CHAPTTER VII

INTUTIONISTIC FUZZY T-IDEAL OF TM-ALGEBRA

Introduction: .Jun [2004] introduced the concept on fuzzy h-ideal in hemi ring and investigated the idea of anti-fuzzy left h-ideal in hemi ring. Some structure properties of Q-fuzzy left h-ideal is established in a hemi ring. Similar results in the intuitionistic anti fuzzy ideal in a hemi ring are obtained.

Consequently, Molodtsov [1999] proposed a completely new approach for modeling vagueness and uncertainty. This is so-called soft theory that is free from the difficulties affecting existing methods. In soft set theory, the problem of setting the membership function among other related problems simply does not arise. This makes the theory very convenient and easy to apply in practice soft set theory that has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Person integration, Probability theory and Measurement theory. Most of these applications have already been demonstrated in Molodtsov's book [1999].

Section 7.1 – Previous works:

At present, work on the soft set theory is progressing rapidly. Maji et al [2002a,2002b, 2002c] described the application of soft set theory to a decision making problem using rough sets. The same authors have also published a detailed theoretical study on soft sets. The algebraic structure of set theories dealing with uncertainties has also been studied by some authors. Our definition of soft groups is similar to the definition of rough groups, but constructed using different methods.

This chapter begins by introducing the basic concepts of fuzzy soft set theory, and then a basic version of fuzzy soft group theory is discussed, and it is extended the notion of a group to the algebraic structures of fuzzy soft sets. A fuzzy soft group is a parameterized family of fuzzy subgroups.

Since then a number of research works has been done in the area of fuzzy algebra, Gau and Bueher [1993] have initiated the study of vague sets as an improvement over the theory of fuzzy sets to interpret and solve real life problems which are in general vague. Biswas [2006] defined the notion of vague groups analogous to the idea of Rosenfeld [1971].

Solairaju and Nagarajan [2009a] fuzzified a new class of algebraic structures. In this fuzzification, they introduced the notion of Q-fuzzy groups (QFG) and investigated some of their related properties. The purpose of this study was to implement the fuzzy set theory and group theory in Q- fuzzy groups. This fuzzification leads to development of new notions over fuzzy groups. Characterization of Q-fuzzy groups (QFCG) and normal Q- fuzzy groups (QFNG) were given.

Demirci [1999] studied the definitions like vague groups (VG), Vague normal groups (VNG) and made some characterizations of them. .The notion of Vague theory explained is of interest to us. In most cases of judgments, evaluation is done by human beings (or by an intelligent agent) where there certainly is a limitation of knowledge or intellectual functionaries. Naturally, every decision-maker hesitates more or less, on every evaluation activity.

To judge whether a patient has cancer or not, a doctor (the decision-maker) will hesitate because of the fact that a fraction of evaluation he thinks in favor of truthiness', another fraction in favor of falseness and rest part remains undecided to him. This is the breaking philosophy in the notion of vague set theory. The notion of vague algebra is defined by vague groups of a group, vague normal groups.

It has initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems which were in general vague, than the theory of fuzzy sets do. The objective of the study of vague groups was by introducing concepts of vague normalizer, vague centralizer and vague homologous group. Certain theorems were on vague normal groups by imposing finiteness condition which was removed. If A, B are homologous vague groups of a group G, then centralizers C(A); C(B) ,and normalizers N(A); N(B) are also homologous subgroups of G.

Gau and Buechrer [1993] analyzed that both rough sets theory and vague sets theory were emerging as powerful tool for managing uncertainty, incomplete and imprecise information. An integration between vague sets theory and rough sets theory is developed. They constructed the lower and upper approximation operators of a vague set in the universe, which is partitioned by an indiscernibility relation. Furthermore, a parameterized roughness measure of a vague set is given and followed by an analysis of its properties.

There were a number of generalizations of Zadeh's fuzzy set theory so far reported in the literature viz., *i-v* fuzzy theory, two-fold fuzzy theory, intuitionistic fuzzy theory, *L*-fuzzy theory, etc. to list a few. Vague sets and vague groups were got. Sup-property of vague sets and some characterizations of vague groups were done. A result on classical groups is proved with the help of vague group theory.

The notion of Q-fuzzy groups was introduced by Solairaju and Nagarajan [2009a]. Their objective was to contribute further to the study Q-Vague groups and introducing concepts of Q-Vague normalize, Q-Vague centralizer and Q-Vague homologous group by imposing fitness condition that can be removed, and characterized the Q-Vague normal groups. Homologous Q-Vague group were characterized, and admitted a particular type of Q-fuzzy groups.

After the introduction of the concept of fuzzy sets by Zadeh [1965] several researches were conducted on the generalizations of the notion of fuzzy sets. The idea of "Intuitionistic fuzzy set" was first published by Atanassov [1986], as a generalization of the notion of the fuzzy set.

In this chapter, the intuitionistic fuzzification of the concept of sub-algebra and T-ideal are established in TM-algebra using the Atanassov's idea [1986], and investigated some of their properties.

Section 7.2: Preliminaries and definitions

Definition 7.2.1: A TM-algebra (X, $*$, 0) is a non-empty set X with a constant "0" and a binary operation " $*$ " satisfying the following axioms:

(i). $x * 0 = x$; (ii) $(x * y) * (x * z) = z * y$, for all x, y, z $\in$ X.

In X a binary relation $\leq$ is defined by $x \leq y$ if and only if $x * y = 0$.

Definition 7.2.2: Let (X, $*$, 0) be a TM-algebra. A non-empty set I of X is called an ideal of X if it satisfies $0 \in I$; (ii) $x * y \in I$ and $y \in I$ imply $x \in I$, for all x, y $\in$ X.

Definition 7.2.3: An ideal A of a TM-algebra X is closed if $0 * x \in A$ for all $x \in A$.

Definition 7.2.4: The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all x $\in$ X.

Definition 7.2.5: A fuzzy set μ in a TM-algebra X is called a fuzzy sub-algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all x, y $\in$ X.

Definition 7.2.6: A fuzzy set μ in a TM-algebra X is called a fuzzy ideal of X if

(i). $\mu(0) \geq \mu(y)$

(ii). $\mu(x) \geq \min\{\mu(x * y), \mu(y))\}$ for all x, y $\in$ X.

Definition 7.2.7: An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(X, \mu_A(x), \lambda_A(x)) : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely,$\lambda_A(x)$) of each element x ∈ X to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all x ∈ X. For the sake of simplicity, we use the symbol $A = (X, \mu_A, \lambda_A)$ for the intuitionistic fuzzy set $A = \{(X, \mu_A(x), \lambda_A(x)/x \in X\}$

Definition 7.2.8: Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy set in X. Then

(i). $\Omega A = (X, \mu_A, \mu_A^c)$ and (ii) $\Psi A = (X, \lambda_A^c, \lambda_A)$

Definition 7.2.9: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ is called an intuitionistic fuzzy subalgebra of X if it satisfies

(i). $\mu_A(x * y) \geq \min \{\mu_A(x),\ \mu_A(y)\}$

(ii). $\lambda_A(x * y) \leq \max \{\lambda_A(x),\ \lambda_A(y)\}$, for all x, y ∈ X.

Theorem 7.2.10: Every intuitionistic fuzzy sub-algebra $A = (X, \mu_A, \lambda_A)$ of X satisfies the inequalities $\mu_A(0) \geq \mu_A(x)\ and\ \lambda_A(0) \leq \lambda_A(x)$

Proof: $\mu_A(0) = \mu_A(x * x) \geq \min \{\mu_A(x),\ \mu_A(x)\}$ = $\mu_A(x)$ and

$\lambda_A(0) = \lambda_A(x * x) \leq \max \{\lambda_A(x), \lambda_A(x\} = \lambda_A(x)$ for all x, y ∈ X.

Section 7.3: Properties on Intuitionistic fuzzy T-ideal

Definition 7.3.1: A non-empty set I of a TM-algebra X is called T-ideal of X, if

(i). $0 \in I$; (ii) $(x * y) * z \in I$ and $y \in I$ imply that $x * z \in I$, for all $x, y, z \in X$.

Definition 7.3.2: A fuzzy subset μ in a TM-algebra X is called a fuzzy T-ideal of X, if

(i). $\mu(0) \geq \mu(x)$; (ii) $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\}$ for all $x, y, z \in X$.

Definition 7.3.3: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in a TM-algebra X is called an intuitionistic fuzzy T-ideal of X if,

(i). $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$

(ii). $\mu_A(x^* z) \geq \min\{\mu_A(((x^* y)^* z), \mu_A(y))\}$

(iii). $\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z), \lambda_A(y)\}$, for all $x, y, z \in X$.

Definition 7.3.4: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in a TM-algebra X is called an intuitionistic fuzzy closed T-ideal of X, if it satisfies

(i). $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$

(ii). $\mu_A(x * z) \geq \min\{\mu_A((x * y) * z), \mu_A(y)\}$

(iii). $\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z)), \lambda_A(y)\}$, for all $x, y, z \in X$.

Definition 7.3.5: Let $A = (X, \mu_A, \lambda_A)$ can be an intuitionistic fuzzy set in a TM-algebra X. The set $\cup(\mu_A; s) = \{x \in X: \mu_A(x) \geq s\}$ is called upper s-level of μ_A and the set $L(\lambda_A; t) = \{x \in X: \lambda_A(x) \leq t\}$ is called lower t-level of λ_A

Now the following lemmas and theorems are obtained:

Lemma 7.3.6: Let $A = (\mu_A, \lambda_A)$ in X be an intuitionistic fuzzy T-ideal of X. If $x * y \leq z$ then: $\mu_A(x * z) \geq \min\{\mu_A(y), \mu_A(z)\}$ and $\lambda_A(x * z) \leq \max\{\lambda_A(y), \lambda_A(z)\}$

Proof: Let x, y, z ∈ X such that$x * y \leq z$. Then $(x * y) * z = 0$, and thus

$$\mu_A(x * z) \geq \min\{\min\{\mu_A((x * y) * z), \mu_A(z)\}, \mu_A(y)\}$$

$$= \min\{\min\{\mu_A(0), \mu_A(z)\}, \mu_A(y)\}$$

$$= min\{\mu_A(y), \mu_A(z)\}$$

and similarly $\lambda_A(x^* z) \leq \max\{\lambda_A(y), \lambda_A(z)\}$.

Lemma 7.3.7: Let $A = (\mu_A, \lambda_A)$ in X be an intuitionistic fuzzy ideal of X. If $x \leq y$ then: $\mu_A(x * z) \geq \mu_A(y), \lambda_A(x * z) \leq \lambda_A(y)$ that is, μ_A is order-reserving and λ_A is order-preserving.

Proof: Let x, y, z ∈ X be such that (x ∗ y) ∗ z = 0 and so

$$\mu_A(x * z) \geq \min\{\mu_A((x * y) * z), \mu_A(y)\}$$

$$= \min\{\mu_A(0), \mu_A(y)\}$$

$$= \mu_A(y)$$

and

$$\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z), \lambda_A(y)\}$$

$$= \max\{\lambda_A(0), \lambda_A(y)\}$$

$$= \lambda_A(y)$$

Definition 7.3.8: A mapping $f: X \to Y$ on TM-algebras is called a homomorphism if $f(\mathrm{x} * y) = f(x) * f(y)$ for all x, y ∈ X. Note that if $f: X \to Y$ is a homomorphism of TM-algebras, then $f(0) = 0$.

Let $f: X \to Y$ be a homomorphism of TM-algebras for any IFS $A = (\mu_A, \lambda_A)$ in Y, define a new IFS $A^f = (\mu_A^f, \lambda_A^f)$ in X by:

$$\mu_A^f(\mathrm{x} * z) = \mu_A\, f(\mathrm{x} * y);\ \ \lambda_A\ (\mathrm{x} * y) = \lambda_A\left(f(x^* z)\right) for\ all\ \ x \in X.$$

Theorem 7.3.9: Let $f: X \to Y$ be a homomorphism on BG-algebras. If an IFS $A = (\mu_A, \lambda_A)$, is an intuitionistic fuzzy ideal of Y, then an IFS $A^f = (\mu_A^f, \lambda_A^f)$ in X is an intuitionistic fuzzy ideal of X.

Proof: It follows that

$$\mu_A^f(\mathrm{x} * z) = \mu_A\left(f(\mathrm{x} * z)\right)$$

$$\leq \mu_A(0) = \mu_A\big(f(0)\big) = \mu_A^f(0)$$

$$\lambda_A^f(\mathrm{x} * z) = \lambda_A\left(f(\mathrm{x} * z)\right)$$

$$\leq \lambda_A(0) = \lambda_A\big(f(0)\big) = \lambda_A^f(0) \text{ for all x, z} \in \text{X}.$$

Let x, y, z ∈ X.

Then $\min\left\{\mu_A^f\left((\mathrm{x} * y) * z),\ \mu_A^f(y)\right)\right\}$

$$= \min\{\mu_A(f(\mathrm{x} * y) * z),\ \mu_A^f(y)\}$$

$$= \min\{\mu_A(\mathrm{f(x)} * \mathrm{f}(y)) * f(z),\ \mu_A\, f(y)\}$$

$$\leq\ \mu_A\big(f(x)\big) = \mu_A^f(x)$$

and

$\max\left\{\lambda_A^f\left((x * y) * z),\ \lambda_A^f(y)\right)\right\}$

$= \max\{\lambda_A f((x * y) * z)),\ \lambda_A\, f(y)\}$

$= \max\{\lambda_A(f(x) * f(y)) * f(z)), \lambda_A\, f(y)\}$

$\leq\ \lambda_A\big(f(x)\big) = \lambda_A^f(x)$

Hence $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of X.

Theorem 7.3.10: Let $f: X \to Y$ be an epimorphism of BG-algebra and $A = (\mu_A, \lambda_A)$, be and IFS in Y. Then f$(A) = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of Y.

Proof: For any x $\in$ X there exist $a \in X$ such that $f(a) = x * z$. Then

$$\mu_A(x * z) = \mu_A(f(a)) = \mu_A^f(0) \leq \mu_A^f(0) = \mu_A\left(f(0)\right) = \mu_A(0)$$

$$\lambda_A(x * z) = \lambda_A\big(f(a)\big) = \lambda_A^f(a) \geq \lambda_A^f(0) = \lambda_A\left(f(0)\right) = \lambda_A(0)$$

Let x, y, z $\in$ X. Then $f(a) = x$, $f(b) = y$ and $f(c) = z$ for some $a, b, c \in X$. Thus

$\mu_A(x * z) = \mu_A\big(f(a)\big) = \mu_A\left(f(a)\right)$

$\geq \min\{\mu_A^f((a * b) * c, \mu_A(f(b))\ \}$

$= \min\{\mu_A(f\ ((a * b) * c), \mu_A(f(b))\}$

$= \min\{\mu_A((f(a) * f(b)) * f(c),\ \mu_A(f(b))\}$

$= \min\{\mu_A\big((x * y)\big),\ \mu_A(y)\}$

and

$$\lambda_A\big((x*z)\big) = \lambda_A\big(f(a)\big) = \lambda_A\,\big(f(a)\big)$$

$$\leq \max\{\lambda_A^f\big((a*b)*c\big), \lambda_A(f(b))\}$$

$$= \max\{\lambda_A(f\big((a*b)*c\big)), \lambda_A(f(b))\}$$

$$= \max\{\lambda_A(\big(f(a)*f(b)\big)*f(c)), \lambda_A(f(b))\}$$

$$= \min\{\lambda_A\big((x*y)\big), \lambda_A(y)\}.$$

Section 7.4: Anti fuzzy ideals in any BCI-Algebra

Theorem 7.4.1: Union of two anti fuzzy ideals is again an anti fuzzy ideal.

Proof: Let A and B be two anti fuzzy ideals.

$$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

$$\leq \max\{\max[\mu_A(x*y), \mu_A(y)], \max[\mu_B(x*y), \mu_B(y)]\}$$

$$\leq \max\{\max[\mu_A(x*y), \mu_B(x*y)], \max[\mu_A(y), \mu_B(y)]\}$$

$$\leq \max\{\mu_{A\cup B}(x*y), \mu_{A\cup B}(y)\} \text{ for every } x, y \in X.$$

Theorem 7.4.2: If A is an anti-fuzzy ideal then its compliment is a fuzzy ideal.

Proof: A is an anti fuzzy ideal. It follows that

$$\mu_{\mathbf{A}}(x) \leq \max\{\mu_{\mathbf{A}}(x*y), \mu_{\mathbf{A}}(y)\}$$

$$1-\mu_{\mathbf{A}}(x) \geq \mathbf{1-}\max\{\mu_{\mathbf{A}}(x*y), \mu_{\mathbf{A}}(y)\}$$

$= \min\{1 - \mu_A(x * y), 1 - \mu_A(y)\}$

$\mu_{A^I}(x) \geq \min\{\mu_{A^I}(x * y), \mu_{A^I}(y)\}$ for every $x, y \in X$.

Theorem 7.4.3: Every ideal of a BCK algebra X can be realized as a level ideal of a anti fuzzy ideal of X.

Proof: Let A be an ideal of a BCK algebra X and B be a fuzzy set with membership function μ defined by

$\mu(x) = \{ \alpha$ if $x \in A$

0 Otherwise where $\alpha \in (0,1]$ is fixed .

If $(x * y) \in A$ and $y \in A$ then $x \in A$

Then $\mu(x * y) = \alpha$, $\mu(y) = \alpha$. and $\mu(x) = \alpha$.

$\mu(x) = \alpha = \max\{\alpha, \alpha\}$

$= \max\{\mu(x * y), \mu(y)\}$.

If at least one of $(x * y)$ and y does not belongs to A, then at least one of $\mu(x * y)$and $\mu(y)$ is 0.

$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$. for every $x, y \in X$.

Also it is clear that $\cup(\mu;\alpha) =$ A.

Theorem 7.4.4: If A is an anti fuzzy ideal then prove that A^m is an anti fuzzy ideal.

Proof: Given that A is an anti fuzzy ideal.

$\mu_A(x) \leq \max\{ \mu_A(x * y) , \mu_A(y) \}$.

Now define $*$ in A^m by coordinate-wise-multiplication

$\mu_{A^m}(x) = \mu_{A^m}((x_1 * x_2 * .... * x_m)$

$= \mu_A(x_1) * \mu_A(x_2) *....* \mu_A(x_m)$

$\leq \max\{\mu_A(x_1 * y_1), \mu_A(y_1)\} * \max\{\mu_A(x_2 * y_2), \mu_A(y_2)]\}$

$*.... * \max\{\mu_A(x_m * y_m), \mu_A(y_m)]\}$

$= \max\{ \mu_{A^m}[(x_1 * x_2 *... * x_m) * (y_1 * y_2 * ... * y_m)], \mu_{A^m}(y_1 * y_2 * ... * y_m)\}$

$= \max\{ \mu_{A^m}(x * y) , \mu_{A^m}(y) \}$

Thus $\mu_{A^m}(x) \leq \max\{ \mu_{A^m}(x * y) , \mu_{A^m}(y) \}$ for every $x, y \in X$.

Theorem 7.4.5: An onto homomorphic image of an anti fuzzy ideal with sup property is an anti fuzzy ideal.

Proof: Let $f: G \rightarrow G^l$ be an onto homomorphism of a BCK algebra and let μ_G be a fuzzy BCK algebra with sup property.

Given $x^l, y^l \in G^l$ .Let $x_0 \in f^{-1}(x) \in G$,$y_0 \in f^{-1}(y)$ be such that

$\mu_G(x_0) = \sup_{t \in f^{-1}(x^l)} \mu_G(t)$, $\mu_G(y_0) = \sup_{t \in f^{-1}(y^l)} \mu_G(t)$

Then it finds that

$$\mu_{f(G)}(x^l) = \sup_{t \in f^{-1}(x^l)} [\mu_G(t)]$$

$$= \mu_G(x_0)$$

$$\leq \max\{ \mu_G(x_0 * y_0) , \mu_G(y_0) \}.$$

$$= \max\{ \sup_{t \in f^{-1}(x^l * y^l)} \mu_G(t) , \sup_{t \in f^{-1}(y^l)} \mu_G(t) \}$$

$$= \max\{ \mu_{f(G)}(x^l * y^l) , \mu_{f(G)}(y^l)\} \text{ for every } x^l, y^l \in G^l.$$

Theorem 7.4.6: An onto homomorphic pre image of an anti fuzzy ideal is an anti fuzzy ideal.

Let f: $G \to G^l$ be an onto homomorphism of a BCK algebra.

Let A be an anti fuzzy ideal of G^l.

For any x, y ,z $\in$G, it follows that

$$\mu_{f^{-1}(A)}(x) = \mu_A(f(x))$$

$$\leq \max\{ \mu_A(f(x) * f(y)), \mu_A(f(y))\}$$

$$= \max\{ \mu_A(f(x * y)) , \mu_A(f(y))\}$$

$$= \max\{ \mu_{f^{-1}(A)}(x * y), \mu_{f^{-1}(A)}(y) \}$$

Hence $\mu_{f^{-1}(A)}(x) \leq \max\{ \mu_{f^{-1}(A)}(x * y), \mu_{f^{-1}(A)}(y) \}$ for every x, y $\in$G.

Section 7.5: Properties of anti-fuzzy prime ideals:

Definition 7.5.1: μ **i**s anti-fuzzy prime ideal in BCI – algebra if

$\mu(x \wedge y) \geq \min \{ \mu(x), \mu(y) \}$ and $\mu(y * (y * x)) \geq \min \{ \mu(x), \mu(y) \}$ for every $x, y \in G$.

Theorem 7.5.2: If one is contained in another, then $A \cup B$ is an anti fuzzy prime ideal.

Let $x, y \in A \cup B$. Then $x, y \in A$ or $x, y \in B$

Since A and B are anti fuzzy prime ideal it gives that $\mu(x \wedge y) \geq \min \{ \mu(x) , \mu(y) \}$

and $\mu(y * (y * x)) \geq \min \{ \mu(x) , \mu(y) \}$ for every $x, y \in A \cup B$.

Theorem 7.5.3: If A and B are anti fuzzy prime ideals then prove that $A \cap B$ is anti fuzzy prime ideals.

Proof: Let $x , y \in A \cap B$ Then $x , y \in A$ and $x , y \in B$

Since A and B are anti fuzzy prime ideals, it finds that $\mu(x \wedge y) \geq \min \{ \mu(x) , \mu(y) \}$

$\mu(y * (y * x)) \geq \min \{ \mu(x) , \mu(y) \}$ for every $x, y \in A \cap B$.

Theorem 7.5.4: If A is an anti fuzzy prime ideal then prove that A^m is an anti fuzzy prime ideal

Proof: Given that A is an anti fuzzy prime ideal.

$\mu_A(y * (y * x)) \geq \min \{ \mu_A(x) , \mu_A(y) \}$ for every $x, y \in X$.

Define $*$ in A^m by coordinate-wise-multiplication.

$\mu_A{}^m (y * (y * x)) = \mu_A{}^m [(y_1 * y_2 * .... * y_m) * \{ (y_1 * y_2 * ... * y_m) * (x_1 * x_2 * ... * x_m) \}$

$= \mu_A(y_1 * (y_1 * x_1)) * \mu_A(y_2 * (y_2 * x_2)) * ..... * \mu_A(y_m * (y_m * x_m))$

$\geq \min\{\mu_A(x_1) , \mu_A(y_1) \} * \min \{ \mu_A(x_2) , \mu_A(y_2)\}$

$* \; ..... * \min \{\mu_A(x_m), \mu_A(y_m) \}$

$= \min \{\mu_A(x_1) * \mu_A(x_2) * ... * \mu_A(x_m) , \mu_A(y_1) * \mu_A(y_2) \; .... * \mu_A(y_m)\}$

$\geq \min \{ \mu_A{}^m(x) , \mu_A{}^m(y) \}$ for every $x, y \in X$.

Theorem 7.5.5: Direct product of two anti fuzzy prime ideals is again an anti fuzzy prime ideal.

Proof: Let μ_1 and μ_2 be the anti fuzzy prime ideals of G_1 and G_2

Then μ in G is defined by $\mu(x) = \mu(x_1 , x_2) = (\mu_1 X \mu_2)(x_1 , x_2)$

$= \min (\mu_1(x_1), \mu_2 (x_2))$

$\mu(x \wedge y) = \min (\mu_1(x_1 \wedge y_1) , \mu_2(x_2 \wedge y_2))$

$\geq \min \{ \min (\mu_1(x_1), \mu_1 (y_1)) , \min (\mu_2(x_2), \mu_2 (y_2))\}$

$\geq \min\{ \min (\mu_1(x_1), \mu_2 (x_2)) , \min (\mu_1(y_1), \mu_2 (y_2))$

$= \min \{ \mu(x) , \mu(y) \}$ for every $x,y \in G$.

Theorem 7.5.6: An onto homomorphic image of an anti fuzzy prime ideal with sup property is an anti fuzzy prime ideal.

Proof: Let $f: G \rightarrow G^l$ be the homomorphic image of BCK algebra and let A be an anti fuzzy prime ideal of G^l $\mu_G(x \wedge y) \geq \min \{ \mu_G(x) , \mu_G(y) \}$ for every $x, y \in G$.

Given that $x^l , y^l \in G^l$. Let $x_0 \in f^{-1}(x^l)$, $y_0 \in f^{-1}(y^l)$ be such that

$$\mu_G(x_0) = \sup_{t \in f^{-1}(x^l)} \mu_G(t) , \quad \mu_G(y_0) = \sup_{t \in f^{-1}(y^l)} \mu_A(t)$$

$$\mu_{f(G)}(x^l \wedge y^l) = \sup_{t \in f^{-1}(x^l \wedge y^l)} \mu_G(t)$$

$$\geq \mu_G(x_0 \wedge y_0)$$

$$\geq \min \{ \mu_G(x_0) , \mu_G(y_0) \}$$

$$\geq \min \{ \sup_{t \in f^{-1}(x^l)} \mu_G(t) , \sup_{t \in f^{-1}(y^l)} \mu_G(t) \}$$

$\mu_{f(G)}(x^l \wedge y^l) \geq \min \{ \mu_{f(G)}(x^l) , \mu_{f(G)}(y^l) \}$ for every $x^l, y^l \in G^l$.

Theorem 7.5.7: Let f be a homomorphism of a BCK algebra G_1 into G_2 and B be an anti fuzzy prime ideal of G_2, then $f^{-1}(B)$ is an anti fuzzy prime ideal of G_1.

Proof: For any x , y $\in G_1$, it gives that $\mu_{f^{-1}(B)}(x \wedge y) = \mu_B(f(x \wedge y))$

$$\geq \min \{ \mu_B(f(x)) , \mu_B(f(y)) \}$$

$$= \min \{ \mu_{f^{-1}(B)}(x), \mu_{f^{-1}(B)}(y) \} \text{ for every } x, y \in G_l.$$

Theorem 7.5.8: If A and its compliment is an anti fuzzy prime, then A is a constant function.

Proof: Given that A and A^l is an anti fuzzy prime ideals.

$$\mu_A(x \wedge y) \geq \min \{ \mu_A(x) , \mu_A(y) \}$$

$$\mu_{A^l}(x \wedge y) \geq \min \{ \mu_{A^l}(x) , \mu_{A^l}(y) \}$$

$$1-\mu_A(x \wedge y) \geq \min \{1- \mu_A(x) ,1- \mu_A(y) \}$$

$$= 1- \max \{ \mu_A(x) , \mu_A(y) \}$$

$$\mu_A(x \wedge y) \leq \max \{ \mu_A(x) , \mu_A(y) \}$$

Without loss of generality, it implies that

$$\max \{ \mu_A(x) , \mu_A(y) \} \geq \mu_A(x \wedge y) \geq \min \{ \mu_A(x) , \mu_A(y) \}$$

$$\mu_A(x) \geq \mu_A(y * (y * x)) \geq \mu_A(y) \Rightarrow \mu_A(x) \geq \mu_A(y * (y * x))$$

Put y = 0 implies that $\mu_A(x) \geq \mu_A(0)$.Then $\mu_A(x) \geq \mu_A(y)$

Put x= 0 and y = x $\Rightarrow$ $\mu_A(0) \geq \mu_A(x)$ and $\mu_A(0) = \mu_A(x)$

Hence A is a constant function.

Section 7.6: Closed ideals in a BCI algebra

Definition 7.6.1: An ideal A in a BCI algebra X is closed if $0 * x \in A$ then $x \in A$ for every $x \in X$.

Theorem 7.6.2: Union of two closed ideals is closed if one is contained in other.

Proof: Let A and B be two closed ideals. Then $0 * x \in A \cup B$.

Then $0 * x \in A$ or $0 * x \in B \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$ for every $x \in X$.

Theorem 7.6.3: Intersection of two closed ideals is closed.

Proof: Let A and B be two closed ideals. Then $0 * x \in A \cap B$.

Then $0 * x \in A$ and $0 * x \in B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B$ for every $x \in X$.

Theorem 7.6.4: Union of two fuzzy closed ideals is closed.

Proof: $\mu_{A \cup B}{}^{(0* x)} = \max\{\mu_A(0 * x), \mu_B(0 * x)\}$

$\geq \max\{\mu_A(x), \mu_B(x)\}$

$= \mu_{A \cup B}{}^{(x)}$

$\mu_{A \cup B}{}^{(0* x)} \geq \mu_{A \cup B}{}^{(x)}$ for every $x \in X$.

Theorem 7.6.5: Intersection of two fuzzy closed ideals is closed.

Proof: $\mu_{A \cap B}{}^{(0* x)} = \min\{\mu_A(0 * x), \mu_B(0 * x)\}$

$\geq \min\{\mu_A(x), \mu_B(x)\}$

$= \mu_{A \cap B}{}^{(x)}$

$\mu_{A \cap B}(0 * x) \geq \mu_{A \cap B}{}^{(x)}$ for every $x \in X$.

Theorem 7.6.6: If A is a fuzzy closed ideal then prove that A^m is fuzzy closed ideal.

Proof: Define $*$ in A^m by coordinate-wise multiplication.

$\mu_A{}^m (x) = \mu_A{}^m (x_1 * x_2 * \ldots * x_m)$

$= \mu_A(x_1) * \mu_A(x_2) * \ldots * \mu_A(x_m)$

$\mu_A{}^m (0 * x) = \mu_A{}^m ((0 * 0 * 0 \ldots m \text{ times}) * (x_1 * x_2 * \ldots * x_m))$

$= \mu_A{}^m ((0 * x_1) * (0 * x_2) * \ldots * (0 * x_m))$

$= \mu_A(0 * x_1) * \mu_A(0 * x_2) * \ldots * \mu_A(0 * x_m)$

$\geq \mu_A(x_1) * \mu_A(x_2) * \ldots * \mu_A(x_m)$

$= \mu_A{}^m (x_1 * x_2 * \ldots * x_m)$

$= \mu_A{}^m (x)$

$\mu_A{}^m (0 * x) \geq \mu_A{}^m (x)$ for every $x \in X$.

Theorem 7.6.7: An onto homomorphic image of a closed fuzzy ideal with sup property is an closed fuzzy ideal .

Proof: Let $f: G \to G^l$ be an onto homomorphic image of BCI algebra and μ_G be a fuzzy BCI algebra of G with sup property.

Given $x^l \in G^l$ and let $0 * xo^l \in f^{-1}(0 * x^l)$ be such that

$$\mu_{f(G)}(0 * x^l) = \sup_{t \in f^{-1}(0 * x^l)} [\mu_G(t)]$$

$= \mu_G(0 * xo^l)$

$\geq \mu_G(xo^l)$

$$= \sup_{t \in f^{-1}(x^l)} [\ \mu_G(t)\ ]$$

$$= \mu_{f(G)}(x^l)$$

Thus $\mu_{f(G)}(0 * x^l) \geq \mu_{f(G)}(x^l)$ for all $x^l \in G^l$

Theorem 7.6.8: An onto homomorphic pre-image of an closed fuzzy ideal is closed.

Proof: Let $f: G_1 \rightarrow G_2$ be an onto homomorphic pre-image of BCI algebra.

For any $x \in G_l$, it follows that

$$\mu_{f^{-1}(A)}(0 * x) = \mu_A(f(0 * x))$$

$$= \mu_A(f(0) * f(x))$$

$$\geq \mu_A(f(x))$$

$$= \mu_{f^{-1}(A)}(x) \text{ for every } x \in G_1.$$

Conclusion: In this chapter, homomorphic image and homomorphic pre-image of intuitionistic fuzzy T-ideals of TM-algebra are studied. Also algebraic properties on anti fuzzy ideals and anti-fuzzy prime ideals in BCI-algebra is discussed. Further basic properties of closed ideals in a BCI-algebra are analyzed. A part of this chapter has published in American Journal of Mathematics and Mathematical Science, Volume 3, Number 2, (2014).