CHAPTER - V

CONVERGING CYLINDRICAL DETONATION WAVES IN A NON-IDEAL GAS WITH AN AZIMUTHAL MAGNETIC FIELD

1. INTRODUCTION

Converging shock and detonation waves offer interesting possibilities of attaining extremely high temperature, pressure and density. In fact, even the applications to thermonuclear fusion, synthesizing of materials, phenomena of sonoluminescence and treatment of stones in the human body (lithotripsy) were considered (Glass and Sagie [1], Glass and Sharma [2], Roberts and Wu [3, 4], Takayama [5], Delius [6]). The problem of contracting cylindrical or spherical shock front propagating into a uniform gas at rest was investigated by Guderley [7] and Stanyukovich [8] by using the method of self-similarity. Nigmatulin [9], Welsh [10] and Teipel [11] replaced the shock front by a contracting detonation front propagating into a uniform combustible gas. These studies show that the similarity solution can not be obtained for a general energy release, but it can be used for studying the flow-field only if the detonation front is governed by the Chapman-Jouguet condition (Helliwell [12]). Lee and Lee [13] described the method of generation of cylindrical detonation waves in acetylene-oxygen mixture, and discussed the possibility of theoretical explanation of the process of convergence of detonation waves by means of Chester-Chisnell-Whitham (CCW) method [14, 15, 16]. The CCW method is a very simple and effective
method for the analysis of imploding shocks and detonation waves. Although this method is approximate one, it agrees well with exact solutions and with experimental results (Lee and Lee [13], Lee [17], Jumper [18]).

Tyl and Wlodarczyk [19] studied cylindrical and spherical detonation waves converging in gaseous explosive mixture by CCW method. They applied the Chapman-Jouguet condition on the detonation wave in the initial position only, and obtained analytical solution describing its propagation in the absence of magnetic field. Their solutions agreed very well with the experimental results. Vishwakarma and Vishwakarma [20] extended the case of converging detonation waves of Tyl and Wlodarczyk [19] to include the effects of the presence of an azimuthal magnetic field. They studied both the cases (i) when the gas is strongly ionized before and behind the detonation front and (ii) when the non-ionized gas undergoes intense ionization as a result of passage of the detonation front. The combustible gas was assumed to obey the equation of state of a perfect gas.

When the flow takes place in extreme conditions, the assumption that the gas is ideal is no more valid. Anisimov and Spiner [21] have taken an equation of state for low density non-ideal gases in a simplified form, and investigated the effect of parameter for non-idealness on the problem of a strong point explosion. Roberts and Wu [3, 4] have used an equivalent equation of state to discuss the shock wave theory of sonoluminescence. In the present work, we analyse the convergence of a strong cylindrical detonation wave in a non-ideal gas (combustible) in the presence of an azimuthal magnetic field. The initial density is variable. It is assumed that the detonation wave is initially Chapman-Jouguet,
i.e. initially it travels with velocity of propagation of small disturbances relative to the burnt gas (Helliwell [12]). The effects of the non-idealness of the gas and the azimuthal magnetic field are investigated. To our knowledge, the problem of converging detonation wave in a non-homogeneous non-ideal gas, which takes into account the effects of a variable magnetic field, has not been studied previously.

During the experiments involving the implosion of a detonation wave in a gas, the following states may occur:

(i) The gas is weakly ionized before and behind the detonation front, i.e. \( R_m << 1 \), where \( R_m \) is the magnetic Reynolds number.

(ii) The gas is strongly ionized before and behind the detonation front, i.e. \( R_m >> 1 \) or \( \sigma \to \infty \), where \( \sigma \) is the electrical conductivity.

(iii) Non-ionized (or weakly ionized) gas undergoes intense ionization as a result of the passage of detonation front, i.e. \( \sigma \) increases in a jump like manner from 0 to \( \infty \).

In our study, we analyse all the three cases when the initial magnetic field is azimuthal and variable. CCW method is employed to determine the velocity of detonation front and the other flow variables just behind the front.

2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The equation of state for a non-ideal gas is borrowed from the statistical physics (Landau and Lifshitz [22]) which has been simplified by Anisimov and
Spiner [21] in the form
\[ p = \overline{R} \rho T (1 + \overline{b} \rho), \tag{2.1} \]
where \( \overline{b} < 1 \) is internal volume of the molecules, \( \overline{R} \) is gas constant, and \( p, \rho \) and \( T \) are pressure, density and temperature of the gas, respectively.

The total energy \( e \) per unit mass is given by (Ojha [23], Roberts and Wu [3, 4])
\[ e = \frac{p}{(\gamma - 1)\rho(1 + \overline{b}\rho)} \approx \frac{p(1 - \overline{b}\rho)}{(\gamma - 1)\rho}, \tag{2.2} \]
which implies that
\[ c_p - c_v = \overline{R} \left( 1 + \frac{\overline{b}\rho^2}{1 + 2\overline{b}\rho} \right) \approx \overline{R}, \tag{2.3} \]
neglecting the term \( \overline{b}^2 \rho^2 \). Here, \( c_p, c_v \) are the specific heats of gas at constant pressure and constant volume processes, respectively and \( \gamma = \frac{c_p}{c_v} \).

The basic equations governing the unsteady and cylindrically symmetric motion of a weakly conducting non-ideal gas (Case I, \( R_m << 1 \)) are given by (Tyl [24], Sakurai [25])
\[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \rho \frac{\partial u}{\partial r} + \rho u \frac{\partial u}{\partial r} = 0, \tag{2.4} \]
\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) + \frac{\partial p}{\partial r} = -\sigma B_0^2 u, \tag{2.5} \]
\[ \left( \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right) - a^2 \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = (\gamma - 1)\sigma B_0^2 u^2, \tag{2.6} \]
\[ \frac{\partial B}{\partial r} = \mu \sigma B_0 u, \tag{2.7} \]
where \( u, B \) are velocity and azimuthal magnetic induction at time ‘\( t \)’ and at distance ‘\( r \)’ from the axis of symmetry, \( \mu \) is the magnetic permeability, \( B_0 \) is
the initial magnetic induction and ‘a’ the speed of sound in the non-ideal gas is given by (Vishwakarma and Chaubey [26])

\[ a^2 = \frac{\gamma p}{\rho} \left( \frac{1 + 2b\rho}{1 + b\rho} \right) \approx \frac{\gamma p}{\rho(1 - b\rho)}. \] (2.8)

The density and the magnetic induction of the gas ahead of the converging detonation front are assumed to be varying and obeying the laws:

(a) \[ \rho_0 = Kr_0^\alpha, \] (2.9)

(b) \[ h_0 = Ar_0^{\alpha_1}, \]

where \( r_0 \) is the radius of detonation front and \( K, A, \alpha \) and \( \alpha_1 \) are constants.

Equations (2.4) to (2.7) can be combined to form the characteristic equation (Whitham [16], Tyl [24]),

\[ dp - \rho a du + \frac{\rho a^2 u}{u - a} \frac{dr}{r} = \frac{[(\gamma - 1)(1 + b\rho)u^2 + ua]\sigma B_0^2 dr}{(u - a)} \] (2.10)

along the negative characteristic

\[ \frac{dr}{dt} = u - a. \] (2.11)

The fundamental equations governing the unsteady flow behind a cylindrical magnetogasdynamic (Case II, \( R_m >> 1 \)) or gas ionizing (Case III, \( \sigma : 0 \rightarrow \infty \)) detonation front are given by (Whitham [16], Vishwakarma and Yadav [27])

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\rho a}{r} = 0, \] (2.12)

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) + \frac{\partial p}{\partial r} + \frac{B \partial B}{\mu} = 0, \] (2.13)

\[ \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) - a^2 \left( \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right) = 0, \] (2.14)

\[ \frac{\partial B}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(rBu) = 0. \] (2.15)
Equations (2.12) to (2.15) can be combined to obtain Characteristic equation (Whitham [16])

$$dp + \mu h \, dh - \rho c \, du + \frac{\rho c^2 u \, dr}{u - c} \frac{dr}{r} + \frac{\mu h^2 (c - u) \, dr}{u + c} \frac{dr}{r} = 0 \quad (2.16)$$

along the negative characteristic

$$\frac{dr}{dt} = u - c; \quad (2.17)$$

where \( h = \frac{B}{\mu} \) is the azimuthal magnetic field and \( c \) is the effective speed of sound given by

$$c^2 = a^2 + b^2 \quad \text{and} \quad b^2 = \frac{B^2}{\mu \rho},$$

\( b \) being the Alfvén speed.

Since \( \sigma \) is small in the Case I, and \( \sigma \) is zero ahead of the detonation front in the case III, the magnetic induction may be taken continuous in these cases (Sakurai [25], Ranga Rao and Ramana [28]). The conditions across the detonation front in the cases I and III are therefore (Tyl and Wlodarczyk [19], Vishwakarma and Vishwakarma [20])

$$\rho_1 (D - u_1) = \rho_0 D,$$

$$p_1 = p_0 + \rho_0 Du_1,$$

$$e_1 = e_0 + \frac{1}{2} (p_1 + p_0) \left( \frac{1}{\rho_0} - \frac{1}{\rho_1} \right) + Q, \quad (2.18)$$

where \( D, Q \) and \( e \) denote the velocity of detonation front, heat energy released per unit mass and the internal energy per unit mass, respectively. The indices ‘1’ and ‘0’ refer to the states just behind and just ahead of the detonation front.

In the pure magnetogasdynamic case (the case II), the gas is strongly ionized, i.e. highly conducting, before and behind the detonation front, upon which
the magnetic induction may be discontinuous at the front resulting from a sheet current there (Sakurai [25]). The conditions across the detonation front, in this case, may be written as (c.f. Whitham [16])

\[
\begin{align*}
    h_1(D - u_1) &= h_0 D \\
    \rho_1(D - u_1) &= \rho_0 D, \\
    p_1 + \frac{\mu h_1^2}{2} + \rho_1(D - u_1)^2 &= p_0 + \frac{\mu h_0^2}{2} + \rho_0 D^2, \\
    \frac{1}{2}(D - u_1)^2 + e_1 + \frac{p_1}{\rho_1} + \frac{\mu h_1^2}{\rho_1} &= \frac{1}{2} D^2 + e_0 + \frac{p_0}{\rho_0} + \frac{\mu h_0^2}{\rho_0} + Q. 
\end{align*}
\] (2.19)

The detonation front is assumed to be strong, i.e., \(p_0 << p_1\), therefore we take \(p_0 = e_0 = 0\) in the relations (2.18) and (2.19).

3. SOLUTION OF THE PROBLEM

The detonation front is assumed to be initially of radius \(R\) and in the Chapman-Jouguet state. The Chapman-Jouguet condition requires that the downstream flow will be sonic in the shock fixed coordinates, i.e.

\[|D_{cj} - u_{cj}| = a_{cj},\] (3.1)

where ‘\(cj\)’ refers to the Chapman-Jouguet state. Therefore, the conditions across the strong Chapman-Jouguet front, in the case I and III, are expressed as

\[
\begin{align*}
    p_{cj} &= \frac{1 - \delta}{\gamma + 1} \rho_0 D_{cj}^2, \\
    u_{cj} &= \frac{1 - \delta}{\gamma + 1} D_{cj}, \\
    \rho_{cj} &= \frac{\gamma + 1}{\gamma + \delta} \rho_0, \\
    h_{cj} &= h_0, \\
    a_{cj} &= \frac{\gamma + \delta}{\gamma + 1} |D_{cj}|, \\
\end{align*}
\] (3.6)
\[ D_{cj} = \frac{\sqrt{2Q(\gamma^2 - 1)}}{1 - \delta}, \quad (3.7) \]

where \( \delta = \bar{\nu} \rho_0 \) is the parameters of non-idealness of the gas. In the case II, the magnetic field is also discontinuous across the front and, therefore, the condition (3.5) is replaced by

\[ h_{cj} = \frac{\gamma + 1}{\gamma + \delta} h_0. \quad (3.8) \]

Making use of equation (2.18) and (3.2) to (3.8) the conditions across the strong detonation front can be expressed in terms of velocity of detonation products (burnt gas), in the case I, by the equations

\[ \frac{D}{D_{cj}} = \frac{1}{2}(q + q^{-1}), \quad (3.9) \]

\[ \frac{p_1}{p_{cj}} = \frac{1}{2}(q^2 + 1)x^\alpha, \quad (3.10) \]

\[ \frac{\rho_1}{\rho_{cj}} = \frac{(\gamma + \delta)(q^2 + 1)x^\alpha}{(\gamma + 1) + (2\delta + \gamma - 1)q^2}, \quad (3.11) \]

\[ \frac{a_1}{a_{cj}} = \left[ \frac{1}{2(\gamma + 2\delta)}(\gamma + 1) + (2\delta + \gamma - 1)q^2 - \delta(\gamma + 1)(q^2 + 1) \right]^{1/2}, \quad (3.12) \]

where \( q = \frac{u_1}{u_{cj}} \) and \( x = \frac{r_0}{R} \).

Using the relations (2.19) and (3.2) to (3.8), the conditions across the strong magnetogasdynamic detonation front (case II) can be expressed by the equations

\[ \frac{D}{D_{cj}} = \frac{1}{2}[q + q^{-1}], \quad (3.13) \]

\[ \frac{p_1}{p_{cj}} = \frac{1}{2}(q^2 + 1)x^\alpha, \quad (3.14) \]

\[ \frac{\rho_1}{\rho_{cj}} = \frac{(\gamma + \delta)(q^2 + 1)}{L} x^\alpha, \quad (3.15) \]

\[ \frac{h_1}{h_{cj}} = \frac{(\gamma + \delta)(q^2 + 1)}{L} x^{\alpha_1}, \quad (3.16) \]
\[
\frac{c_1}{a_{cj}} = \left[ \frac{(1-\delta)(\gamma + \delta)L^3 + 2(\gamma + 2\delta)(\gamma + 1)^2(q^2 + 1)M_{cj}^{-2} \times \\
\{L - \delta(\gamma + 1)(q^2 + 1)\}^{x^{2\alpha_1-a}}}{2(\gamma + 2\delta)(\gamma + \delta)L \times \\
\{L - \delta(\gamma + 1)(q^2 + 1)\}^{1/2}} \right].
\]

where \(L = (\gamma + 1) + (2\delta + \gamma + 1)q^2\) and the Alfven Mach number \(M_{cj}\) of the detonation front in the Chapman-Jouguet state is given by

\[
M_{cj}^2 = \frac{D_{cj}^2}{(\mu h_{cj}^2/\rho_{cj})}
\]

In the case III (strong gas-ionizing detonation front), the equations (3.16) and (3.17) are replaced by

\[
h_1 = h_{ej}, \quad (3.18)
\]

\[
\frac{c_1}{a_{cj}} = \left[ \frac{L}{2(\gamma + 2\delta)} \left\{ \frac{(1-\delta)L}{\{L - \delta(\gamma + 1)(q^2 + 1)\}^{1/2}} \right\}^{1/2} \right. \\
\left. + \frac{2(\gamma + 1)^2(\gamma + 2\delta)M_{cj}^{-2}x^{2\alpha_1-a}}{(\gamma + \delta)^3(q^2 + 1)} \right].
\]

Now, we shall use CCW method [14, 15, 16] to obtain speed of detonation front from other flow variables just behind the front in all the three cases. For converging fronts, the method is to apply the characteristic equation (valid along a negative characteristic) to flow quantities just behind the front.

**Case I: \(R_m \ll 1\) (Detonation wave in a weakly conducting non-ideal gas)**

Using the flow variables just behind the detonation front, into the characteristic equation (2.10) (keeping in mind that \(u_1\) is negative), we obtain

\[
d \left( \frac{p_1}{p_{ej}} \right) + \frac{\rho_1}{p_{ej}} \frac{\alpha_1}{a_{cj}} - q \frac{q^{\frac{\alpha_1}{a_{cj}}}}{\left( \frac{1-\delta}{\gamma+\delta} \right) \left( \frac{\gamma+1}{\gamma+\delta} \right)} dx = \\
- \left[ \frac{(\gamma^2-1)}{(\gamma+\delta)} q^2 \left( 1 + \delta \left( \frac{\gamma+1}{\gamma+\delta} \right) \frac{\rho_1}{p_{ej}} \right) - q \frac{a_{ej}}{\left( \frac{1+\gamma}{1-\delta} \right)} \right] \frac{R_m M_{cj}^{-2} x^{2\alpha_1}}{\left[ q + \frac{a_{ej}}{\left( \frac{\gamma+1}{\gamma+\delta} \right)} \right]},
\]

(3.20)
where the magnetic Reynolds number $R_m$ is given by

$$R_m = \sigma \mu |D_{cj}| R.$$

Using the values of flow variables, given by equations (3.9) to (3.12) into the characteristic equation (3.20), we obtain the differential equation between the velocity of the motion of the detonation products and the location of wave front as

$$\frac{dq}{dx} = -\frac{1}{(\gamma + \delta)\{q(1-\delta) + (\gamma + \delta)\sqrt{G_1}\}\{Lq + (\gamma + \delta)(q^2 + 1)\sqrt{G_1}\} \times}
\left\{[(\gamma^2 - 1)\{L + \delta(\gamma + 1)(q^2 + 1)\}(1-\delta)xq^2 - Lq\sqrt{G_1}(\gamma + 1)\times
(\gamma + \delta)R_mM_{cj}^{-2}x^{2a_1-\alpha} + (\gamma + \delta)\frac{\alpha}{2}(q^2 + 1)\{q(1-\delta) + (\gamma + \delta)\times
\sqrt{G_1}\}Lx^{-1} + (\gamma + \delta)^3 q(q^2 + 1)G_1x^{-1}\right\}. \text{(3.21)}$$

where $G_1 = \left[\frac{(1-\delta)L^2}{2(\gamma + 2\delta)\{L - \delta(\gamma + 1)(q^2 + 1)\}}\right]^{1/2}$.

Numerical integration of the differential equation (3.21), along with the equations (3.9, 3.10, 3.11) with the initial condition $q = 1, x = 1$ gives the values of $q, \frac{D}{D_{cj}}, \frac{p_1}{\rho_{cj}}, \frac{p_1}{\rho_{cj}}$ as $x$ decreases from 1 to zero.

**Case II: $R_m >> 1$ (Pure magnetogasdynamic detonation wave)**

Using the flow variables just behind the detonation front into the characteristic equation (2.16) (keeping in mind that $u_1$ is negative), we obtain

$$\frac{d}{dx}\left(\frac{p_1}{\rho_{cj}} \frac{c_1}{a_{cj}}\right) + \frac{(\gamma + 1)^2}{(1-\delta)(\gamma + \delta)}\left(\frac{h_1}{h_{cj}}\right)^{\gamma - 2}d\left(\frac{h_1}{h_{cj}}\right) + dq + \frac{(\frac{\gamma + \delta}{1 - \delta})\frac{c_1}{a_{cj}} qx^{-1} dx}{q + \left(\frac{\gamma + \delta}{1 - \delta}\right)\frac{c_1}{a_{cj}}} +
\frac{(\gamma + 1)^2}{(1-\delta)(\gamma + \delta)}\left(\frac{h_1}{h_{cj}}\right)^{\gamma - 2}M_{cj}^{-2}\frac{\left(\frac{\gamma + \delta}{1 - \delta}\right)\frac{c_1}{a_{cj}} - q}{q + \left(\frac{\gamma + \delta}{1 - \delta}\right)\frac{c_1}{a_{cj}}} x^{-1} dx = 0. \text{(3.22)}$$

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On using the equations (3.14, 3.15, 3.16, 3.17), in equation (3.22), we obtain the following differential equation

\[
\frac{dq}{dx} = -\frac{1}{(q + \frac{\gamma + \delta}{1 - \delta} G_2) \left[ \frac{L^2 q}{(q^2 + 1)} + 4(\gamma + \delta)(\gamma + 1)^2 q M_{cj}^{-2} x^{2\alpha_1 - \alpha} + (\gamma + \delta) G_2 L^2 \right]}
\]

\[
\left\{ \frac{\alpha}{2} \left( q + \frac{\gamma + \delta}{1 - \delta} G_2 \right) L^3 x^{-1} + (\gamma + \delta)(\gamma + 1)^2(q^2 + 1)L M_{cj}^{-2} x^{2\alpha_1 - (\alpha + 1)} \right\} + \left\{ \frac{\gamma + \delta}{1 - \delta} G_2 - q \right\} + \frac{(\gamma + \delta)^2}{(1 - \delta)q} L^2 G_2 x^{-1},
\]

(3.23)

where \( G_2 = \left[ \frac{(1 - \delta)(\gamma + \delta)L^2 + 2(\gamma + 2\delta)(\gamma + 1)^2(q^2 + 1)M_{cj}^{-2}x^2}{2(\gamma + 2\delta)(\gamma + \delta)L(L - \delta(\gamma + 1)(q^2 + 1))} \right] \)

\[
\left\{ \frac{L - \Delta(\gamma + 1)(q^2 + 1)}{x^{2\alpha_1 - \alpha}} \right\}^{1/2}
\]

Integrating the differential equation (3.23), numerically and using (3.13) to (3.17) we can obtain \( q, \frac{D}{D_{cj}}, \frac{p_1}{p_{cj}}, \frac{\rho_1}{\rho_{cj}} \) and \( \frac{h_1}{h_{cj}} \) in terms of \( x \).

Case III: \( \sigma : 0 \rightarrow \infty \) (Gas-ionizing detonation wave)

On using the equations (3.14, 3.15, 3.16, 3.17) into the characteristic equation (2.16) and simplifying, we obtain the differential equation

\[
\frac{dq}{dx} = -\frac{1}{q + \left( \frac{\gamma + \delta}{1 - \delta} \right) \sqrt{\frac{L G_3}{2(\gamma + 2\delta)}}} \left[ qL + (\gamma + \delta)(q^2 + 1)\sqrt{\frac{L G_3}{2(\gamma + 2\delta)}} \right] \times
\]

\[
\left\{ q + \frac{\gamma + \delta}{1 - \delta} \sqrt{\frac{L G_3}{2(\gamma + 2\delta)}} + \frac{(\gamma + \delta)^2}{(1 - \delta)(\gamma + 2\delta)} q G_3 \right\} + \frac{L(q + 1)^2}{(1 - \delta)(\gamma + \delta)} M_{cj}^{-2} \times
\]

\[
x^{2\alpha_1 - \alpha} \left\{ q(\alpha_1 - 1) + \left( \frac{\gamma + \delta}{1 - \delta} \right) \sqrt{\frac{L G_3}{2(\gamma + 2\delta)}}(\alpha_1 + 1) \right\},
\]

(3.24)

where \( G_3 = \left[ \frac{(1 - \delta)L}{L - \Delta(\gamma + 1)(q^2 + 1)} + \frac{2(\gamma + 1)^2(\gamma + 2\delta)M_{cj}^{-2}x^{2\alpha_1 - \alpha}}{(\gamma + \delta)^3(q^2 + 1)} \right]^{1/2} \).

Numerical integration of the differential equation (3.24), and use of equation (3.13, 3.14, 3.15) gives the variation of \( q, \frac{D}{D_{cj}}, \frac{p_1}{p_{cj}}, \) and \( \frac{\rho_1}{\rho_{cj}} \) with \( x \).
4. RESULTS AND DISCUSSION

For the purpose of numerical calculations, we used the values of $R_m$, $\gamma$, $\delta$, $M_{cj}^{-2}$, $\alpha$ and $\alpha_1$ given by $R_m = 0.01$ (in the case I only); $\gamma = 3.0$; $\alpha_1 = -1$; $\delta = 0, 0.1$; $M_{cj}^{-2} = 0, 0.01$ and $\alpha = 0, -1$. The value $\delta = 0$ corresponds to the case of a perfect gas, $M_{cj}^{-2} = 0$ to the non-magnetic case and $\alpha = 0$ to the case of constant initial density.

In the case I, the velocity of detonation front $\frac{D}{D_{cj}}$ and the pressure behind it $\frac{p_1}{p_{cj}}$ are plotted against the radius $x$ in figures 1 and 2. It is shown that a decrease in the value of index for variable initial density $\alpha$, decelerates the convergence of the front (figure 1) and decreases the pressure behind it (figure 2). A change in the value of the parameter $\delta$ characterizing the non-idealness of the gas and in the value of magnetic parameter $M_{cj}^{-2}$ show almost negligible effect on $\frac{D}{D_{cj}}$ and $\frac{p_1}{p_{cj}}$.

In the case II, figures 3 and 4 show that an increase in the value of $M_{cj}^{-2}$, highly decelerates the convergence of the front and decreases sufficiently the pressure behind it, when $\alpha = 0$, on the other hand when $\alpha = -1$, an increase in $M_{cj}^{-2}$ has small effects on the velocity of front and on the pressure behind it. A decrease in $\alpha$, increases the front velocity near the axis in the magnetic cases, and also it increases the pressure behind the front, in both, the magnetic and non-magnetic cases. When $\alpha = 0$ an increase in $\delta$ increases the front velocity, whereas it decreases when $\alpha = -1$. Also, the effect of an increase in $\delta$ is to increase slightly the pressure behind the front.

In case III, figures 5 and 6 show that an increase in the value of $M_{cj}^{-2}$ decelerates the detonation front near the axis when $\alpha = 0$, whereas the front
velocity increases rapidly when $\alpha = -1$. Also, by an increase in $M_{cj}^{-2}$, the pressure behind the front starts to decrease near the axis when $\alpha = 0$.

On decreasing $\alpha$ in the absence of initial magnetic field, there is an increase in the velocity of detonation front, whereas in the presence of initial magnetic field, the velocity of detonation front increases rapidly near the axis. Also, on decreasing $\alpha$, in presence and in absence of initial magnetic field, i.e. in both the cases, the pressure behind the detonation front increases rapidly. On increasing the value of $\delta$ in the absence of initial magnetic field, there is slight effect on the velocity of detonation front, but in the presence of initial magnetic field it decreases. The pressure behind the detonation front is slightly affected by an increase in the value of $\delta$.

5. CONCLUSION

In this work we have studied converging cylindrical detonation waves in a non-ideal gas under the influence of a variable azimuthal magnetic field. The initial density of the medium is assumed to obey a power law. The effects of non-idealness of the gas, variation of initial density and variation of initial magnetic field on the propagation of the detonation front and the pressure behind it are investigated. It is found that there is a slight effect of the presence of magnetic field on the velocity of detonation front when the gas is weakly conducting (case I). In case II (pure magnetogasdynamic case), on increasing $M_{cj}^{-2}$ the velocity of detonation front and the pressure behind it decrease near the axis when $\alpha = 0$, whereas when $\alpha = -1$, the velocity of detonation front increases. In case III, there is similar effect of $M_{cj}^{-2}$ on the velocity of detonation front as in case II. In case III, there is a similar effect of increasing $M_{cj}^{-2}$ as in case II. On increasing
the value of $\delta$ in case I, there is a negligible effect on the velocity of detonation front and the pressure behind it. In case II, on increasing $\delta$ the velocity of detonation front and the pressure behind it decrease near the axis. In case III, there is a similar behaviour of increment of $\delta$ as in case II. On decreasing the value of $\alpha$, the velocity of detonation front and the pressure behind it decrease near the axis as in the case I. In case II, on decreasing the value of $\alpha$, the velocity of detonation front and the pressure behind it increase. In case III, in the presence of magnetic field (i.e. when $M^{-2}_{cj} = 0.01$), on decreasing the value of $\alpha$, the velocity of detonation front and the pressure behind it increase near the axis.

Thus we can conclude that

(i) In the presence of magnetic field, on increasing the value of $\delta$, the velocity of detonation front and the pressure behind it decrease in cases II and III, near the axis.

(ii) On decreasing the value of $\alpha$, (i) in case I, velocity of detonation front and the pressure behind it decrease in both the magnetic and non-magnetic cases. (ii) in case II, the velocity of detonation front decreases in non-magnetic case but increases in magnetic case, where as pressure behind it increases in both the cases. In case III, velocity of detonation front increases in magnetic case, whereas pressure behind it increases in both the cases.

(iii) On increasing $M^{-2}_{cj}$, (i) in case II, velocity of detonation front and the pressure behind it decrease when $\alpha = 0$ but velocity of detonation front increases for $\alpha = -1$, (ii) in case III, there is a similar effect as in case II.
Figure 1: Variation of velocity of detonation front with its radius in the case I ($R_m \ll 1$) for $\gamma = 3.0$, $R_m = 0.01$ and $\alpha = -1$. 

<table>
<thead>
<tr>
<th>$M_{c_j}^2$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>3.0</td>
<td>-1</td>
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</tr>
<tr>
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<td>-1</td>
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</tr>
<tr>
<td>5.0</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
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<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>7.0</td>
<td>0.01</td>
<td>-1</td>
</tr>
<tr>
<td>8.0</td>
<td>0.01</td>
<td>-1</td>
</tr>
</tbody>
</table>
Figure 2: Variation of pressure behind the detonation front with its radius in case I ($R_m \ll 1$) for $\gamma = 3.0, R_m = 0.01$ and $\alpha_i = -1$. 

<table>
<thead>
<tr>
<th>$M_{ij}^2$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
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<td>-1</td>
<td>0</td>
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</tbody>
</table>
Figure 3: Variation of velocity of detonation front with its radius in the case II ($R_m \gg 1$) for $\gamma = 3.0$ and $\alpha_i = -1$. 

<table>
<thead>
<tr>
<th>$M_{c_i}^2$</th>
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<tbody>
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<tr>
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Figure 4: Variation of pressure behind the detonation front with its radius in case II \((R_m \gg 1)\) for \(\gamma = 3.0\) and \(\alpha = -1\).
Figure 5: Variation of velocity of detonation front with its radius in the case III ($\sigma: 0 \rightarrow \infty$) for $\gamma = 3.0$ and $\alpha_j = -1$. 

<table>
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<tr>
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<td>0</td>
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<tr>
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<td>0.01</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>
Figure 6: Variation of pressure behind the detonation front with its radius in case III ($\sigma : 0 \to \infty$) for $\gamma = 3.0$ and $\alpha_i = -1$. 

<table>
<thead>
<tr>
<th>$M_{ij}^{-2}$</th>
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<tr>
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<tr>
<td>6. 0.01</td>
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<tr>
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<tr>
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References


