3.1. Introduction

In optimization problems pertaining to real-life situations, there may be unidentified factors affecting the values of the objective function and the constraints, which cannot be modelled by the classical mathematical tools. However, when formulating the Linear Programming problem which closely describes and represents the real decision situation, various factors of the real system should be reflected in the description of the objective function and constraints. Naturally this objective function and constraints involve many parameters whose possible values may be assigned by experts. In the conventional approach, such parameters are assigned some values in an experimental and/or subjective manner. And from this information and from our knowledge about the problem we may be able to formulate the impacts of such unknown factors.

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Practically, in most of the optimization problems the objective functions, constraints or parameters are determined on the basis of certain forecasting. For instance, the demand of a market is determined on the basis of sales forecasting, considering the experiences in the market during the previous years or by the current trends in the market. Solving such problems, IFO techniques seem to be more appropriate than fuzzy or crisp one. We illustrate that such optimization under the IF environment is more effective than crisp or fuzzy case with the help of an example.

We introduce a general linear non-membership function to solve LP and Multi Objective LP problems in IF environment and prove by an example that this gives a more general method for IFO problems with higher degree of satisfaction.

3.2. Formulation of IFLP Problem

The most general type of IFLP problem is as follows:

Maximize $\tilde{Z} = \tilde{C}X$

subject to $\tilde{A}X \preceq \tilde{B}, \quad X \geq 0. \quad (16)$
In general for solving IFLP problems they are first converted into equivalent crisp linear problems, which are then solved by standard methods. Here we calculate the lower and upper bounds of the optimal values \( \inf(Z) \) and \( \sup(Z) \) respectively. The lower bound \( \inf(Z) \) or \( Z_l \) is obtained by solving the standard LP problem:

\[
\text{Maximize } \quad Z = CX
\]

subject to \( AX \leq B, \quad X \geq 0. \)

Similarly, the upper bound \( \sup(Z) \) or \( Z_u \) is obtained by replacing \( B \) values by corresponding \( b_i + p_i - q_i \), \( i = 1, \ldots, m \), where \( p_i \) and \( q_i \) are the violations in the constraints respectively. Next we determine the IF set of optimal values.

The membership and non-membership function respectively of the objective is defined as

\[
\mu(Z) = \begin{cases} 
0 & \text{if } Z \leq \inf(Z) \\
\frac{Z - \inf(Z)}{\sup(Z) - \inf(Z)} & \text{if } \inf(Z) < Z < \sup(Z) \\
1 & \text{if } Z \geq \sup(Z) 
\end{cases}
\]
\[ \nu(Z) = \begin{cases} 
1 & \text{if } Z \leq \inf(Z) \\
\frac{\sup(Z) - (Z + l)}{\sup(Z) - (\inf(Z) + l)} & \text{if } \inf(Z) < Z \leq \sup(Z) \\
0 & \text{if } Z > \sup(Z) 
\end{cases} \]

where 0 < l < \sup(Z) - \inf(Z) so that \inf(Z) + l is less than or equal to the next minimum value of \( f(x) \).

Now the above problem (16) is transformed via Plamen Angelov to the crisp optimization: To maximize the degree of acceptance of IF objective and constraints and to minimize the degree of rejection of IF objective and constraints. That is

Maximize \( \lambda - \mu \)

subject to

\[ \lambda \leq \frac{Z - \inf(Z)}{\sup(Z) - \inf(Z)} \]
\[ \mu \geq \frac{\sup(Z) - (Z + l)}{\sup(Z) - (\inf(Z) + l)} \]

(17)

\[ \lambda p - \mu q + AX \leq B + p - q, \]

\[ \lambda \geq \mu, \lambda + \mu \leq 1, \lambda, X \geq 0 \]
$p$ and $q$ are the violation in the constraints. Here we introduce a general linear non-membership function which is not exactly the complement of the membership function of the IF objective function.

**Remark 3.2.1** In some cases all or some of the constraints may be fuzzy or crisp. Then the corresponding $q$ or $p$ & $q$ can be chosen as zeros respectively.

### 3.3. Formulation of IFMOLP Problem

A typical Multiple-Objective Linear Programming problem in classical theory is

\[
\text{Maximize } \{f_1(x), \ldots, f_k(x)\} \\
\text{subject to } AX \leq B, \quad X \geq 0
\]

where $A$, $B$ and $X$ are defined as in Chapter 1, section 1.4 and $f_i$ is the $i^{th}$ objective function. $i = 1, 2, \ldots, k$.

We now formulate the IF version of this problem by modifying the degree of acceptance and rejection of objectives/constraints as:
Maximize \{ \tilde{f}_1(x), \ldots, \tilde{f}_k(x) \}

subject to \quad \tilde{A}X \preceq \tilde{B}, \quad X \geq 0 \quad (18)

where the degree of acceptance of the objectives are defined by

\[
\mu_{\tilde{f}_i}(x) = \begin{cases} 
0 & \text{if } f_i(x) \leq \inf(f_i) \\
\frac{f_i(x) - \inf(f_i)}{\sup(f_i) - \inf(f_i)} & \text{if } \inf(f_i) < f_i(x) < \sup(f_i) \\
1 & \text{if } f_i(x) \geq \sup(f_i) 
\end{cases}
\]

and the degree of rejection by

\[
\nu_{\tilde{f}_i}(x) = \begin{cases} 
1 & \text{if } f_i(x) \leq \inf(f_i) \\
\frac{\sup(f_i) - (f_i(x) + l_i)}{\sup(f_i) - (\inf(f_i) + l_i)} & \text{if } \inf(f_i) < f_i(x) \leq \sup(f_i) \\
0 & \text{if } \sup(f_i) < f_i(x) 
\end{cases}
\]

where \( i = 1, 2, \ldots, k \) and \( 0 < l < \sup(f_i) - \inf(f_i) \). So that \( \inf(f_i) + l_i \) is less than or equal to the next minimum value of \( f_i(x) \) for each \( i \).

Now the above problem (18) transforms to the following crisp single objectives optimization problem
Maximize $\lambda - \mu$

subject to

\[
\lambda \leq \frac{f_i(x) - \inf(f_i)}{\sup(f_i) - \inf(f_i)}
\]

\[
\mu \geq \frac{\sup(f_i) - (f_i(x) + l_i)}{\sup(f_i) - (\inf(f_i) + l_i)}, \quad i = 1, 2, \ldots k
\]  \(19\)

\[
\lambda p - \mu q + AX \leq B + p - q
\]

\[
\lambda + \mu \leq 1, \quad \lambda \geq \mu,
\]

\[
\lambda, X \geq 0.
\]

**Remark 3.3.1.** Given a decision situation characterised by IF sets $A$ and $\mu_{\tilde{G}_i}, \mu_{\tilde{C}_j} (i \in \mathbb{N}_n, j \in \mathbb{N}_m)$ the membership functions of goals and constraints respectively and $\nu_{\tilde{G}_i}, \nu_{\tilde{C}_j}$ the respective non-membership functions then the IF decision $\tilde{D}$ is conceived as an IF set on $A$ that simultaneously satisfies the given goals and constraints with membership

\[
\mu_{\tilde{D}}(x) = \min \left\{ \inf_{i \in \mathbb{N}_n} \mu_{\tilde{G}_i}(x), \inf_{j \in \mathbb{N}_m} \mu_{\tilde{C}_j}(x) \right\}
\]

and membership

\[
\nu_{\tilde{D}}(x) = \max \left\{ \sup_{i \in \mathbb{N}_n} \nu_{\tilde{G}_i}(x), \sup_{j \in \mathbb{N}_m} \nu_{\tilde{C}_j}(x) \right\}
\]

for all $x \in A$.  

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3.4. Illustration

We illustrate the proposed IF optimization by a simple numerical example and see that it gives a general method for Intuitionistic Fuzzy Optimization with higher degree of satisfaction.

Consider the following IF optimization problem:

**Example 3.4.1.** A company makes two products, $P_1$ and $P_2$. Product $P_1$ has a Rs. 0.40 per unit profit and product $P_2$ has Rs. 0.30 per unit profit. Each unit of product $P_1$ requires twice as many labour hours as each product $P_2$. It seems by previous experience that the total available labour hours may at least 500 hrs. per day, and may possibly be extended to 600 hrs. per day, due to special arrangements for overtime work. Similarly the supply of materials may be at least sufficient for 400 units of both products, but may possibly be extended to 500 units per day according to previous experience. The problem is, to decide how many units of products $P_1$ and $P_2$ should be made per day in order to maximize the total production.
Formulating the above problem mathematically we have the following IFO problem:

Maximize \[ \tilde{Z} \approx 0.4x_1 + 0.3x_2 \]

subject to \[ x_1 + x_2 \preceq B_1 \]
\[ 2x_1 + x_2 \preceq B_2, \ x_1, x_2 \geq 0 \]

Here the total supply of materials \( B_1 \) and the available labour hours \( B_2 \) are given only by forecasting. Hence the membership of \( B_1 \) and \( B_2 \) are defined by,

\[ \mu_{B_1}(x) = \begin{cases} 
0 & \text{when } x \leq 400, \\
\frac{500-x}{100} & \text{when } 400 < x \leq 500, \\
1 & \text{when } 500 < x 
\end{cases} \]

\[ \mu_{B_2}(x) = \begin{cases} 
0 & \text{when } x \leq 500, \\
\frac{600-x}{100} & \text{when } 500 < x \leq 600, \\
1 & \text{when } 600 < x.
\end{cases} \]
And the degrees of non-membership or rejection for $B_1$ and $B_2$ are defined by:

\[
\nu_{B_1}(x) = \begin{cases} 
1 & \text{when } x \leq 400, \\
\frac{x-450}{50} & \text{when } 400 < x \leq 500, \\
0 & \text{when } 500 < x 
\end{cases}
\]

\[
\nu_{B_2}(x) = \begin{cases} 
1 & \text{when } x \leq 500, \\
\frac{x-550}{50} & \text{when } 500 < x \leq 600, \\
0 & \text{when } 600 < x.
\end{cases}
\]

Here $Z_l = 130$ and $Z_u = 160$. Calculating $\mu(Z)$ and $\nu(Z)$ respectively, the IFLP problem is transformed to the equivalent crisp problem as follows:
Maximize \( \lambda - \mu \)

subject to \(30\lambda - 0.4x_1 - 0.3x_2 \leq -130\)

\(27\mu + 0.4x_1 + 0.3x_2 \geq 157\)

\(100\lambda - 50\mu + x_1 + x_2 \leq 550\)

\(100\lambda - 50\mu + 2x_1 + x_2 \leq 650\)

\(\lambda + \mu \geq 1, \quad \lambda \geq \mu, \quad \lambda, x_1, x_2 \geq 0.\)

Solving this by Linear Programming technique (MATLAB) we get the solution as \(\lambda = 0.7826, \mu = 0.1304, x_1 = 100.00, x_2 = 378.2609\) and the optimum value of \(Z = 153.4788\).

The analogous crisp LP problem and Fuzzy Linear Programming problem give the value of \(Z\) as 138 and 145 respectively. Here the degree of satisfaction in the fuzzy case is 0.5 while it is 0.78 in the IF case.

3.5. Conclusion

A new concept to the optimization problem in a IF environment is introduced here. The IF maximizing set introduced in the chapter
yields higher degree of satisfaction than the one obtained from the existing Fuzzy optimization or crisp optimization techniques. However, the optimization model made using the newly proposed IF maximizing set is found to be superior to the existing optimization techniques. This can be considered as an extension of fuzzy optimization and as an application of IF sets.