endomorphism of R, then μ[θ] is Fuzzy soft ring of R; (2) let t be continuous t-norm and f be a soft homomorphism on R. If μ is fuzzy soft ring of R, then μf is a fuzzy soft ring of f (R); (3) onto homomorphic image of a fuzzy soft ring with sup property is a fuzzy soft ring of R.

Molodtsov [1999] initiated the concept of soft set theory as a new approach for Modeling uncertainties. Then Maji et.al [2001a] expanded this theory to fuzzy soft set theory. The algebraic structures of soft set theory have been studied increasingly in recent years. Aktas and ca'gman [2007] defined the notion of soft groups. Feng et.al [2008] initiated the study of soft semi rings and finally soft rings are defined by Acar et.al [2010]. The application of soft set was analyzed in [Chen & Tsang, 2005; Zhiming Zhang, 2012].

Nagarajan et.al. [2012a] introduced homomorphic image of fuzzy soft ring, which is a generalization of soft rings introduced by Acar et.al. [2010] and some of their properties were studied. Hemirings were discussed in [Anitha and Arjunan, 2012a, 2012b]. Fuzzy quotient rings and fuzzy isomorphism were studied in [Areez Tawfeek Hameed, 2010]. Jayanta Ghosh et. al. [2011] found some algebraic results on fuzzy soft ring and fuzzy ideal. Pazar Varol et.al. [2012] investigated few properties on fuzzy soft ring.

Definition 1.3.1: Let X be a group and (f, A) be a soft set over X. Then (f, A) is a soft group over X if and only if f(a) ≤ x, for all a ∈ A.

Union of two fuzzy soft sets (f, A) and (g, B) over universal set X is the fuzzy soft set (h, C) where C = A ∪ B. Then (f, A) ∪ (g, B) = (h, C) is noted.

Intersection of two fuzzy soft sets (f, A) and (g, B) over universal X is the fuzzy soft set (h, C), where C = A ∩ B. In this case (f, A) ∩ (g, B) = (h, C).

Product of two fuzzy soft sets (f, A) and (g, B) denoted (f, A) ∩ (g, B) is defined as (h, A × B) where h (a, b) = f (a) ∩ g (b).

Definition 1.3.2: Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is fuzzy soft group if: (i) $f_a(xy) \geq \min \{f_a(x), f_a(y)\}$; (ii) $f_a(x^{-1}) \geq f_a(x)$; (iii) $f_a(x) = e$, for all a ∈ A, and x, y ∈ X.

Definition 1.3.3: Let 'f_a' be a fuzzy soft set in X and ϕ: X → Y be a map. Then a map $f_a^{\phi} : X \to [0, 1]$ is defined by $f_a^{\phi}(x) = f_a(\phi(x))$, for all x ∈ X.

Definition 1.3.4: Let (f, A) be a fuzzy soft group over X and λ ∈ [0, 1]. Then (i) (f, A) is λ-identity fuzzy soft group over X if $f_a(x) = \lambda$ if x ≠ e for all x ∈ X; = 0 otherwise; (ii) (f, A) is λ-absolute fuzzy soft group over X if $f_a(x) = \lambda$ for all x ∈ X, and a ∈ A.

Definition 1.3.5: Let X be a group and let $f_{a_1}, f_{a_2}, \ldots, f_{a_n}$ be n-fuzzy soft subgroups of X_1, X_2,... X_n. Define a map $f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n}$: $(x_1, x_2, \ldots, x_n) \to [0, 1]$ by $(f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})(x_1, x_2, \ldots, x_n) = \min \{ f_{a_1}(x_1), f_{a_2}(x_2), \ldots, f_{a_n}(x_n) \}$(this is called generalized product of fuzzy soft groups).

Throughout chapter II, fuzzy soft set (f,A) is recalled as f_a

Definition 1.3.6: Let 'μ_A' be a Q- fuzzy soft set of G, and θ: G × Q → G be a map. The map μ_A^θ: G × Q → [0, 1] is defined by $\mu_A^\theta(x, q) = \mu_A(\theta(x, q))$.

Definition 1.3.7: A Q-fuzzy soft group A of G is Q-fuzzy soft characteristics of G if $\mu_A^\theta = \mu_A$.

Definition 1.3.8: A Q- fuzzy soft group A of G is normal if there exists x ∈ G and q ∈ Q such that A(x, q) = 1. Note that if μ is normal Q- fuzzy soft group of G, then A(e, q) = 1 and so A is normal if and only if A(e, q) = 1.

Definition 1.3.9: Let A be a Q- fuzzy soft group of G. Then A is a Q-fuzzy soft normal group (QFSNG) if for all x, y ∈ G, A(xy, q) = A(yx, q), q ∈ Q. Then a Q- fuzzy soft group A is a Q-fuzzy soft normal if $A(x, q) = A(yxy^{-1}, q)$ for x, y ∈ G and q ∈ Q. The notation [x, y] is used to stand for the expression $x^{-1}y^{-1}xy$.

The following contributions are found:

Our work in this section is to define Q- fuzzy soft characteristic group (QFSCG) and to study their properties. For this first of all, the notion of μ_A^θ that is defined, will be useful in our discussion.

Theorem 1.3.10: If A is a Q-fuzzy soft group of G, A^C is also a Q-fuzzy soft group of G.

Theorem 1.3.11: If A is a Q- fuzzy soft group of G, then the set U (A; t) is also a Q-fuzzy soft group for all $q \in Q$, and $t \in Im(A)$.

Definition 1.3.12: Let θ be a mapping from X to Y. If A and B are Q- fuzzy soft sets in X and Y respectively, then the inverse image of B under θ denoted by $\theta^{-1}(B)$ is a Q-fuzzy soft set in X defined by $\theta^{-1}(B) = \mu_\theta^{-1}(B)$ where $\mu_{\theta^{-1}(B)}(x, q) = \mu_B(\theta(x), q)$ for all $x \in X$, and $q \in Q$ and the image of A under θ denoted by θ(A), where $\mu_{\theta(A)}(y, q) = \{\sup \mu_A(x, q)\}$ if $\theta^{-1}(y) \neq 0$ where $x \in \theta^{-1}(y)$; 0, otherwise for all $y \in Y$, and $q \in Q$.

Theorem 1.3.13: Let G and G^1 be two soft groups and $\theta: G \to G^1$ be a homomorphism. If B is a Q-fuzzy soft group of G^1, then the pre-image $\theta^{-1}(B)$ is a Q-fuzzy soft group of G.

Theorem 1.3.14: Let $\theta: G \to G^1$ be a soft epimorphism and B be Q-fuzzy soft set in G^1. If $\theta^{-1}(B)$ is a Q-fuzzy soft group of G, then 'B' is a Q-fuzzy soft group of G^1.

Theorem 1.3.15: If $\{A_i\}_{i \in A}$ is a family of Q-fuzzy soft groups of G, then $\cap A_i$ ($i \in A$) is a Q-fuzzy soft group of G where $\cap A_i (i \in A) = \{ ((x, q), \wedge \mu_{A_i}(x, q)) : x \in G, \text{ and } q \in Q\}$.

Theorem 1.3.16: If 'A' is a Q-fuzzy soft set in G such that each non- empty level subset U (A; t) is a Q-fuzzy soft group of G, then A is a Q- fuzzy soft group of G.

Theorem 1.3.17: A set of necessary and sufficient condition for a Q-fuzzy soft set of a group G to be a Q-fuzzy soft group of G is that $A(xy^{-1}, q) \geq \min (A(x, q), A(y, q)\}$ for all x, y in G and q in Q.

Theorem 1.3.18: If 'A' is a Q-fuzzy soft group of G and θ is a soft homomorphism of G, then the Q-fuzzy soft set A^{θ} of G given by $A^{\theta} = \{ < (x, q), \mu_A^{\theta} (x, q) > : x \in G, q \in Q\}$ is a Q-fuzzy soft group of G.

Theorem 1.3.19: Let 'A' be a Q-fuzzy soft group of G. Let A^+ be a Q-fuzzy soft set in G defined by $A^+ (x, q) = A(x, q) + 1 - A(e, q)$ for all $x \in G$. Then A^+ is a normal Q-fuzzy soft group of G containing A.

Theorem 1.3.20: Let A be a QFSNG of a soft group G .Then $A ([x, y], q) = A (e, q)$ for all $x, y \in G$,

Theorem 1.3.21: If A is QFSCG of a soft group G, then A is QFSNG of G.

Section 1.4: Construction of Q-fuzzy left H-ideal

Lekkoksung [2012b] found (1) Let $f : S \rightarrow T$ be a homomorphism of Γ-semigroups. If $B = (\mu B, \gamma B)$ is an intuitionistic fuzzy bi-ideal of T, then the pre-image $f^{-1}(B) = (f^{-1}(\mu B), f^{-1}(\gamma B))$ of B under f is an intuitionistic fuzzy bi-ideal of S; (2) Let $A = (\mu A; \gamma A)$ be an intuitionistic fuzzy ideal of S. If S is an intra-regular, then $A(a) = A(a\beta a)$ for all $a \in S$, and $\beta \in \Gamma$. They investigated some properties of a *Q*-fuzzy interior ideal of a semigroup, and also considered *Q*-fuzzy characteristic interior ideals.

Lekkoksung [2012c] verified (1) a Q-fuzzy set μ is a Q-fuzzy right k-ideal in a semiring R if and only if $1 - \mu$ is an anti Q-fuzzy right (left) k-ideal in a semiring *R;* (2) An intuitionistic Q-fuzzy set $A = \{< (x, q), \mu_A(x, q), \lambda_A(x, q) >: x \in R, q \in Q\}$ in R is an intuitionistic Q-fuzzy right k-ideal in R if and only if $U(\mu A; t)$ is a right k-ideal in a semiring R, and $L(\lambda A; t)$ is a right k-ideal in a semiring R for all $t \in R$ whenever nonempty.

They applied the concept of intuitionistic *Q*-fuzzy set to semiring, and introduced the notion of anti *Q*-fuzzy right ideal, anti fuzzy right *k*-ideal and intuitionistic *Q*-fuzzy right *k*-ideal in semiring. Their properties and relations with right *k*-ideal, *Q*-fuzzy right *k*-ideal, anti *Q*-fuzzy right *k*-ideal are investigated.

Ece Yetkin and Necati Olgun [2011] obtained the following contributions: (1) Let (G1, J1) and (G2, J2) be two fuzzy groups and let H1 and H2 be two nonempty subsets of G1 and G2, respectively. Then H1 × H2 is a fuzzy subgroup of G1 × G2 if and only if both H1 and H2 are fuzzy subgroups of G1 and G2; (2) Let f: G1 × G2 → H1 × H2 be a fuzzy group homomorphism. Then **(i).** If A × B is a fuzzy subgroup of G1 × G2, then f (A × B) is a fuzzy subgroup of H1 × H2, (ii)**.** If A × B is a fuzzy subgroup of H1 × H2, then f^{-1} (A × B) is a fuzzy subgroup of G1 × G2; (3) Let f : R1 × R2 → K1 × K2 be a fuzzy ring homomorphism. Then (i) If f is surjective and I1 × I2 is a fuzzy ideal of R1 × R2, then f (I1 × I2) is a fuzzy ideal of K1×K2 and (ii) If I1×I2 is a fuzzy ideal of K1× K2, then f^{-1}(I1×I2) is a fuzzy ideal of R1×R2.

Chandrasekhara Rao and Swaminathan [2010] got (1) the anti homomorphic image and pre-image of a fuzzy groupoid are a fuzzy groupoid.; (2) An anti homomorphic pre-image of a right (left) ideal is a left (right) ideal. (3); Let $f : N \to N'$ be an anti-epimorphism of near-rings. If v is a fuzzy (left/right) ideal in the right (left) near-ring N', then $f^{-1}(v)$ is a fuzzy (left/right) ideal in the left (right) near-ring N.; (4) Let $f : N \to N'$ be an anti-epimorphism of near-rings. If μ is a fuzzy (left/right) ideal in the left (right) near-ring N with sup property, then $f(\mu)$ is a fuzzy (left/right) ideal in the right (left) near-ring N'; (5) Let $f : N \to N'$ be an anti epimorphism. If μ is an anti fuzzy (left/right) ideal of left (right) near-ring N with sup property, $f(\mu)$ is an anti fuzzy (left/right) ideal of right (left) near-ring N'.

Kul Hur et. al. [2011] studied algebraic natures on fuzzy sub-semigroup and fuzzy ideal with operators in a semigroup. Akram and Shum [2012] discussed fuzzy Lie ideal in a fuzzy field.

Ren [1985] discussed fuzzy quotient rings, and relation between fuzzy ideals and their quotient rings. Palaniappan, N., Arjunan, K., and Veeramani, V [2007] investigated algebraic properties in terms of homomorphism, anti-homomorphism, their images and pre-images, level cut-set, and their powers of intuitionistic fuzzy normal subrings.

Yunqiang Yin et. al. [2011] introduced the notion of Intuitionistic fuzzy ideals with thresholds of an ordered semigroups, and studied the homomorphic images and inverse image of Intuitionistic fuzzy ideals with thresholds of an ordered semigroups.

Manemaran and Chellappa [2011] considered the Intuitionistic fuzzification of the concepts of subalgebra and d-ideals in BG-Algebra, and investigated some properties.

Manemaran and Chellappa [2010a] introduced the notion of bipolar-valued fuzzy groups / fuzzy d-ideals of groups under T-norms, and investigated few algebraic properties. They also gave relation between bipolar fuzzy groups and bipolar fuzzy d-ideals.

The following definitions are needed for the discussions in chapter IV:

Definition 1.4.1: An algebra $(R, +, \cdot)$ is said to be a semi ring if it satisfies the following conditions: $(R, +)$ is a semi group; $(R, \cdot)$ is a semi group; $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$, for all $a, b, c \in R$.

Definition 1.4.2: A semi ring (R, +, ·) is a hemi ring if (**1)**: + is commutative and (2): there exists an element $0 \in R$ such that 0 is the identity of (R, +) and 1 is the unit element of (R, ·) with 0.a = a.0 = 0, for all $a \in R$. A subset I of a semi ring R is a left ideal of R if I is closed under addition and $RI \subseteq I$ .A left ideal of R is a left K-ideal of R if $y, z \in I$ and $x \in R$, and x + y = z implies $x \in I$.

A left h-ideal of a hemi ring R is defined a left ideal A of R such that ($x + a + z = b + z \rightarrow x \in A$, for all $x, z \in R$, and $a, b \in A$. Right h-ideals are defined similarly.

Definition 1.4.3: A mapping $f: R_1 \rightarrow R_2$ is a hemi ring homomorphism of R_1 is to R_2 if f (x + y) = f (x) + f (y) and f (xy) = f (x).f (y) for all $x, y \in R$.

Definition 1.4.6: A fuzzy subset is of a semi ring R is a Q-fuzzy left h–ideal of R if (i) $\mu (x + y, q) \geq \min\{ \mu(x, q), \mu(y, q)$ and (ii) $\mu (xy, q) \geq \mu (y, q)$, for all $x, y \in R, q \in Q$. Note that if μ is a Q-fuzzy left h–ideal in a hemi ring R, then $\mu(0, q) \geq \mu(x, q)$ for all $x \in R$.

Definition 1.4.7: A fuzzy subset μ of a hemi ring R is a Q - fuzzy left h – ideal of R if the following conditions hold:

1. $\mu (x + y, q) \geq \min\{ \mu (x, q), \mu (y, q)\}$ for all $x, y \in R$, and $q \in Q$
2. $\mu (xy, q) \geq \mu(y, q)$, for all $x, y \in R, q \in Q$
3. $x + a + z = b + z \rightarrow \mu(x, q) \geq \min\{ \mu (a, q), \mu (b, q)\}$.

The following are obtained on properties of Q-anti fuzzy left h–ideals in chapter IV:

Theorem 1.4.8: Let R be a hemi ring and 'μ' be a Q-fuzzy set in R. Then μ is Q-fuzzy left h-ideal in R if and only if μ^c is a Q-fuzzy left h-ideal in R.

Theorem 1.4.9: Let ‘μ’ be Q – fuzzy ideal in a hemi ring R such that $L(\mu:\alpha)$ is a left h-ideal of R for each $\alpha \in [0, 1]$. Then μ is Q-fuzzy left h-ideal in R.

Corollary 1.4.10: Let μ be Q-fuzzy left h-ideal in R. Then μ is a Q-fuzzy left h-ideal in R if and only if $L(\mu; \alpha)$ is a left h-ideal in R for every $\alpha \in [\mu (0, q),1]$ with $\alpha \in [0, 1]$.

Theorem 1.4.11: Let ‘μ’ be Q – fuzzy set in a hemi ring R. Then two lower level subsets $L(\mu; t_1)$ and $L(\mu; t_2)$,($t_1<t_2$) are equal if and only if there is no $x \in R$ such that $t_1 \leq \mu(x, q) \leq t_2$.

Now few definitions are given for further results:

Definition 1.4.12: A left h – ideal A of hemi ring R is characteristic if and only if f (A) = A for all f ∈ Aut (R), where Aut (R) is the set of all automorphisms on R. A Q-fuzzy left h–ideal μ of hemi ring R is Q-fuzzy characteristic if $\mu^f (x ,q)=\mu(x ,q)$ for all f ∈ Aut (R).

Theorem 1.4.13: Let μ be a Q-fuzzy left h-ideal of a hemi ring R and $x \in R$. Then $\mu (x, q) = s$ if and only if $x \in L(\mu ; s)$ and $x \notin L(\mu ; t)$ for all $s > t$.

Theorem 1.4.14: Let ‘μ’ be a Q-fuzzy left h-ideal of a hemi ring R. Then each level left h-ideal of μ is characteristic if and only if μ is Q-fuzzy characteristic.

The following result is got on homomorphic image of fuzzy left h–ideal of hemi ring:

Theorem 1.4.15: Let f: R1 $\rightarrow$ R2 be an epimorphism of hemiring. If V is a Q-fuzzy left h-ideal of R2 and μ be the pre-image of V under f. Then μ is anti Q-fuzzy left h-ideal of R1.

Definition 1.4.16: Let R_1 and R_2 be two hemi rings and f be a function of R_1 into R_2. If μ is a Q – fuzzy in R_2, then the pre-image of μ under f (defined by $f^{-1}(\mu)(x, q) = \mu(f(x, q))$, for all $x \in R_1$, $q \in Q$) is a Q – fuzzy in R_1.

Theorem 1.4.17: Let f: $R_1 \rightarrow R_2$ be an onto homomorphism of hemi rings. If μ is a Q-fuzzy left h-ideal of R2, then $f^{-1}(\mu)$ is a Q-anti fuzzy left ideal of R1.

Definition 1.4.18: Let R_1 and R_2 be any sets and f: $R_1 \rightarrow R_2$ be any function. A Q-fuzzy subset μ of R_1 is f- invariant if $f(x) = f(y) \Rightarrow \mu(x, q) = \mu(y, q)$), for all x, y ∈ R, and q ∈ Q.

Theorem 1.4.19: Let f: $R_1 \rightarrow R_2$ be an epimorphism of hemi rings. Let μ be f–invariant Q-fuzzy left h-ideal of R_1. Then f (μ) is a Q-fuzzy left h-ideal of R_2.

Definition 1.4.20: A Q- fuzzy left h-ideal μ of a hemi ring R is said to be normal if there exists x ∈R such that μ(x, q) = 1. Note that if μ is a normal Q-anti fuzzy left h-ideal of R_1, then μ (0, q) = 1 and so μ is normal if and only if μ(0, q) = 1 for some q ∈ R.

Theorem 1.4.21: Let μ be a Q – fuzzy left h – ideal of a hemi ring R. Let μ^+ be a Q-fuzzy set in R defined by $\mu^+(x, q) = \mu(x, q) = 1 - \mu(0, q)$ for all x ∈ R. Then μ^+ is a normal Q-fuzzy left h-ideal of R containing μ.

Definition 1.4.22: Let N (R) denote the set of all normal Q-fuzzy left h-ideals of R. Note that N(R) is a poset under the set inclusion. A Q-fuzzy set μ in a hemi ring R is a maximal Q-fuzzy left h-ideal of R if it is non–constant and μ^+ is a maximal element of (N (R), ⊆).

Theorem 1.4.23: Let $\mu \in N(R)$ be non–constant such that it is a maximal element of $(N(R), \subseteq)$. Then it takes only two values $\{0, 1\}$.

Section 1.5: Intuitionistic L-fuzzy sub-algebra of BG-algebra with respect to TS-norm

Introduction: Since then its application have been growing rapidly over many disciplines. As a generalization of this, Intuitionistic fuzzy subset was defined by Atanassov [1986]. Rosenfield [1971] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. Liu [1982] introduced the concepts of fuzzy subrings and fuzzy ideals. Indira et. al. [2013 studied about Q-intuitionistic L-fuzzy sub-semigroup on a semigroup. Further Jiayin Peng [2012] analyzed properites on intuitionistic fuzzy B-algebra. Jun et. al. [1999] investigated algebraic structures on intuitionistic fuzzy ideals of near-ring.

Young Sin Ahn [2005] discussed partial relation in a lattice obtained from intuitionistic fuzzy ideal of a ring. Yuming Feng et. al. [2012] found (L, M) – anti-fuzzy sub-group. There are two classes of abstract algebras; BCK-algebras and BCI-algebra. It is known that the BCK-algebra is proper sub class of the class of BCI-algebra. Neggers and Kim [2002] introduced the notion of B-algebra. Kim and Kim [2005] introduced the notion of BG-algebra, which are the generalization of B-algebra.

Atanassov and Stoeva [1984] constructed intuitionistic L-fuzzy set, and intuitionistic L-fuzzy group. In the sense of homomorphism and isomorphism between two classical groups, image and pre-image of intuitionistic L-fuzzy groups were studied.

Fathi and Salleh [2009] is to the notion of introduced Intuitionistic fuzzy groups based on fuzzy spaces, and discussed a relation between Intuitionistic fuzzy groups and classical Intuitionistic fuzzy subgroups. With these ideas, fuzzy sub-algebra of BG-algebra was developed by Sun Shin Ahn and Hyun Deok Lee [2004].

In this chapter, intuitionistic L-fuzzy sub-algebra of BG-algebra is investigated with respect to TS-norm and established some of their basic properties.

The basic definitions are here given:

Definition 1.5.1: A non-empty set X with a constant 0 and a binary operation *, is said to be a BG-algebra if it satisfies the following:

1. $x * x = 0$; (2). $x * 0 = x$; (3). $(x * y) * (0 * y) = x$, for all $x, y \in X$.

Definition 1.5.2: A binary relation $<$ on X is defined as $x < y$ if and only if $x * y = 0$.

Definition 1.5.3: A non-empty subset S of a BG-algebra X is said to be a sub-algebra if $x * y \in S$ for all $x, y \in S$.

Definition 1.5.5: Let $(L, <)$ be a complete lattice with least element 0 and greatest element 1. A L-fuzzy subset B in non-empty set X is defined as a function $B : X \rightarrow L$.

Example 1.5.6: Let Z be the ring of integers. Define $A: Z \rightarrow L$ by $A(x) = 1$, if $x = 0$; 1/3, if $x \in Z - \{0\}$; 0, if $x \in Z - \{2\}$. Then A is a L-fuzzy subset of Z.

Definition 1.5.7: An intuitionistic fuzzy subset (IFS) A in a non-empty set X is defined as an object of the form A = {< x, $\mu_A(x)$, $\nu_A(x)$ > / x ∈ X} where $\mu_A : X \rightarrow [0,1]$ is the degree of membership and $\nu_A : X \rightarrow [0,1]$ is the degree of non-membership of the element x ∈ X satisfying $0 < \mu_A(x) + \nu_A(x) < 1$.

Definition 1.5.8: Let (L, <) be a complete lattice with least element 0 and greatest element 1 and an involution order reversing operation N : L → L. Then an intuitionistic L-fuzzy subset (ILFS) A in a non-empty set X is defined as an object of the form A = {< x, $\mu_A(x)$, $\nu_A(x)$ > : x ∈ X} where $\mu_A : X \rightarrow L$ is the degree of membership and $\nu_A : X \rightarrow L$ is the degree of non-membership of the element x ∈ X satisfying $\mu_A(x) < N(\nu_A(x))$.

Example 1.5.9: Let L be a complete lattice and Z be the ring of integers. Let A : Z → L be the subset A = {< x, $\mu_A(x)$, $\nu_A(x)$ > : x ∈ X} of X defined by for all x ∈ Z. Then A is an intuitionistic L fuzzy subset of Z.

Definition 1.5.10: [9, 10, 11] Let A and B be two intuitionistic L-fuzzy subsets of a non-empty set X. Then

1. A ⊂ B iff $\mu_A(x) < \mu_B(x)$ and $\nu_A(x) > \nu_B(x)$ for all x Є X.
2. A = B iff A ⊆ B and B ⊆ A.
3. A ∪ B = {< x, $\mu_A(x) \vee \mu_B(x)$, $\nu_A(x) \wedge \nu_B(x)$ >} for all x ∈ X.
4. A ∩ B = {<x, $\mu_A(x) \wedge \mu_B(x)$, $\nu_A(x) \vee \nu_B(x)$ >} for all x ∈ X.

5. $A = \{< x, \nu_A(x), \mu_A(x) >\}$ for all $x \in X$.

6. $*A = \{< x, \mu_A(x), \mu_A(x) >\}$ for all $x \in X$.

7. $\#A = \{< x, \nu A(x), \nu_A(x) >\}$ for all $x \in X$.

Definition 1.5.12: Let $\{A_i : i \in I\}$ be a family of intuitionistic L-fuzzy sets with respect to TS-norm in a non-empty set X. Then min (Ai) = T(x, min μ_Ai (x), min ν_Ai (x)) for all $x \in X$.

Intuitionistic L-fuzzy sub-algebra of BG-algebra

In this section, the notion of intuitionistic L-fuzzy sub-algebra in a BG-algebra X is introduced. Hereafter X denotes a BG-algebra, unless otherwise specified.

Definition 1.5.13: A fuzzy subset μ in a BG-algebra X is said to be a T-fuzzy sub-algebra of X if $\mu(x * y) \geq T\{\mu(x),\mu(y)\}$ for all $x, y \in X$.

Definition 1.5.14: An intuitionistic fuzzy subset A in a BG-algebra X is said to be an intuitionistic fuzzy sub-algebra with respect to TS-norm of X if $\mu(x * y) \geq T\{\mu(x),\mu(y)\}$; (2). $\nu(x * y) \leq S\{\nu(x),\nu(y)\}$ for all $x, y \in X$.

Definition 1.5.15: A L-fuzzy subset A in a BG-algebra X is said to be an L-fuzzy sub-algebra with respect to T-norm of X if $A(x * y) \geq T\{A(x), A(y)\}$ for all $x, y \in X$.

Definition 1.5.16: An intuitionistic L-fuzzy subset A in a BG-algebra X is said to be an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if

1. $\mu_A(x * y) \geq T\{\mu_A(x), \mu_A(y)\}$; (2). $\nu_A(x * y) \leq S\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in X$.

Lemma 1.5.17: In an intuitionistic L-fuzzy sub-algebra with respect to TS-norm A of X,

1. $\mu_A(0) \geq \mu_A(x)$; (2) $\nu_A(0) \leq \nu_A(x)$ for all $x \in X$.

Theorem 1.5.18: Intersection of any two intuitionistic L-fuzzy sub-algebras of X is again an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

Theorem 1.5.19: Intersection of any family of intuitionistic L-fuzzy sub-algebras with respect to TS-norm of X is again an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X.

In the same way and by the definition of A, the theorem (1.5.18) is followed

Theorem 1.5.20: If A is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X, then so is $*A$.

Theorem 1.5.21: An intuitionistic L-fuzzy subset A of X is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if and only if the L-fuzzy subsets μ_A and ν_A are L-fuzzy sub-algebras with respect to TS-norm of X.

Theorem 1.5.22: An intuitionistic L-fuzzy subset A of X is an intuitionistic L-fuzzy sub-algebra with respect to TS-norm of X if and only if the *A and #A are L-fuzzy sub-algebras with respect to TS-norm of X.

Conclusion: The fuzzy sub-algebras of BG-algebras are extended into intuitionistic L-fuzzy sub-algebras with respect to TS-norm of BG-algebras. These concepts can further be generalized.

Bi polar anti Q-fuzzy left R-subgroups of near-rings with respect to T, S-norms

Introduction: Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1].

Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property and its counter property. In a bipolar valued fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0, 1] indicate that elements some what satisfy the property, and the membership degrees on [-1, 0) indicate that elements somewhat satisfy the implicit counter property. In the definition of bipolar-valued fuzzy sets, there are two kinds of representations so called canonical representation and reduced representation.

In this chapter, the canonical representation of bipolar valued Q- fuzzy sets is used. In this paper, The theory of fuzzy sets which was introduced by Zadeh [1965] is applied to many mathematical branches. Abou-Zoid [1991] , introduced the notion of a fuzzy sub near-ring and studied fuzzy ideals of near-ring.

This concept discussed by many researchers among Cho and Jun [2005], Davvaz, Dudek [2005], Kim and Yun [2001], Kim [2002], Pankajam [2009]. Kim and Jun [1999] discussed anti-fuzzy R-subgroup of near-ring. Kim and Jun [2000, 2001] considered the intuitionistic fuzzification of a right (resp left) R- subgroup in a near-ring.

Also Cho.and Jun [2005] the notion of normal intuitionistic fuzzy R- subgroup in a nearing is introduced and related properties are investigated. The notion of intuitionistic Q- fuzzy semi primality in a semi group is given by Kim [2006].

In this chapter, the notion of Q- fuzzification of bipolar left R- subgroups is introduced in a near ring and investigate some related properties. Characterization of bipolar Q- fuzzy left R- subgroups are given.

Preliminaries and basic concepts

Definition 1.5.23: A non empty set with two binary operations '+' and '.' is called a near-ring if it satisfies the following axioms (i) (R ,+) is a group; (ii) (R, .) is a semi group.

(iii) $x . (y + z) = x .y + x . z$ for all $x, y, z \in R$.

Precisely speaking it is a left near-ring because it satisfies the left distributive law.

As R–subgroup of a near- ring 'R' is a subset 'H' of 'R' such that (i) (H , +) is a subgroup of (R, +); (ii) RH $\subset$ H; (iii) HR $\subset$ H. If 'H' satisfies (i) and (ii), then it is called a left R-subgroup of 'R'; if 'H' satisfies (i) and (iii), then it is called a right R-subgroup of 'R'.

A map f : R→ S is called homomorphism if f(x+ y) = f(x) + f (y) and f(xy) = f(x)f (y) for all x, y in R.

Definition 1.5.25: Let 'G' be a non-empty set. A bipolar-valued Q-Fuzzy set A in G is an object having the form. A = {((x, q) μ_A^+(x, q), μ_A^-(x, q)) / x $\in$ G, q $\in$ Q } where μ_A^+: G × Q à [0, 1] and μ_A^+: G X Q $\rightarrow$ [-1,0] are mapping.

The positive membership degree μ_A^+ (x, q) denotes the satisfaction degree of an element x to the property corresponding to 'A' and the negative membership degree μ_A^+ (x, q) denotes the satisfaction degree of x to some implicit counter property of A.

Definition 1.5.26: A fuzzy set 'μ' in R is called fuzzy sub-near ring in a near ring 'R' if (i) $\mu(x - y) \geq \min \{\mu(x), \mu(y)\}$; (ii) $\mu(xy) \geq \min \{\mu(x), \mu(y)\}$ for all x, y in R.

Definition 1.5.27: An anti Q-fuzzy set 'μ' is called a Bi polar Q fuzzy left R- subgroup of R over Q if 'μ' satisfies

(i) $\mu^+(x-y , q) \leq \max \{ \mu^+ (x, q), \mu^+ (y, q) \}$

(ii) $\mu^-(x - y, q) \geq \min \{ \mu^- (x, q) , \mu^- (y, q) \}$

(iii) $\mu^+ (rx, q) \geq \mu^+ (x q)$. (iv) $\mu^- (rx, q) = \mu^- (x q)$ for all x, y $\in$ R, and q $\in$ Q.

Definition 1.5.28: For a bipolar anti Q- fuzzy set 'A' and (β, α) $\in$ [-1.0] x [0,1], define $A_t^+ = \{x \in X / \mu_A^+ (x, q) \leq \alpha\}$ and $A_s^- = \{x \in X / \mu_A^- (x,q) \geq \alpha\}$ which are called the positive t-cut and negative s-cut of A respectively.

Definition 1.5.29: Let λ and μ be two anti Q-fuzzy subsets in X, and S, T are s-norm and T-norm respectively. The cartesian product of $\lambda^+ \times \mu^+ : X \times X \to [0,1]$ is defined by $\lambda^+ \times \mu^+ (x, y) = S\{\lambda^+(x, q), \mu^+(y, q)\}$ and $\lambda^+ \times \mu^+ : X \times X \to [0, 1]$ is defined by $\lambda^- \times \mu^- (x, y) = T\{\lambda^-(x, q), \mu^-(y, q)\}$ for all $x, y \in X$ and $q \in Q$.

Definition 1.5.30: Let $f : X \to Y$ be a mapping of groups and 'μ' be a bipolar anti Q-fuzzy set of Y. The map μ^f is the pre-image of μ_1 and μ_2 under f if $\mu_1^{f+}(x, q) = \mu^{f+}(x, q)$, and $\mu_2^{f--}(x, q) = \mu^{f--}(x, q)$.

In this chapter, the notion of anti Q-fuzzification of bipolar left R- subgroup is introduced in a near-ring and some related properties are investigated. Characterization of anti-Bipolar Q-fuzzy left R-subgroups with respect to S-norm is given.

Properties of anti Q-fuzzy left subgroups

Theorem 1.5.31: Every imaginable Bi polar anti Q-fuzzy left R- subgroup μ^+ of a near ring R is an anti Q-fuzzy left R subgroup of R.

Theorem 1.5.32: If μ^+ is an anti Bipolar Q-fuzzy left R subgroups of a near ring R and 'θ' is an endomorphism of R, then $\mu[\Theta]$ is Bipolar anti Q-fuzzy left R- subgroup of R.

Theorem 1.5.33: An onto homomorphism f of an anti bipolar Q-fuzzy left R- subgroup of near ring R is also anti bipolar Q-fuzzy left R1-subgroup.

Theorem 1.5.34: An onto homomorphic image of a bipolar anti Q-fuzzy left R-subgroup with the sup property is a bipolar anti Q- fuzzy left R'- subgroup.

Theorem 1.5.35: Let T, S be continuous t-norm and s norm respectively. Let f be a homomorphism on a near ring R. If 'μ^{+}' is bipolar anti Q- fuzzy left R- subgroup of R, then μ^{+f} is a bipolar anti Q- fuzzy left R- subgroup of f(R).

Conclusion: Osman kazanci , Sultanyamark and Serifeyilmaz [2007b] introduced the intutionistic Q- fuzzy R-subgroups of near rings. In this chapter, the notion of bipolar Q fuzzy left R- subgroup of near ring with respect to (T, S)-norms is introduced and investigated few characterization properties.

Section 1.6: Structures on anti Q-fuzzy left N-subgroups of near-rings under triangular norms

Introduction: The theory of fuzzy sets which was introduced by Zadeh [1965] is applied to many mathematical branches. Abou-Zoid [1991] introduced the notion of a fuzzy sub near ring and studied Fuzzy ideals of near ring. This concept discussed by many researchers among Cho & Jun [2005], Kim & Jun [2000], Kim & Jun [2001], and Davvaz [2006], Dudek [2005].

Osman Kazari et.al. [2007a, 2007b] considered the intuitionistic Fuzzification of a right crisp left R-subgroup in a near ring. Solairaju and Nagarajan [2009a] introduced a notion of Q-fuzzy group. Also Cho. and Jun [2005] introduced the notion of normal intuitionistic fuzzy R-sub group in a near ring is introduced and related are investigated. The notion of intuitionistic Q-fuzzy semi primality in a semi group is given by Kim [2006].

Manemaran and Chellappa [2010b] introduced the notion of Q-fuzzification left M-N-subgroups in a near ring and investigated their algebraic properties.

Akram and Dudek [2008] described the structure of intuitionistic fuzzy left k-ideals of semirings. Li Hongxing [1987] introduced the notion of HX group.

In this chapter, the notion of Q-fuzzification of left N-Subgroups is introduced in a near ring and some related properties are investigated. Characterization of Q-fuzzy left N-subgroup is given.

Basic concepts and definition

As N-Subgroup of a near ring R' is a subset 'H' of R such that (i) (H, +) is a subgroup of (R, +); (ii) RH $\subset$ H; (iii) HR $\subset$ H. If 'H' satisfies (i) and (ii) then it is called left N Subgroup of R and if 'N' satisfies (i) and (iii) then it is called right N-subgroup of R. A map f : R$\rightarrow$ R1 is called homomorphism if f (x + y) = f(x) + f(y) for all x, y in R.

Definition 1.6.4: Let R be a near ring. A fuzzy set I in R is called anti Q-fuzzy sub near ring in R if (i) μ (x-y, a) $\leq$ max { μ(x,a), μ (y,a) }l; (ii) μ(xy , a) $\leq$ max { μ(x,a), μ (y,a)} for all x, y in R, and for all a in Q

Definition 1.6.5 : (T-norm) A triangular norm is a function T : [0,1] $\times$ [0,1] $\rightarrow$ [0,1] that satisfies the following conditions for all x, y, z in [0,1].

(T1) T (x,1) = x; (T2) T (x,y) = T (y,x); (T3) T [x, T (y,z)] = T (T(x,y) , z)

(T4) T (x, y) $\leq$ T (x,z) when y $\leq$ z.

Definition 1.6.6: A Q-fuzzy set 'μ' is called a Q-fuzzy left N-subgroup of R over Q if μ obeys (i) μ (n(x-y, q) $\leq$ S{μ(x, q), μ(y, q)}; (ii) μ (nx, q) $\leq${ μ(x, q)} for all x, y, n $\in$ R and q $\in$ Q.

Aim: In this chapter, the notion of anti Q-fuzzification of left N-subgroups is introduced in a near ring and some related properties are investigated. Characterization of anti Q-fuzzy left N-subgroup with respect to a triangular norm is given.

Properties of Q-fuzzy left N-subgroup

The theorems mentioned below are proved in this section:

Theorem 1.6.7: Let S be an s-norm. Then every imaginable anti Q-fuzzy left N-subgroup 'μ' of a near ring R is an anti Q-fuzzy left N-subgroup of R.

Theorem 1.6.8: If 'μ' is anti Q-fuzzy left N-subgroups of a near ring R and f is a endomorphism of R, then $\mu_{(f)}$ is a Q-fuzzy left N-subgroup of R.

Theorem 1.6.9: An onto homomorphism of an anti Q-fuzzy left N-subgroup of near ring R is anti Q-fuzzy left N-subgroup.

Theorem 1.6.10: An onto homomorphic image of an anti Q-fuzzy left N-subgroup with the supremum property is anti Q-fuzzy left N-subgroup.

Theorem 1.6.11: Let S be a continuous s-norm and f be a homomorphism on a near ring R. If μ is anti Q-fuzzy left N-subgroup of R, then μ^f is an anti Q-fuzzy left N-subgroup of f(R).

Theorem 1.6.12: Let 'μ' be an anti Q-fuzzy left N-subgroup of R. Then anti Q-fuzzy subset < μ > is anti Q-fuzzy left N-subgroup of R generated by ' μ' moreover < μ > is the smallest anti Q-fuzzy left N-subgroup containing it.

Theorem 1.6.13: Let U be an anti Q-fuzzy left N-subgroup near ring R and let μ^+ be an anti Q-fuzzy set in N defined by $\mu^+ (x, a) = \mu (x, a) + 1 - \mu(0, q)$ for $x \in N$. Then μ^+ is a normal anti Q-Fuzzy left N-subgroup of S containing 'μ'.

Conclusion: Osman Kozanci et. al. [2007] introduced the intuitionistic Q-Fuzzy R-Subgroups of near rings. In this chapter, the notion of Q-fuzzy left N-subgroup of near ring is introduced with respect to t-norm, and few characterization properties are discussed.

Chapter 1.7: Intuitionistic fuzzy T-ideals of TM-algebra, and closed ideal in BCI-algebra

Introduction: Babushri Srinivas Kedukodi et. al. [2009] presented the notions of equiprime fuzzy ideal, 3-prime fuzzy ideal and c-prime fuzzy ideal of a near-ring. They characterized these fuzzy ideals using level subsets and fuzzy points.

If $f: N \rightarrow M$ is an onto near-ring homomorphism, they showed that the map defines a one-to-one correspondence between the set of all f-invariant (alternatively with sup property) equiprime (3-prime and c-prime, respectively) fuzzy ideals of N and the set of all equiprime (3-prime and c-prime, respectively) fuzzy ideals of M. Finally, they defined fuzzy cosets determined by generalized fuzzy ideals; and obtained fundamental results and isomorphism theorems.

Azam et. al. [2011] defined an anti fuzzy ideal and lower level ideals of a ring X. The fuzzification of lower level subset of fuzzy set is referred and some properties are proved.

Srinivas, and Nagaiah [2012a, 2012b] introduced the notion of fuzzy ideals of a Γ-near-ring with respect to a t-norm, and investigated some related properties. This concept of *T*-fuzzy ideals of a Γ-near-ring is a generalization of the concept of *T*-fuzzy ideals in near-rings. Also the notions of *T*-fuzzy ideals of a Γ-near-ring, quotient Γ-near-ring with respect to a t-norm and the sum of *T*-fuzzy ideals of a Γ-near-ring are introduced. Further, it is shown that an onto homomorphic image of a *T*-fuzzy ideal with Sup property is a *T*-fuzzy ideal and an epimorphic pre-image of a *T*-fuzzy ideal of a Γ-near-ring is a *T*-fuzzy ideal.

After the introduction of the concept of fuzzy sets by Zadeh [1965] several researches were conducted on the generalization of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1986], as a generalization of the notion of the fuzzy set. In this work, using the Atanassov's idea[1986], he was established the intuitionistic fuzzification of the concept of subalgebras and T-ideals in TM-algebras, and investigated some of their properties.

Nagaiah [2011] got (1) let the pair of mapping f: m $\rightarrow$ m1, h: $\gamma \rightarrow \gamma$1 be homomorphism of γ - semigroups. If a is an anti fuzzy bi- γ - ideal of m1, then the preimage f^{-1} (a) of a under f is an anti fuzzy bi - γ - ideal of m;   (2) Let a be an anti fuzzy bi - γ - ideal of a γ - semigroup m and f: [0, a(0)] $\rightarrow$ [0, 1] be an increasing function. Then the fuzzy set af : m $\rightarrow$ [0, 1] defined by af (x)= f (a(x)) is an anti-fuzzy bi - γ - ideal of m. In particular, if f [a (0)] = 1, then af is normal; if f (t) $\geq$ t for all t $\in$ [0, a (0)], then A $\subseteq$ af. The notion of an anti-fuzzy bi- γ - ideals in γ - semigroups is introduced, and investigated some of their related properties. Further complete and Normal anti fuzzy ideals in γ - semigroups are investigated.

Tariq Shah & Muhammad Saeed [2013] proved (1) If every primary fuzzy ideal of a ring R is a strongly irreducible Fuzzy ideal, then every fuzzy minimal primary decomposition for each fuzzy Ideal of R is unique; (2) Let D be a laskerian domain of dimension 1. Then every non-zero fuzzy ideal of D can be uniquely expressed as a product of primary fuzzy ideals with distinct radicals; (3) In a ring R, the following are equivalent. (1) every fuzzy ideal of R is a strongly irreducible fuzzy ideal. (2) any two fuzzy ideals of R are comparable. They introduced strongly primary fuzzy ideals and strongly irreducible fuzzy ideals In a unitary commutative ring and fixed their role in a laskerian ring.

They established that: a finite intersection of prime fuzzy ideals (resp. primary fuzzy ideals, irreducible fuzzy ideals and strongly irreducible fuzzy ideals) is a prime fuzzy ideal (resp. primary fuzzy ideal, irreducible Fuzzy ideal and strongly irreducible fuzzy ideal). They also found that, a fuzzy ideal of a ring is Prime if and only if it is semi-prime and strongly irreduci ble. Furthermore they characterized that: (1) every nonzero fuzzy ideal of a one dimensional laskerian domain can be uniquely expressed as a product of primary fuzzy ideals with distinct radicals, (2) a unitary commutative ring is (strongly) laskerian if and only if its localization is (strongly) laskerian with respect to every fuzzy ideal.

The following definitions are now stated:

Definition 1.7.1: A TM-algebra (X, $*$, 0) is a non-empty set X with a constant “0” and a binary operation “*” satisfying the following axioms:

(i). $x * 0 = x$; (ii) $(x * y) * (x * z) = z * y$, for all $x, y, z \in X$.

In X, a binary relation $\leq$ is defined by $x \leq y$ if and only if $x * y = 0$.

Definition 1.7.2: Let (X, $*$, 0) be a TM-algebra. A non-empty set I of X is called an ideal of X if it satisfies 0 $\in$ I; (ii) x $*$ y $\in$ I and y $\in$ I imply x $\in$ I, for all x, y $\in$ X. An ideal A of a TM-algebra X is closed if 0 $*$ x $\in$ A, for all x $\in$ A.

Definition 1.7.3: A fuzzy set μ in a TM-algebra X is called a fuzzy sub-algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all x, y $\in$ X.

Definition 1.7.4: A fuzzy set μ in a TM-algebra X is called a fuzzy ideal of X if

(i). $\mu(0) \geq \mu(y)$; (ii) $\mu(x) \geq \min\{\mu(x * \text{y}), \mu(y))\}$ f or all x, y $\in$ X.

Definition 1.7.5: An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(X, \mu_A(x), \lambda_A(x)) | x \in X\}$ where the function $\mu_A: X \to [0,1]$ and $\lambda_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely,$\lambda_A(x)$) of each element x $\in$ X to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all x $\in$ X. For the sake of simplicity, the symbol $A = (X, \mu_A, \lambda_A)$ is used for the intuitionistic fuzzy set $A = \{(X, \mu_A(x), \lambda_A(x)/x \in X\}$.

Definition 1.7.6: Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy set in X. Then $\Omega A = (X, \mu_A, \mu_A^c)$ and $\Psi A = (X, \lambda_A^c, \lambda_A)$

Definition 1.7.7: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ is called an intuitionistic fuzzy sub-algebra of X if it satisfies

(i). $\mu_A(x^* y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii). $\lambda_A(x^* y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, for all x, y $\in$ X.

Theorem 1.7.8: Every intuitionistic fuzzy sub-algebra $A = (X, \mu_A, \lambda_A)$ of X satisfies the inequalities $\mu_A(0) \geq \mu_A(x)$ *and* $\lambda_A(0) \leq \lambda_A(x)$

Aim: The intuitionistic fuzzification of the concept of sub-algebras and T-ideals is considered in TM-algebra, and some of their properties are investigated.

Properties on intuitionistic fuzzy T-ideal

Definition 1.7.9: A non-empty set I of a TM-algebra X is called T-ideal of X, if

(i). $0 \in I$; (ii) $(x * y) * z \in I$ and $y \in I$ imply that $x * z \in I$, for all x, y, z $\in$ X.

Definition 1.7.10: A fuzzy subset μ in a TM-algebra X is called a fuzzy T-ideal of X, if

(i). $\mu(0) \geq \mu(x)$; (ii) $\mu(x * z) \geq \min\{\mu(x * y) * z)), \mu(y)\}$ for all x, y, z $\in$ X.

Definition 1.7.11: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in a TM-algebra X is called an intuitionistic fuzzy T-ideal of X if,

(i). $\mu_A(0) \geq \mu_A(x)$ *and* $\lambda_A(0) \leq \lambda_A(x)$

(ii). $\mu_A(x * z) \geq \min\{\mu_A((x * y) * z)), \mu_A(y))\}$

(iii). $\lambda_A(x^* z) \leq \max\{\lambda_A(x * y) * z)), \lambda_A(y)\}$, for all x, y, z $\in$ X.

Definition 1.7.12: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in a TM-algebra X is called an intuitionistic fuzzy closed T-ideal of X, if it satisfies

(i). $\mu_A(0 * z) \geq \mu_A(x)\ and\ \lambda_A(0 * x) \leq \lambda_A(x)$

(ii). $\mu_A(x^* z) \geq \min\{\mu_A((x^* y)^* z), \mu_A(y)\}$

(iii). $\lambda_A(x^* z) \leq \max\{\lambda_A((x^* y)^* z), \lambda_A(y)\}$, for all x, y, z ∈ X.

Definition 1.7.13: Let $A = (X, \mu_A, \lambda_A)$ can be an intuitionistic fuzzy set in a TM-algebra X. The set $\cup\ (\mu_A; s) = \{x \in X/\mu_A(x) \geq s\}$ is called upper s-level of μ_A and the set $L(\lambda_A; t) = \{x \in X/\lambda_A(x) \geq t\}$ is called lower t-level of λ_A.

Now the following theorems are obtained:

Lemma 1.7.14: Let $A = (\mu_A, \lambda_A)$ in X be an intuitionistic fuzzy T-ideal of X. If $x^* y \leq z$ then: $\mu_A(x^* z) \geq \min\{\mu_A(y), \mu_A(z)\}$ and $\lambda_A(x^* z) \leq \max\{\lambda_A(y), \lambda_A(z)\}$.

Lemma 1.7.15: Let $A = (\mu_A, \lambda_A)$ in X be an intuitionistic fuzzy ideal of X. If $x \leq y$ then $\mu_A(x^* z) \geq \mu_A(y), \lambda_A(x^* z) \leq \lambda_A(y)$ (μ_A is order-reserving and λ_A is order-preserving).

Definition 1.7.16: A mapping $f: X \to Y$of TM-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all x, y ∈ X. Note that if $f: X \to Y$ is a homomorphism of TM-algebras, then $f(0) = 0$. Let $f: X \to Y$ be a homomorphism of TM-algebras for any IFS $A = (\mu_A, \lambda_A)$ in Y, define a new IFS $A^f = (\mu_A^f, \lambda_A^f)$ in X by:

$$\mu_A^f(x^* z) = \mu_A\left(f(x^* z)\right); \quad \lambda_A^f(x^* z) = \lambda_A\left(f(x^* z)\right) for\ all\ x \in X.$$

Theorem 1.7.17: Let $f: X \to Y$ be a homomorphism of BG-algebras. If an IFS $A = (\mu_A, \lambda_A)$, is an intuitionistic fuzzy ideal of Y. So IFS $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of X.

Theorem 1.7.18: Let $f: X \to Y$ be an epimorphism of BG-algebra and $A = (\mu_A, \lambda_A)$, be and IFS in X. Then f $(A) = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of Y.

Anti-fuzzy ideals in any BCI-algebra

Theorem 1.7.19: Union of two anti-fuzzy ideals is again an anti fuzzy ideal.

Theorem 1.7.20: If A is an anti-fuzzy ideal, then its complement is a fuzzy ideal.

Theorem 1.7.21: Every ideal of a BCK algebra X can be realized as a level ideal of a anti fuzzy ideal of X.

Theorem 1.7.22: If A is an anti fuzzy ideal, then A^m is an anti fuzzy ideal.

Theorem 1.7.23: An onto homomorphic image of an anti-fuzzy ideal with sup property is an anti fuzzy ideal.

Theorem 1.7.24: An onto homomorphic pre image of an anti-fuzzy ideal is an anti-fuzzy ideal.

Properties on anti-fuzzy prime ideals:

Definition 1.7.25: μ is anti-fuzzy prime ideal in BCI – algebra if

$\mu(x \wedge y) \geq \min \{ \mu(x) , \mu(y) \}$ and $\mu(y * (y * x)) \geq \min \{ \mu(x) , \mu(y) \}$ for every $x, y \in G$.

Theorem 1.7.26: If one is contained in another, then $A \cup B$ is an anti fuzzy prime ideal.

Theorem 1.7.27: If A and B are anti-fuzzy prime ideals, then $A \cap B$ is anti fuzzy prime ideals.

Theorem 1.7.28: If A is an anti-fuzzy prime ideal, then A^m is an anti fuzzy prime ideal

Theorem 1.7.29: Direct product of two anti-fuzzy prime ideals is again an anti-fuzzy prime ideal.

Theorem 1.7.30: An onto homomorphic image of an anti-fuzzy prime ideal with sup property is an anti-fuzzy prime ideal.

Theorem 1.7.31: Let f be a homomorphism of a BCK algebra G_1 into G_2 and B be an anti-fuzzy prime ideal of G_2 then $f^{-1}(B)$ is an anti-fuzzy prime ideal of G_1.

Theorem 1.7.32: If A and its complement is an anti-fuzzy prime, then A is a constant function.

Closed ideals in a BCI algebra

Definition 1.7.33: An ideal A in a BCI algebra X is closed if $0 * x \in A \Rightarrow x \in A$ for all $x \in X$.

Theorem 1.7.34: Union of two closed ideals is closed if one is contained in the other.

Theorem 1.7.35: Intersection of two closed ideals is closed.

Theorem 1.7.36: Union of two fuzzy closed ideals is closed.

Theorem 1.7.37: Intersection of two fuzzy closed ideals is closed.

Theorem 1.7.38: If A is a fuzzy closed ideal, then A^m is fuzzy closed ideal.

Theorem 1.7.39: An onto homomorphic image of a closed fuzzy ideal with sup-property is an closed fuzzy ideal .

Theorem 1.7.40: An onto homomorphic pre-image of a closed fuzzy ideal is closed.