CHAPTER 5
EXPERIMENTAL INVESTIGATION II
MULTI PRODUCT SUPPLY CHAIN
INVENTORY OPTIMIZATION

5.1 INTRODUCTION

The inventory and supply chain managers are mainly concerned about the estimation of the exact amount of inventory at each point in the supply chain, free of excesses and shortages, although the overall objective is to minimize the total supply chain cost. Owing to the fact that shortage of inventory yields to lost sales, whereas excess of inventory may result in pointless storage costs, the precise estimation of optimal inventory is indispensable. In other words, there is a cost involved in manufacturing any product in the factory, as well as in holding any product in the distribution center and in the agent’s shop. More the products are manufactured or held, the higher will be the holding cost. Meanwhile, there is a possibility for the shortage of products. For the shortage of each product there will be a shortage cost. Holding excess stock levels as well as the occurrence of shortage for products, leads to the increase in the total supply chain cost.
This study supplements the previous study outlined in Chapter 5 that focuses only on a single product inventory optimization. In this chapter, the situation of multiple products and multiple members of the supply chain is considered. Thus the complexity of the problem has been increased. A novel and efficient approach using Genetic Algorithm to solve this complexity has been developed. In order to minimize the total supply chain cost, the proposed approach clearly determines the most probable excess stock level and shortage level that are required for inventory optimization in the supply chain. In practice, the dynamic nature of the excess stock level and shortage level, over all the periods is the typical problem occurring in inventory management. The determination of the stock level that occurs at a maximum rate is the vital operation to be performed. Thus, the maximum occurrences of similar stock levels should be considered in order to optimize effectively. The employed fitness function of the genetic algorithm is formulated in such a way that it will consider the past periods to determine the necessary actions for future stock level. The proposed approach of genetic algorithm predicts the excess/shortage of stock levels at each member of the supply chain by considering the stock levels of the past years. This is vital information such that the total supply chain cost will be maintained as minimum.
5.2 GA MODEL FOR MULTI-PRODUCT INVENTORY OPTIMIZATION

The inventory control for more number of products along with different levels of supply chain is a complex task. To make the inventory control effective, the most primary objective is to predict where, why and how much of the control is required. Such a prediction is to be made here through the methodology proposed. To accomplish the same Genetic algorithm is used and the optimal number of units of a specific product at each member of supply chain that needs to be kept in the desired level of control is determined on the basis of the knowledge of the past records. This leads to an easy estimation of the stock levels of the respective products to be maintained at different members of the chain in the upcoming period. For example, a three stage supply chain having seven members is depicted in Fig: 5.1.

![Fig: 5.1 Three stage-7 member supply chain](image-url)
As shown in Fig: 5.1, a factory is the parent of the chain and it is having two distribution centers Distribution center 1 and Distribution center 2. Each Distribution center further comprises of several agents but as stated in the example case, each Distribution center is having two agents. So, in aggregate there are four agents, Agent 1 and Agent 2 for Distribution center 1 and Agent 3 and Agent 4 for Distribution center 2. The factory manufactures different products that would be supplied to the distribution centers. From the distribution center, the stocks will be moved to the corresponding agents. The proposed methodology is aimed at in determining the specific product that needs to be concentrated on and the amount of stock levels of the product to be maintained by the different members of the supply chain. Also, the methodology analyses whether the stock level of the particular product needs to be in abundance, in order to avoid shortage of the product or needs to be held minimal in order to minimize the holding cost.

The methodology as shown in Fig: 5.2 would analyze the past records very effectively and thus facilitate efficient inventory management with the help of Genetic Algorithm. The analysis is initiated by the selection of valid records. The validation of records are done over the records of past periods, for instance, a record of ten years, from July 1999 to June 2009 can be considered as past years’ record set.
The data from Jan 1999 to Jan 2009 is simulated for a hypothetical company and the simulated data set represents the daily position of excess/shortage of inventory levels occurring in the organization along with its supply chain partners and this is done for generalization purpose. For an individual organization, the actual data representing the excess/shortage inventory levels occurring in the supply chain network during the planning period at desired intervals will replace the simulated data and the proposed technique can be applied for optimization.

The stock levels at the different supply chain members are held in the dataset for different products, namely P1, P2, P3, P4, P5, P6, P7, etc. If seven products have been considered for the analysis, the stock levels for the seven products at each member of the chain throughout the ten years period are considered as data set as shown in the Table 6.1. For the valid record set selection, records having nil values are neglected and the records having positive or negative values are selected for the analysis. This can be done by means of clustering algorithms, extraction algorithms or by any of the data mining functions. Hence the extraction function results in data sets having either positive or negative values.
Fig: 5.2 Genetic Algorithm flow for the proposed inventory management analysis

<table>
<thead>
<tr>
<th>Products</th>
<th>Factory</th>
<th>center1</th>
<th>center2</th>
<th>Agent1</th>
<th>Agent2</th>
<th>Agent3</th>
<th>Agent4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2000</td>
<td>500</td>
<td>-300</td>
<td>30</td>
<td>120</td>
<td>-130</td>
<td>200</td>
</tr>
<tr>
<td>P3</td>
<td>-4000</td>
<td>200</td>
<td>400</td>
<td>200</td>
<td>-300</td>
<td>-450</td>
<td>-160</td>
</tr>
<tr>
<td>P6</td>
<td>3000</td>
<td>-700</td>
<td>-600</td>
<td>-450</td>
<td>80</td>
<td>230</td>
<td>400</td>
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</table>

Table 5.1: The dataset for the analysis taken from the past periods

The record set having positive values represents excess stock levels of a particular product and the negative values represent shortage level of the particular product. This will be carried out for each product and for every
member of the chain. Then the data set is subjected to Genetic Algorithm and the various steps performed in the genetic algorithm are discussed below.

5.2.1 Generation of Chromosomes

Each individual which is constituted by genes is generated with random values. Here, the chromosome of seven genes with random values occupying each gene is generated along with the product representation. A random individual generated for the genetic operation is shown in Fig: 5.3.

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<thead>
<tr>
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<tbody>
<tr>
<td>P4</td>
<td>6000</td>
<td>-100</td>
<td>-500</td>
<td>700</td>
<td>-300</td>
<td>600</td>
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<td>P1</td>
<td>2000</td>
<td>200</td>
<td>-400</td>
<td>600</td>
<td>-800</td>
<td>-300</td>
</tr>
<tr>
<td>P5</td>
<td>5000</td>
<td>300</td>
<td>-600</td>
<td>400</td>
<td>900</td>
<td>-800</td>
</tr>
</tbody>
</table>

Fig: 5.3 Random individual generated for the genetic operation

Each gene of the chromosome displayed in Fig. 5.3 is the stock level of a specific product with the respective members of the supply chain. As for the three individuals, Individual 1 deals with product 4, Individual 2 deals with product 1 and Individual 3 deals with product 5. So, the first individual P4 represents 6000 excess stocks in factory, shortage of 100 stocks in distribution center 1, shortage of 500 stocks in Distribution center 2 and so on. In such a manner the other individuals represent stock levels corresponding to product P1 and Product P5 at the respective members of the chain.
After the generation of the individuals, the number of occurrences of the individual in the past records is determined. This is performed by the function `count()` and the total number of occurrences of that individual for the particular product is determined. This is equivalent to the number of occurrences of such situation of stock levels for the particular product in all the members throughout the period of ten years under consideration.
5.2.2 Evaluation of Fitness Function

A specific kind of objective function that enumerates the optimality of a solution in a genetic algorithm in order to rank certain chromosome against all the other chromosomes is known as Fitness function. Optimal chromosomes, or at least chromosomes which are near optimal, are permitted to breed and merge their datasets through one of the several techniques available in order to produce a new generation that will be better than the ones considered thus far.

The fitness function is given by:

\[ f(i) = \log\left(1 - \frac{n_{occ}(i)}{n_{tot}}\right), i = 1, 2, 3, \ldots, n, \]

where:

- \( n_{occ}(i) \) = The number of occurrences of the chromosome \( i \) in the record set
- \( n_{tot} \) = The total number of records that have been collected from the past or total number of data present in the record set

\( n \) is the total number of chromosomes for which the fitness function is calculated.

In the fitness function, the ratio \( \frac{n_{occ}(i)}{n_{tot}} \) plays the role of finding the probability of occurrence of a particular chromosome; and \( \log [1 - \frac{n_{occ}(i)}{n_{tot}}] \) will ensure minimum value corresponding to the maximum probability; So, the fitness function is structured to retain the minimum value
corresponding to the various chromosomes being evaluated iteration after iteration and this in turn ensures that the fitness function evolution is towards optimization.[Appendix 10.5]

The fitness function is carried out for each chromosome and the chromosomes are sorted on the basis of the result of the fitness function and ranked. The chromosome generating value as minimum as possible will be selected by the fitness function and will be subjected further to the genetic operations, crossover and mutation.

**Genetic operations:** Once fitness calculation is done, Genetic operations are performed. Selection, Crossover and mutation comprise Genetic operations.

**Selection:** The selection operation is the initial genetic operation which is responsible for the selection of the fittest chromosome for further genetic operations. This is done by offering ranks based on the calculated fitness to each of the prevailing chromosome. On the basis of this ranking, best chromosome with minimum fitness function value is selected for further proceedings.
5.2.3 Crossover

Among the numerous crossover operators in practice, for the complex operation, a two point crossover is adopted. From the matting pool, two chromosomes are subjected for the two point crossover. The crossover operation performed in the analysis is pictured in Fig: 5.4.

![Diagram of Crossover](image)

**Fig: 5.4 Chromosomes are subjected for two point crossover operation**

As soon as the crossover operation is completed, the genes of the two chromosomes present within the two crossover points get interchanged. The genes before the crossover point C1 and the genes beyond the crossover point C2 remain unaltered even after the crossover operation.

5.2.4 Mutation

The crossover operation is succeeded by the final stage of genetic operation known as Mutation. In the mutation, a new chromosome is obtained. This chromosome is totally new from the parent chromosome. The concept
behind this is the child chromosome thus obtained will be fitter than the parent chromosome. The performance of mutation operation is shown in Fig: 5.5.

**Before Mutation:**

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<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>P1</td>
<td>2000</td>
<td>200</td>
<td>-500</td>
<td>700</td>
</tr>
<tr>
<td>Mp1</td>
<td>Mp2</td>
<td>Mp3</td>
<td>Mp4</td>
<td></td>
</tr>
</tbody>
</table>

**After Mutation:**

<p>| | | | | |</p>
<table>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>-500</td>
<td>200</td>
<td>2000</td>
<td>700</td>
</tr>
<tr>
<td>Mp1</td>
<td>Mp2</td>
<td>Mp3</td>
<td>Mp4</td>
<td></td>
</tr>
</tbody>
</table>

**Fig: 5.5 Chromosome subjected for mutation operation**

As in Fig: 5.5, four mutation points Mp1, Mp2, Mp3 and Mp4 are chosen. The mutation is done on the particular gene present at the Mutation points. This pointing of gene is done randomly. Hence, the four mutation points may point to any of the seven genes.

The process explained so far will be repeated along with the new chromosome obtained from the previous process. In other words, at the end of each of the iteration, a best chromosome will be obtained. This will be included
with the newly generated random chromosome for the next iteration. Eventually, an individual which is the optimal one among all the possible individuals is obtained. This best chromosome obtained has the optimal information about the stock levels of a particular product corresponding to each member of the supply chain. From the information it can be concluded that the particular product and its corresponding stock levels play a significant role in the increase of supply chain cost. By controlling the stock level of that particular product at each member of the supply chain, the supply chain cost can be minimized in the upcoming periods.

5.3 EXPERIMENTAL RESULTS

The approach suggested for the optimization of inventory level and thereby efficient supply chain management has been implemented in the platform of MATLAB (MATLAB 7.4). The database consists of the records of stock levels held by each member of the supply chain for every period. For implementation, five different products in circulation with the seven member supply chain network have been considered. The sample database which consists of the past records is shown in Table 5.2.
Table 5.2: A sample data set constituted by the product Identification along with its stock levels in each member of the supply chain

In the database tabulated in Table 5.2, the first field comprises of the product Identification (PI) and the other fields are related with the stock levels that were held by the respective seven members of the supply chain network.
For example, the first attribute and first field of the database is ‘5’ which refers the Product I.D. ‘5’. The corresponding fields of the same attribute denote the stock levels of the product I.D. ‘5’ in the respective members of the supply chain. Similarly, different sets of stock levels are held by the database.

As per the proposed analysis based on GA, a random initial chromosome ‘[4 895 -732 485 -213 -270 314 -850]’ is generated. This will represent the database content as shown in the Table 6.3.

<table>
<thead>
<tr>
<th>PI</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>F7</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>895</td>
<td>-732</td>
<td>485</td>
<td>-213</td>
<td>-270</td>
<td>314</td>
<td>-850</td>
</tr>
</tbody>
</table>

**Table 5.3: Random inventory generated initially for the GA based analysis**

Table 5.3 clearly describes that the first element of the chromosome generated randomly points to the product I.D. of the database. Similarly, all the other elements of the generated chromosome point to the corresponding fields of the database. In this manner two different random chromosomes have been generated and they will be subjected to genetic operations like Fitness evaluation, Selection, Crossover and Mutation.

An iteration involving all these processes was carried out so as to obtain the best chromosome. Here for instance, the iteration value of ‘100’ is chosen
and so hundred numbers of iterative steps will be performed. The best chromosome obtained as a result is ‘[1  697  -906  304  257  849  -444  -845]’ and its database format is depicted in the Table 5.4.

<table>
<thead>
<tr>
<th>P1</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>F7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>697</td>
<td>-906</td>
<td>304</td>
<td>257</td>
<td>849</td>
<td>-444</td>
<td>-845</td>
</tr>
</tbody>
</table>

Table 5.4: The final best chromosome obtained from the analysis

A simulation example explaining the procedure for using the proposed algorithm towards identification of emerging optimal solution is illustrated in Appendix 10.5. As long as minimization of the fitness function is still possible, then the iteration continues till such a time that no improvement in the fitness function value is noticeable. After a certain number of iterations, if the fitness function value is not improving from the previous iterations, then this is an indication that the fitness function value is stabilizing and the algorithm has converged towards optimal solution. This inference is useful for deciding the number of iterations for running the GA simulation as well as this may be used as the stopping criteria for the algorithm. For greater accuracy, the number of iterations should be sufficiently increased and run on the most frequently updated large database of past records.
5.4 DISCUSSION OF RESULTS

The final chromosome obtained from the GA based analysis shown in the Table 5.4 is the inventory level that has the potential to cause maximum increase of supply chain cost. By this we mean that if the emerging excess/shortage of inventory levels based on the past data is not taken care of, then the organization will incur shortage cost/ excess storage cost thus resulting in the maximum increase in the supply chain cost.

It is inferred that controlling this resultant chromosome is sufficient to reduce the loss either due to the holding of excess stocks or due to the shortage of stocks. By focusing on the excess/shortage inventory levels and initiating appropriate steps to eliminate the same at each member of the chain, it is possible to optimize the inventory levels in the upcoming period and thus minimize the supply chain cost. That is, the organization should take necessary steps to decrease the production of product 1 in the factory by 697 units to make up for the predicted excess; increase the inventory level of product 1 by 906 units in distribution centre 1 to make up for the predicted shortage and reduce inventory level of product 1 by 304 in distribution centre 2 to make up for the predicted excess; decrease inventory level of product 1 by 257 units in agent1 to make up for the predicted excess; decrease inventory level of product 1 by 849 units in agent2 to make up for the predicted excess; increase the inventory level of product 1 by 444 units in agent3 to make up for the predicted shortage; increase the inventory level of product 1 by 845 units in agent 4 to make up for the predicted shortage. Thus by following the
predicted stock levels, the excess/shortage inventory levels can be avoided in the upcoming period and thus the increase of supply chain cost can also be avoided. The analysis extracts an inventory level that made a remarkable contribution towards the increase of supply chain cost, and in turn enabled to predict the future optimal inventory levels to be maintained in all the supply chain members with the aid of these levels. Therefore it is possible to minimize the supply chain cost by maintaining the optimal stock levels that was predicted from the inventory analysis, and thus making the inventory management more effective and efficient.