CHAPTER 6

THE THEORY OF APPROXIMATE REASONING IN
INTUITIONISTIC FUZZY ENVIRONMENT

6.1. Introduction

L. A. Zadeh introduced the theory of Approximate Reasoning in 1979. He represented the propositions as statements assigning fuzzy sets as values of variables. This theory provides a framework for the reasoning in the context of imprecise and uncertain information. Later, C. Carlsson and Robert Fuller followed by Zimmermann interpreted FLP problems with fuzzy coefficients and fuzzy inequality relations as Multiple Fuzzy Reasoning Schemes (MFR), where the antecedents of the scheme correspond to the constraints of FLP problem and the fact is the objective function [RF; Z 93a], [RF; Z 93b], [C; RF 02].

Here we interpret the Intuitionistic Fuzzy Linear Programming (IFLP) problems as Intuitionistic Fuzzy Multiple Reasoning Schemes (IFMR). This is an extension of the work of R. Fuller and C. Carlsson on Fuzzy Reasoning for solving LP problems into IF environment in which, the degree of non-membership or vagueness of objective(s)
and of constraints are considered together with the degrees of membership or satisfaction. The result will clearly show the superiority of IF approach over the solutions of analogous fuzzy and crisp problems.

The sections 6.2 and 6.3 are taken from [C; RF 02].

6.2. Preliminaries

To provide a powerful framework for the reasoning in the context of uncertain information, Zadeh introduced the theory of approximate reasoning in 1979. He represented the propositions as statements assigning fuzzy sets as values to variables.

Suppose we have two interactive variables $x \in X$ and $y \in Y$ and the casual relationship between $x$ and $y$ is completely known, i.e., $y = f(x)$. Then we can make the inference easily as “$y = f(x)$” and “$x = x_1$” $\rightarrow$ “$y = f(x_1)$”.

Suppose we don’t know the complete casual link $f$ between $x$ and $y$, but know only the value of $f(x)$ for some particular values of $x$, i.e.,

$R_1 : \text{if } x = x_1 \text{ then } y = y_1$

$R_2 : \text{if } x = x_2 \text{ then } y = y_2$

$\cdots \cdots \cdots \cdots \cdots$

$R_n : \text{if } x = x_n \text{ then } y = y_n$
and if we are given an \( x' \in X \) and to find a corresponding \( y' \in Y \) under the rule-base \( \mathcal{R} = \{ \mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_n \} \), then we have an interpolation problem.

Let \( x \) and \( y \) be linguistic variables, e.g. “\( x \) is high” and “\( y \) is small”. The basic problem of approximate reasoning is to find the membership function of the consequence \( C \) from the rule-base \( \{ \mathcal{R}_1, \ldots, \mathcal{R}_n \} \) and the fact \( A \)

\[
\mathcal{R}_1 : \text{if } x \text{ is } A_1 \text{ then } y \text{ is } C_1
\]
\[
\mathcal{R}_2 : \text{if } x \text{ is } A_2 \text{ then } y \text{ is } C_2
\]
\[
\mathcal{R}_n : \text{if } x \text{ is } A_n \text{ then } y \text{ is } C_n
\]
\[
\text{fact : } x \text{ is } A
\]

Consequence \( y \) is \( C \)

**Definition 6.2.1. [C; RF 02]** The crisp inequality relation ‘\( \leq \)’ between two fuzzy quantities \( \tilde{a} \) and \( \tilde{b} \) is defined as \( \tilde{a} \leq \tilde{b} \) if and only if \( \max\{\tilde{a}, \tilde{b}\} = \tilde{b} \), that is \( \mu_{\tilde{a}}(x) \leq \mu_{\tilde{b}}(x) \) where \( \mu_{\tilde{a}} \) and \( \mu_{\tilde{b}} \) are the membership function of \( \tilde{a} \) and \( \tilde{b} \). That is \( \tilde{a} \leq \tilde{b} \) if and only if \( \tilde{a} \subseteq \tilde{b} \).
Definition 6.2.2. [C; RF 02] Let \( \leq \) be a crisp inequality relation in the family of fuzzy sets \( \mathcal{F} \). Then for all pairs \( \tilde{a}, \tilde{b} \in \mathcal{F} \), it induces a crisp binary relation in \( \mathbb{R} \), set of real numbers, defined by

\[
(\tilde{a} \leq \tilde{b})(u, v) = \begin{cases} 
1 & \text{if } u = v \text{ and } \tilde{a} \text{ and } \tilde{b} \text{ are in the relation } \leq \\
0 & \text{otherwise.}
\end{cases}
\]

If the inequality relation \( '\leq' \) is modeled by a fuzzy implication operator then for all pairs \( \tilde{a}, \tilde{b} \in \mathcal{F} \) it induces a fuzzy binary relation in \( \mathbb{R} \) defined by

\[
\mu_{\tilde{a} \leq \tilde{b}}(u, v) = \begin{cases} 
1 & \text{if } \mu_{\tilde{a}}(u) \leq \mu_{\tilde{b}}(v) \\
\mu_{\tilde{b}}(v) & \text{otherwise.}
\end{cases}
\]

Definition 6.2.3. [C; RF 02] Let \( \mathcal{F} \) be the family of Fuzzy sets and \( I \) be an index set. Let \( \tilde{a}_r \in \mathcal{F} \) for \( r \in I \) and \( '\leq' \) be a crisp inequality relation in \( \mathcal{F} \). Then \( \tilde{a} \) is a maximal element of the fuzzy set \( \mathcal{G} = \{ \tilde{a}_r | r \in I \} \) if, \( \tilde{a}_r \leq \tilde{a} \) for all \( r \in I \) and \( \tilde{a} \in \mathcal{G} \). A fuzzy quantity \( \tilde{A} \) is called an upper bound (supremum) of \( \mathcal{G} \) if it is an upper bound and if there exists another upper bound \( \tilde{B} \) such that \( \tilde{B} \leq \tilde{A} \), then \( \tilde{A} \leq \tilde{B} \). If \( \tilde{A} \) is the least upperbound of \( \mathcal{G} \) then \( \tilde{A} = \sup \{ \tilde{a}_r | r \in I \} \).
Definition 6.2.4. [C; RF 02] The compositional rule of inference scheme with several relations called Multiple Fuzzy Reasoning Scheme (MFR) has the general form

Fact : \( X \) has the property \( P \)

Relation 1 : \( X \) and \( Y \) are in relation \( W_1 \)

\[ \cdots \cdots \cdots \]

Relation \( m \) : \( X \) and \( Y \) are in relation \( W_m \)

Consequence : \( Y \) has property \( Q \).

where \( X \) and \( Y \) are linguistic variables taking their values from fuzzy sets in classical sets \( U \) and \( V \) respectively, \( P \) and \( Q \) are unary fuzzy predicates in \( U \) and \( V \). The consequence \( Q \) is determined by

\[ Q = P \circ \bigcap_{i=1}^{m} W_m \]

where

\[ \mu_Q(y) = \sup_{x \in U} \min\{\mu_p(x), \mu_{W_1}(x,y), \ldots, \mu_{W_m}(x,y)\}. \]

In fuzzy logic and approximate reasoning, the most important fuzzy inference rule is the Generalized Modus Ponens.

The classical Modus Ponens inference rule says:
Premise if $p$ then $q$

fact $p$

Consequence $q$

i.e., If $p$ is true and $p \rightarrow q$ is true then $q$ is true.

If we have fuzzy sets, $A \in \mathcal{F}(u)$ and $B \in \mathcal{F}(v)$, a fuzzy implication operator in the premise, and the fact is also a fuzzy set $A' \in \mathcal{F}(u)$, then the consequence $B' \in \mathcal{F}(v)$ can be derived from the premise and the fact using the compositional rule of inference suggested by Zadeh [Za 73].

The Generalised Modulus Ponens inference rule says

Premise if $x$ is $A$ then $y$ is $B$

fact $x$ is $A'$

Consequence $y$ is $B'$

where the consequence $B'$ is determined as a composition of the fact and the fuzzy implication operator.

$$B' = A' \circ (A \rightarrow B)$$

i.e., $$B'(v) = \sup_{u \in U} \min\{A'(u), (A \rightarrow B)(u, v)\}, \quad v \in V$$
In many practical cases we use sup-$T$ composition instead of sup-min composition.

**Definition 6.2.5** (sup-$T$ compositional rule of inference).

<table>
<thead>
<tr>
<th>Premise</th>
<th>if $x$ is $A$ then $y$ is $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fact</td>
<td>$x$ is $A'$</td>
</tr>
<tr>
<td>Consequence</td>
<td>$y$ is $B'$</td>
</tr>
</tbody>
</table>

where the consequence $B'$ is determined as a composition of the fact and fuzzy implication operators $B' = A' \circ (A \rightarrow B)$.

\[
\text{i.e., } B'(v) = \sup_{u \in U} \min \{A'(u), A \rightarrow B(u, v)\}, \ v \in V
\]

\[
= \sup \{T(A'(u), A \rightarrow B(u, v)) | u \in U\}
\]

where $T$ is a t-norm.

### 6.3. Fuzzy Reasoning for FLP [C; RF 02]

The Fuzzy Linear Programming (FLP) problems interpreted in Multiple Fuzzy Reasoning Schemes (MFR) by C. Carlsson and R. Fuller, where the antecedents of the scheme correspond to the constraints and the fact of the scheme is the objective of the FLP problem.
The classical LP problem can be stated as:

\[
\max \langle c, x \rangle,
\]

subject to \( Ax \leq B, \ x \in \mathbb{R}^n \)

Let \( X^* \) be the set of solution and if \( X^* \neq \emptyset \) then let \( v^* = \langle c, x^* \rangle \) denote the optimal solution. The FLP problem corresponding to the above crisp LP is

Maximize \( \text{Goal}(x) := \tilde{c}_1 x_1 + \cdots + \tilde{c}_n x_n \)

subject to \( \text{Constraint}_1(x) := \tilde{a}_{11} x_1 + \cdots + \tilde{a}_{1n} x_n \leq \tilde{b}_1 \)

\[ \vdots \]

\( \text{Constraint}_m(x) := \tilde{a}_{m1} x_1 + \cdots + \tilde{a}_{mn} x_n \leq \tilde{b}_m \)

where \( \tilde{a}_{ij}, \tilde{b}_j \) and \( \tilde{c}_i \) denote the characteristic function corresponding to the crisp coefficients \( a_{ij}, b_j \) and \( c_i \) and the inequality relation is defined by

\[
\tilde{a}_{i1} x_1 + \cdots + \tilde{a}_{in} x_n \leq \tilde{b}_j \iff a_{i1} x_1 + \cdots + a_{in} x_n \leq b_j
\]
i.e. $\langle \tilde{a}_i x_1 + \cdots + \tilde{a}_i x_n \leq \tilde{b}_j \rangle (u,v) = \begin{cases} 1 & \text{if } u = v \text{ and } \langle a_i, x \rangle \leq b_j \\ 0 & \text{otherwise.} \end{cases}$

Then

$$\mu_{\max(X)}(v) = \begin{cases} 1 & \text{if } v = \langle c, x \rangle \text{ and } Ax \leq B \\ 0 & \text{otherwise.} \end{cases}$$

i.e. $\max(X) = \begin{cases} \langle c, x \rangle & \text{if } x \text{ is feasible} \\ 0 & \text{otherwise.} \end{cases}$

Consequently, if $X$ and $X'$ are feasible then

$$\max(X) \leq \max(X') \iff \langle c, x \rangle \leq \langle c, x' \rangle$$

and if $X'$ is feasible, but $X''$ is not feasible then

$$\max(X'') \leq \max(X'), \text{ since } \max(X'') \text{ is empty.}$$

Hence we get $M = v^*$, and $X^*$ satisfies the equality $\max(x^*) = M$ if and only if $v^* = \langle c, x^* \rangle$. This means that the above LP problem and the corresponding FLP have the same solution set, and the optimal
value of the FLP is the characteristic function of the optimal value of the LP problem.

6.4. Intuitionistic Fuzzy Reasoning (IFR)

Here we introduce the IFR schemes and formulate some immediate IF inference rules needed for the proposed solution principle via C. Carlsson and Robert Fuller.

Definition 6.4.1. The crisp inequality relation ‘≤’ between two IF quantities $\tilde{A}$ and $\tilde{B}$ is defined as

$\tilde{A} \leq \tilde{B}$ if and only if $\max\{\tilde{A}, \tilde{B}\} = \tilde{B}$,

$\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \geq \nu_{\tilde{B}}(x)$.

That is $\tilde{A} \leq \tilde{B}$ if and only if $\tilde{A} \subseteq \tilde{B}$.

Definition 6.4.2. Let $\mathcal{F}$ be the family IFS on $\mathbb{R}$, $\leq$ be a crisp inequality order relation on $\mathcal{F}$. Then for all pairs $\tilde{A}, \tilde{B} \in \mathcal{F}$, it induces a crisp binary relation in $\mathbb{R}$, by

$$(\tilde{A} \leq \tilde{B})(u, v) = \begin{cases} 1 & \text{if } u = v \text{ and } \tilde{A} \text{ and } \tilde{B} \text{ are in the relation } \leq \\ 0 & \text{otherwise.} \end{cases}$$
If the inequality relation $\leq$ is modeled by an IF implication operator then for all pairs $\tilde{A}, \tilde{B} \in \mathcal{F}$ it induces an IF binary relation in $\mathbb{R}$ defined by

$$
\mu_{\tilde{A} \leq \tilde{B}}(u, v) = \begin{cases} 
1 & \text{if } \mu_{\tilde{A}}(u) \leq \mu_{\tilde{B}}(v) \\
\mu_{\tilde{B}}(v) & \text{otherwise.}
\end{cases}
$$

and

$$
\nu_{\tilde{A} \leq \tilde{B}}(u, v) = \begin{cases} 
0 & \text{if } \nu_{\tilde{A}}(u) \geq \nu_{\tilde{B}}(v) \\
\nu_{\tilde{B}}(v) & \text{otherwise.}
\end{cases}
$$

**Definition 6.4.3.** Let $\mathcal{F}$ be the family of IFSs and $I$ be an index set. Let $\tilde{A}_\gamma \in \mathcal{F}$ for $\gamma \in I$ and ‘$\leq$’ be a crisp inequality relation in $\mathcal{F}$. We say that $\tilde{A}$ is a maximal element of the IFS $\mathcal{G} = \{\tilde{A}_\gamma | \gamma \in I\}$ if $\tilde{A}_\gamma \leq \tilde{A}$ for all $\gamma \in I$ and $\tilde{A} \in \mathcal{G}$. An IF quantity $\tilde{A}$ is called an upper bound (supremum) of $\mathcal{G}$ if it is an upperbound and if there exists another upper bound $\tilde{B}$ such that $\tilde{B} \leq \tilde{A}$, then $\tilde{A} \leq \tilde{B}$. If $\tilde{A}$ is the least upperbound of $\mathcal{G}$ then $\tilde{A} = \sup\{\tilde{A}_\gamma | \gamma \in I\}$.

We now define the compositional rule of inference scheme with several relations called IFMR schemes as follows:
Let $x$ and $y$ be IF linguistic variables, e.g. “$x$ seems to be high” and “$y$ seems to be small”. Then the basic problem of approximate reasoning is to find the membership and non-membership function of the consequence $C$ from the rule-base $\{R_1, \ldots, R_n\}$ and the fact $A$.

\[ R_1 : \text{if } x \text{ may be } A_1 \text{ then } y \text{ may be } C_1 \]
\[ R_2 : \text{if } x \text{ may be } A_2 \text{ then } y \text{ may be } C_2 \]
\[ \ldots \]
\[ R_n : \text{if } x \text{ may be } A_n \text{ then } y \text{ may be } C_n \]

\[ \text{fact } : x \text{ may be } A \]

\[ \text{Consequence } y \text{ may be } C. \]

If we have IF sets $\tilde{A} \in \mathcal{F}(u)$ and $\tilde{B} \in \mathcal{F}(v)$ where $\mathcal{F}(u)$ and $\mathcal{F}(v)$ are two family of IF sets, an IF implication operator in the premise and the fact $\tilde{A}'$ also belong to $\mathcal{F}(u)$, ($\tilde{A} \neq \tilde{A}'$), then the consequence $\tilde{B} \in \mathcal{F}(v)$ can be derived from the premise and the fact using compositional rule of inference as follows:
Definition 6.4.4.

**Premise:** if $x$ may be $\tilde{A}$ then $y$ may be $\tilde{B}$

**Fact:** $x$ may be $\tilde{A}'$

**Consequence** $y$ may be $\tilde{B}'$.

where $\tilde{B}' = \tilde{A}' \circ (\tilde{A} \rightarrow \tilde{B})$. Also the membership function of $\tilde{B}'$ is defined as

$$
\mu_{\tilde{B}'}(v) = \sup_{u \in U} \min \{ \mu_{\tilde{A}}(u), \mu_{\tilde{A} \rightarrow \tilde{B}}(u, v) \}, \quad v \in V
$$

and the non-membership for $\tilde{B}'$ as

$$
\nu_{\tilde{B}'}(v) = \inf_{u \in U} \max \{ \nu_{\tilde{A}}(u), \nu_{\tilde{A} \rightarrow \tilde{B}}(u, v) \},
$$

where $\mu_{\tilde{A}}$, $\mu_{\tilde{A} \rightarrow \tilde{B}}$ are the membership functions of the fact and premises and $\nu_{\tilde{A}}$ and $\nu_{\tilde{A} \rightarrow \tilde{B}}$ are the respective non-memberships.

Definition 6.4.5. Let $X$ and $Y$ are linguistic variables taking values from IF sets in classical sets $U$ and $V$ respectively, $P$ and $Q$ are unary IF predicates in $U$ and $V$ respectively where $P$ is the fact and $Q$ the consequence. Let $W_i$ are binary IF relations in $U \times V$, $i = 1, 2, \ldots, n$.
The compositional rule of inference scheme with several relations, that is the IMFR scheme has the general form

Fact : $X$ has the property $P$

Relation 1 : $X$ and $Y$ are in relation $W_1$

: 

 Relation $m$ : $X$ and $Y$ are in relation $W_m$

Consequence : $Y$ has property $Q$.

where the consequence $Q$ is determined by $Q = P \circ \bigcap_{i=1}^{m} W_i$ where

$$
\mu_Q(y) = \sup_{x \leq y} \min \{ \mu_P(x), \mu_{W_1}(x, y), \ldots, \mu_{W_m}(x, y) \}
$$

and

$$
\nu_Q(y) = \inf_{x \leq y} \max \{ \nu_P(x), \nu_{W_1}(x, y), \ldots, \nu_{W_m}(x, y) \}.
$$

6.5. IFMR Scheme for Solving IFLP Problems

We consider the LP problem in which objective and constraints are IF of the form

Maximize $\tilde{C}X$  

subject to $\tilde{A}X \preceq \tilde{B}, \quad X \geq 0.$

(30)
Here $\tilde{A}, \tilde{B}, \tilde{C}$ are IF quantities and the inequality relation $\preceq$ is given by a certain IF relation. Also the objective function is to be maximized in the sense of a given crisp inequality relation $\leq$ between IF quantities where $\text{MAX}(X) = \text{Goal}(X) \circ \bigcap_{i=1}^{m} \text{constraints}_i(x)$ whose membership function is

$$
\mu_{\text{MAX}(x)}(v) = \sup_u \min \left\{ \mu_{\text{Goal}(x)}(u), \mu_{\text{cons}_1(x)}(u, v), \ldots, \mu_{\text{cons}_m(x)}(u, v) \right\}
$$

and non-membership is

$$
\nu_{\text{MAX}(x)}(v) = \inf_u \max \left\{ \nu_{\text{Goal}(x)}(u), \nu_{\text{cons}_1(x)}(u, v), \ldots, \nu_{\text{cons}_m(x)}(u, v) \right\}
$$

where $\mu_{\text{MAX}(x)}(u) + \nu_{\text{MAX}(x)}(u) \leq 1$ and

$\mu_{\text{cons}_i(x)}(u, v) + \nu_{\text{cons}_i(x)}(u, v) \leq 1$ for $i = 1, \ldots, m$. 

Then the optimal value of the objective function of the IFLP problem (30) is $M = \max \{ \text{MAX}(X) | X \in \mathbb{R}^n \}$ where $\max$ is defined in
the sense of the inequality relation \( \leq \) in Definition 6.4.1. Then, a solution \( X^* \in \mathbb{R}^n \) to the problem (30) is obtained by solving the equation \( \text{MAX}(X) = M \). The set of solutions of this problem is non-empty if and only if the set of maximizing elements of \( \{\text{MAX}(X) | X \in \mathbb{R}^n\} \) is non-empty.

**Remark 6.5.1.** To determine the maximum of the set 
\[ \{\text{MAX}(X) : X \in \mathbb{R}^n\} \] we can convert it into a crisp LP problem.

Consequently, the IFLP problem (30) can be stated as to find an \( X^* \in \mathbb{R}^n \) such that

\[
\tilde{C}X \leq \tilde{C}X^* \\
\tilde{A}X \preceq \tilde{B}, \quad X \geq 0. \tag{32}
\]

Here we consider the IFLP problem as IFMR schemes, where the antecedents of the scheme correspond to the constraints of the IFLP and the fact as the objective function. Then the solution process consists of two steps:

(i) For every decision variables \( X \in \mathbb{R}^n \), we compute the maximizing IF set, \( \text{MAX}(X) \) where the degree of membership and non-membership of \( \text{MAX}(X) \) is as given in (31).
(ii) An optimal solution to the IFLP problem is any point which will be a maximal element of the set \( \{ \text{MAX}(X) : X \in \mathbb{R}^n \} \) in the sense of the given inequality relation.

The IFMR scheme for the IFLP problem takes the form

\[
\begin{align*}
\text{Fact} & : \quad \text{Goal } (X := \tilde{C}X) \\
\text{Antecedents } 1 & : \quad \text{constraint}_1(X) := a_{11}x_1 + \ldots + a_{1n}x_n \leq b_1 \\
& \vdots \\
\text{Antecedents } m & : \quad \text{constraint}_m(X) := a_{m1}x_1 + \ldots + a_{mn}x_n \leq b_m \\
\text{Consequence} & : \quad \text{MAX}(X).
\end{align*}
\]

Hence the problem transforms to the crisp LP as

\[
\begin{align*}
\text{Maximize} & \quad \lambda - \mu \\
\text{subject to} & \quad \lambda \leq \mu(\text{MAX}(X)) \\
& \quad \mu \geq \nu(\text{MAX}(X)) \\
& \quad \lambda + \mu \leq 1, \quad \lambda \geq \mu; \quad \lambda, X \geq 0.
\end{align*}
\]

**Remark 6.5.2.** In a general IFLP problem (30) the objective(s) and all the constraints are from IF sets and the relation \( \lesssim \) is an IF relation. But in particular problems some of them can be crisp or fuzzy.
The various cases are

(i) Crisp objective and IF constraints.

In this case \( \mu_{\text{Goal}(x)}(u) = 1 \) and \( \nu_{\text{Goal}(x)}(u) = 0 \) and hence the problem can be solved.

(ii) Fuzzy objective and IF constraints.

There is no need to consider the non-membership of the \( \text{Goal}(X) \) or it is the complement of its membership. Hence

\[
\mu_{\text{MAX}(X)}(u) = \sup_u \min \{ \mu_{\text{Goal}(X)}(u), \mu_{\text{cons}_i(X)}(u, v) \}
\]

and

\[
\nu_{\text{MAX}(X)}(u) = \inf_u \max \{ \nu_{\text{cons}_i(X)}(u, v) \}; \quad i = 1, \ldots, m.
\]

(iii) IF objective and crisp constraints.

Here \( \mu_{\text{cons}_i(X)}(u, v) = 1 \) and \( \nu_{\text{cons}_i(X)}(u, v) = 0 \) for \( i = 1, \ldots, m \).

Hence as in the general case we can convert the given IFLP into the corresponding crisp LP (Remark 6.5.1) and optimum can be solved.

Remark 6.5.3. Similar cases arise when some or all constraints are fuzzy or crisp or Intuitionistic Fuzzy.
6.6. IFMR Scheme for MOIFLP Problems

The Multiple Objective Linear Programming Problem whose all objectives and constraints are Intuitionistic Fuzzy has the form,

\[
\text{Maximize } \{\hat{C}_1 X, \ldots, \hat{C}_K X\} \\
\text{subject to } \hat{A} X \preceq \hat{B}, \quad X \geq 0.
\]  

(34)

As in the case of the IFLP problem Multiple IF Reasoning Scheme for the Intuitionistic Fuzzy Multiple Objective Linear Programming problem, for every \( X \in \mathbb{R}^n \) and objectives as the following form

Antecedents 1 : \( \text{constraint}_1(X) := \hat{a}_{11}x_1 + \ldots + \hat{a}_{1n}x_n \preceq \hat{b}_1 \)

\vdots

Antecedents \( m \) : \( \text{constraint}_m(X) := \hat{a}_{m1}x_1 + \ldots + \hat{a}_{mn}x_n \preceq \hat{b}_m \)

Fact : \( \text{Goal}_S(X) := \hat{C}_{S1}x_1 + \ldots \hat{C}_{Sn}x_n \)

Consequence : \( \mathcal{E}_S(X) \).

where \( S = \{1, 2, \ldots, K\} \). Here the \( \mathcal{E}_S(X) \) is the effective attainment of the \( S^{th} \) objective function.

That is

\[
\mathcal{E}_S(X) = \text{Goal}_S(X) \circ \bigcap_{i=1}^{m} \text{constraint}_i(X)
\]
where the degree of acceptance

$$\mu \varepsilon_S(X)(u) = \sup_u \min \{ \mu_{\text{Goal}_S(X)}(u), \mu_{\text{cons}_S(X)}(u, v) \}$$

and the degree of rejection

$$\nu \varepsilon_S(X)(u) = \inf_u \max \{ \nu_{\text{Goal}_S(X)}(u), \nu_{\text{cons}_S(X)}(u, v) \}$$

for all $S = 1, \ldots K$ and $i = 1, \ldots m$.

Then the problem (34) turns to be the IF decision problem

$$\max \left\{ \varepsilon_S(X) : S = 1, \ldots K \right\}$$

which in turn transforms to the crisp single objective linear programming problem

maximize $\lambda - \mu$

subject to $\lambda \leq \mu \varepsilon_S(X)(u); S = 1, \ldots K$

$$\mu \geq \nu \varepsilon_S(X)(u)$$

$$\lambda + \mu \leq 1, \quad \lambda \geq \mu, \quad \lambda, X \geq 0.$$
6.7. Conclusion

Linear Programming problems in the IF environment are considered as Approximate Reasoning Scheme and formulated a model for Intuitionistic Fuzzy Linear Programming Problems as Intuitionistic Fuzzy Multiple Reasoning Schemes. This will give a general method for solving LP problems as Approximate Reasoning Schemes.