CHAPTER 0

INTRODUCTION

Optimization and decision making problems are traditionally handled by either the deterministic or probabilistic approaches. The former provides an approximate solution, completely ignoring uncertainty, while the latter assumes that any uncertainty can be represented as a probability distribution. Both of these approaches only partially capture reality. Uncertainty (such as stock price, commodity cost, and natural resource availabilities) indeed exists but not in the form of known probability distributions. It is in this context optimization using fuzzy mathematical theories becomes more relevant.

‘Fuzzy Set Theory’ introduced by L. A. Zadeh (1965) provides a basic mathematical frame work for dealing with decision problems involving uncertainty due to vagueness associated with linguistic information. In fuzzy decision making problems the idea of optimizing decision was proposed by Bellmann and Zadeh and this idea was used to problems of mathematical programming by T. H. Tanaka and K. Asai. Zimmermann introduced the fuzzy approach to Multi-objective
Linear Programming (MOLP) problems. C. Carlsson, Robert Fuller, P. Korhonen and many authors studied Fuzzy Linear Programming (FLP) models. A. K. Bit, M. P. Biswal, S. S. Alam and many authors applied the fuzzy programming technique with linear membership function to solve Multi-Objective Fuzzy Transportation Problem (MOFTP). Rakesh Varma, M. P. Biswal and A. Biswas developed an algorithm for finding optimal compromise solution for MOFTP with special type of non-linear membership functions (hyperbolic & exponential). Shu Ping Gao and San Yang Liu presented a two-phase fuzzy algorithm to solve MOFTP with linear & non-linear membership functions respectively. C. Carlsson and P. Korhonen have illustrated through example the usefulness of exponential logistic function in FLP and MOLP. C. Carlsson and Robert Fuller demonstrated that the use of interdependence among objectives of an MOLP in the definition of application functions provides more accurate solutions. In 1979 Zadeh introduced the theory of Approximate Reasoning which provides a framework for reasoning in the face of imprecise and uncertain information. Latter followed by Zimmermann they interpret FLP with fuzzy coefficients and fuzzy inequality relations as Multiple
Fuzzy Reasoning Scheme (FMR). Pandian M. Vasant used a modified $S$-curve membership function to find optimal compromise solution with vagueness and degree of satisfaction ranging from 0 to 1.

After the introduction of the concept of fuzzy set by Lotfi A. Zadeh in 1965, several researchers had extended the notion of fuzzy set. An important extension of fuzzy set theory is the theory of Intuitionistic Fuzzy Sets, IFS for short. IFS theory was proposed by Krassimir T. Atanassov in 1983. He introduced a new component degree of non-membership in the definition of these sets with the requirement that the sum of the membership and non-membership function be less than or equal to one. Since then a great number of theoretical and practical results appeared in the area of IFS theory by many authors like Dogan Coker, P. Burillo, H. Bustince, Plamen P. Angelov etc.

IFS are more useful in representing human way of thinking than ordinary fuzzy sets, mainly due to the presence of the non-membership attribute of every element besides its membership. This approach supports further in modeling uncertainty because the sum of membership and non-membership grades lies between 0 and 1 and does not have
to be 1, which is an implicit necessity in ordinary fuzzy sets. IFS introduces a technique in which two separate opinions, supporting or refuting (more or less opposite to each other) of a certain subject can be represented independently. This is particularly useful when one lacks certainty and exact information about a subject.

The concept of optimization under Intuitionistic Fuzzy environment was introduced by Plamen P. Angelov in 1995. He solved some optimization problems by means of IFSs. But here he could not give a general non-membership function for a maximizing/minimizing set. In fuzzy optimization, generally the classical problems are fuzzified and accordingly each action is approximated. Practically in most of the optimization problems the objective function(s), constraints or the parameters are determined on the basis of certain forecasting related to previous experiences or considering the trends in the market. In such situations method of characterizing each action in the Intuitionistic Fuzzy (IF) environment is found more appropriate.

The main objective of our work is to study the optimization problems in Fuzzy and Intuitionistic Fuzzy environment. In the present study, we propose a general linear non-membership function for an
IF maximizing/minimizing set which is not the compliment of the membership function, and using this, it is proved that the solutions to various optimization problems like Linear Programming Problem(s) (both Single and Multiple Objective), Transportation Problem(s) (Single and Multiple Objective), MOP with interdependence (Linear or non-linear) and FLP problems as Intuitionistic Fuzzy Multiple Reasoning Schemes, are more accurate than the analogues in fuzzy and crisp case. We also studied the Fuzzy Transportation Problem(s) considering the parameters in the objective(s) and/or constraints as fuzzy intervals and introduced a new algorithm so solve these problems using $S$-curve membership function.

**Chapter wise summary.**

The thesis is organized in nine chapters.

A general introduction is given in Chapter 0. Chapter 1 consists of the preliminary definitions, terminologies and unknown results required in the subsequent chapters of the thesis.

In Chapter 2, we generalize certain concepts in fuzzy sets to introduce Intuitionistic Fuzzy Maximizing/minimizing set and prove that optimum of the objective function or goal in the IF environment is
the same as the optimum of another function in the restricted do-
main provided that the IF function is convex and its membership and
non-membership functions are continuous. In section 2.2 we define
$\alpha\beta$-cuts, support, convexity etc. of IF sets. Also we introduce the
Intuitionistic Fuzzy Quantities and their properties are studied. A
generalization to the notion of IF functions and IF relation is given
in section 2.3. Here we define IF injective functions, IF continuous
function and introduce the proposed non-membership function to de-
define IF maximizing/minimizing set, which is not the complement of
the membership function. The properties of these functions are also
looked into. We prove the extremum of an IF function can be reduced
to extremum of another function when the function is convex and its
membership and non-membership functions are continuous and use
this idea in the optimization problems in IF environment in the next
chapters.

Chapter 3 deals with the Intuitionistic Fuzzy Optimization (IFO) for
Linear Programming Problems (Both single and multiple objectives).
Here a general linear non-membership function to the objective(s) of
the Linear Programming problem(s) is introduced and prove by an example that this gives a more general method for (IFO) problems with higher degree of satisfaction.

IFO for Transportation problems both single and multiple objectives are discussed in Chapter 4 and formulate a general model for these problems. The superiority of the proposed IFO model over the fuzzy, crisp and the IFO proposed by Angelov is illustrated by an example.

The interdependence is a major issue in any economic theory. In many of the Multiple-Objective Programming (MOP) problems, attainment of one objective helps us to attain better solution to another objective(s). This property of interdependence for Multiple-Objective Programming in the IF environment is studied in Chapter 5. Using the proposed non-membership function we define application functions for the objectives and formulated a corresponding single objective problem for the given MOP and show by a illustrated example the consistency and efficiency of solving it in the IF environment.

In Chapter 6 we interpret the Intuitionistic Fuzzy Linear Programming (IFLP) problem as Intuitionistic Fuzzy Multiple Reasoning
Schemes. It is an extension to the work of C. Calsson and Robert Fuller in the fuzzy environment. We further study this in a general setting considering the various cases that can occur in practical situations and hence this will give a general method for LP problems as Approximate Reasoning Schemes.

In Chapter 7, we consider the optimization of Transportation Problems in the Fuzzy environment. Here we discuss the usefulness of considering the parameters in the objective(s) and/or constrains as fuzzy interval and introduce a new algorithm to solve these problem using the $S$-curve membership function. We prove by illustration that this function is better in the Fuzzy Transportation problems with vagueness and degree of satisfaction ranging from 0 to 1.

A conclusion of the present study and a few open problems for further research comprise the last Chapter.

The author does not claim that the present study is a complete exposition in all respects. Rather, there are various problems connected with the work done here worth investigating, as is often found, any investigation opens up new areas for further exploration.